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A SIMULATION AIDED SOLUTION TO AN MCDM PROBLEM

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ABSTRACT

A forest treatment problem arising in a Northern Arizona region is first formulated as a discrete MCDM problem, in which the payoff values are uncertain. This uncertainty is modeled by randomization considering the uncertain values as random variables with assumed types of distribution depending on the levels of uncertainty. A combination of discrete MCDM methodology and stochastic simulation is used to find the best treatment strategy with respect to criteria including water quantity and quality, wild life, wood production, aesthetics and management costs.

1 INTRODUCTION

Most practical operational problems involve multiple criteria. Therefore, these problems must be effectively formulated as multi-objective optimization problems. Simultaneous optimization of the multiple criteria will then lead to the best decision alternative. Typical examples of such problems are found in industrial planning and control, natural resource management, land use planning, water resources development, and in all other fields of applied sciences.

Multiple Criterion Decision Making (MCDM) is the terminology used in reference to the problems in which two or more non-commensurable and conflicting criteria exist. One such problem is the management of natural resources on watershed lands. In these problems, it is often necessary to consider multiple and simultaneous conflicting objectives and the solution usually requires a proper MCDM approach in order to arrive at the "best" alternative.

The problem of MCDM has been studied extensively in the literature. Pareto optimality, for example, is the basis of the solutions of the MCDM problems. This is an innovative idea first introduced by the Italian economist, Pareto. A significant contribution was later made by Koopmans (1951) using the concept of efficient vectors. He introduced the concept of non-dominated solutions for modern MCDM problems. The introduction of goal programming by Charnes and Cooper (1961) was the most significant work of the 1950s in this area. The 1970s then witnessed the development of a variety of algorithms to solve nonlinear multi-criterion problems. Special algorithms are also developed which have the ability to solve linear as well as nonlinear and discrete as well as continuous problems. A valuable summary of the MCDM methods and game theory specifying their applications in several fields of business, engineering, and natural resources management is presented for example, by Szidarovszky, et al. (1986).

The uncertainty in MCDM problems is usually modeled by randomization or fuzzification. In the case of the first concept the uncertain parameters are considered random variables, and a simulation study is performed. In the case of the second approach the uncertain parameters are modeled as fuzzy numbers, and the resulting fuzzy solution is replaced by a crisp solution by certain defuzzification methodology.

In this paper a simulation model is presented for solving discrete MCDM problems where the objective function values are uncertain. The methodology will be then illustrated by a case study of a specific forestry management problem.

2 DISCRETE MCDM PROBLEMS

Consider a decision problem with N alternatives and M objectives. Let a_{mn} denote the evaluation of alternative n with respect to objective m. The resulting $M \times N$ matrix is called the payoff matrix. Assume that the decision maker (DM) has specified importance weights to all objectives, which will be denoted by $c_1, c_2, \ldots, c_M$. We may assume that all objectives have to be maximized, otherwise the values of the objectives have to multiplied by (-1). The most commonly used methodology is the class of distance based methods.

Let A_m^* and A_{m^*} denote the *m*th components for the ideal and nadir points, respectively, for a given objective *m*. If the DM is unable to provide his or her ideal point and / or nadir selections, they can be chosen as:

$$A_m^* = \max\{a_{mn} : 1 \le n \le N\}$$
$$A_{m^*} = \min\{a_{mn} : 1 \le n \le N\} .$$

Define the scale factor s_m for the objective *m* as:

$$s_m = A_m^* - A_{m^*} .$$

It is noted here that A_m^* and A_{m^*} are the largest and smallest values, respectively, of objective *m* on the set of the given alternatives.

In the present study, we use the family of 1_p metrics, which is one of the most frequently used measures of distance in distance-based methods. This distance, known as Minkowski metric, is described as

$$\rho_p(\underline{a}, \underline{b}) = \left\{ \sum_{i=1}^{I} c_i |a_i - b_i|^p \right\}^{\frac{1}{p}} \quad (p \ge 1),$$

and based on this metric, six methods are most frequently applied. Methods A, C, and E are distance minimizing from the ideal point, and methods B, D, and F maximize the distances form the nadir with $p = 1, 2, \infty$, respectively. Since methods A and B are equivalent to each other, method B will not be applied in this study.

3 SIMULATION METHOD

A distribution is selected for each uncertain value to model the uncertainty of the objective values. Let x_{mn}, X_{mn}, and e_{mn} denote the minimal, maximal, and mean values respectively, presented by the DM for each element of the payoff matrix. Three distribution types are considered in the present study as:

(i) *Mean Value Analysis* with the probability mass function given as

$$f_{mn}(x) = \begin{cases} 1 & \text{if } x = e_{mn} \\ 0 & \text{if } x \ne e_{mn} \end{cases} .$$

It is noted that this distribution type corresponds to the deterministic case in which complete information is available on the a_{mn} values.

(ii) *Triangular Distribution* with the density function given by

$$f_{mn} = \begin{cases} \dfrac{2(x - x_{mn})}{(X_{mn} - x_{mn})(e - x_{mn})} & \text{if } x_{mn} \le x \le e \\ \dfrac{2(X_{mn} - x)}{(X_{mn} - x_{mn})(X_{mn} - e)} & \text{if } e \le x \le X_{mn} \\ 0 & \text{otherwise} \end{cases}$$

where

$$e = 3e_{mn} - x_{mn} - X_{mn};$$

(iii) *Uniform Distribution* whose density function is

$$f_{mn} = \begin{cases} \dfrac{X_{mn} - e_{mn}}{(e_{mn} - x_{mn})(X_{mn} - x_{mn})} & \text{if } x_{mn} \le x \le e_{mn} \\ \dfrac{e_{mn} - x_{mn}}{(X_{mn} - e_{mn})(X_{mn} - x_{mn})} & \text{if } e_{mn} < x \le X_{mn} \\ 0 & \text{otherwise} \end{cases}$$

With the distributions in (i), (ii), and (iii), we may generate random values using a routine method. The A_{mn} value equals e_{mn} in case (i). The distributions in cases (ii) and (iii) are continuous. It is well-known that $V = F_{mn}^{-1}(U)$ follows the given distribution function F_{mn} provided that U is uniformly distributed in $[0,1]$ (see, for example, Rubinstein, 1981). Therefore, the values V for the two cases (ii) and (iii) are calculated, respectively, as

$$V = \begin{cases} x_{mn} + \sqrt{U(X_{mn} - x_{mn})(e_{mn} - x_{mn})} & \text{if } U \le \dfrac{e - x_{mn}}{X_{mn} - x_{mn}} \\ X_{mn} - \sqrt{(1-U)(X_{mn} - x_{mn})(X_{mn} - e_{mn})} & \text{otherwise} \end{cases}$$

and

$$V = \begin{cases} x_{mn} + \dfrac{(e_{mn} - x_{mn})(X_{mn} - x_{mn})}{X_{mn} - e_{mn}} U & \text{if } U \le \dfrac{e - x_{mn}}{X_{mn} - x_{mn}} \\ e_{mn} + \dfrac{(X_{mn} - e_{mn})(X_{mn} - x_{mn})}{e_{mn} - x_{mn}} \left(U - \dfrac{X_{mn} - e_{mn}}{X_{mn} - x_{mn}} \right) & \text{otherwise} \end{cases}$$

as described in Eskandari, et al. (1995).

The overall best alternative for each given distance-based method was selected by the following simulation procedure based on the above observations. Let *S* be the number of simulation runs. Choose *S* to be a large number and generate *S* random payoff matrices with independent elements. Use the selected method to find the best alternative for each payoff matrix and record the number of times each alternative happens to be optimal. We then obtain estimates of the probability that each alternative is the best by dividing these frequencies by *S*. The overall best choice is then the alternative with the highest probability estimate.

4 NUMERICAL STUDY

Four forest treatment strategies are compared in this section. In the first method of treatment, a watershed of 184 ha in size was completely clear cut, removing all merchantable poles and saw timber and felling the remaining noncommercial wood. In order to trap and retain snow, reduce evapotranspiration losses, and increase the surface drainage efficiency of the watershed, all slash and debris were machine windrowed. Once the clearing treatment was completed, the woodland tree species were allowed to sprout and grow.

The second treatment, uniform thinning, was performed be removing 75% of the initial $30\,\mathrm{m}^2/\mathrm{ha}$ of the basal area (a measure of the density of the forest overstory). This procedure left even-aged groups of trees resulting in an average basal area of 7 m^2/ha. All slash was windrowed in this treatment.

The combined strip cutting and thinning treatment started by an irregular strip cut applied to a third watershed of approximately 546 ha in size. Within irregular strips of 20 m in width, all merchantable wood was removed and the remaining nonmerchantable trees were felled. The intervening leave strips which averaged 35 m in width were reduced to 25% of their basal area.

An area of 351 ha watershed land was used as control in order to evaluate the other three treatments. The control was managed with minimal managerial inputs; therefore, the characteristics and resources of the control represent what might be expected to result from a pure custodial management.

There were several decision making groups concerned with the natural resources in this area. Experts from each major group were selected in our study to provide realistic opinions and preferences. These experts include: 1) Water users (downstream concerns), 2) Livestock producers (upstream concerns), 3) Foresters (upstream concerns), 4) Environmentalists (upstream-downstream concerns), and 5) Land use planners (up-stream-downstream concerns). Each of these groups has several criteria, which are presented in Table 1. The table also includes the average, minimal, and maximal values for the first three alternatives. For control no data are given in the table, since all values are equal to zero (all other alternatives are compared to it). The five groups have also assigned importance weights for each criterion. Based on these weights, two combined weight sets have been estimated, where we took into account the importance and power of these groups in public relation.

The numerical results are presented in Table 2. In the first case the distances are tabulated, and in the cases of the triangular and uniform distributions the probability values that each alternative is optimal, are presented. With few exemptions, control is the best choice. The detailed analysis of the results and the other model variants will be presented in a future paper.

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Table 1: Model Data

Alternative Criterion	Data Type	Weights (by five DM groups) 1	2	3	4	5	Combined Weight Case 1	Case 2	Clear Cut	Uniform Thinning 75%	Strip cut and thinning
Water production Stream flow (inch/year)	mean min max	20	10	15	7.5	10	12	12.5	2.95 -18 20.82	2.32 -20.9 23.74	0.92 -1.42 3.25
Sediment yield (ton/year)	mean min max	20	10	10	7.5	7	11	12	-1.65 -13.11 0.06	-0.25 -1.3 0.016	-0.01 -0.06 0.11
Wild life Deer (pellet/group/acre/mo)	mean min max	2	3	2	7	3.4	4	3.5	1.11 -0.87 2.19	2.31 -2.1 5.5	1.5 -0.43 3.21
Elk (pellet/group/acre/mo)	mean min max	2	3	2	7	3.3	4	3.5	-0.15 -3.44 2.42	3.97 -2.25 6.16	0.58 -0.15 2.04
Pygmy nuthatch (pair/100 acres)	mean min max	2	3	2	7	3.3	4	3.5	-13.2 -18.1 -8.3	-9.9 -13.6 -6.2	0.7 -0.16 3.6
Violet-green swallow (pair/100 acres)	mean min max	2	3	2	7	3.3	4	3.5	-6.7 -9.7 -3.7	-5.3 -9.7 -1.7	1.7 -0.5 4.5
Cavity nesters (pair/100 acres)	mean min max	2	3	2	7	3.3	4	3.5	-42.3 -48.6 -31.9	-28.3 -43 -21	-12.9 -28.9 8.1
Aesthetics (rank)	mean min max	10	10	10	30	16.6	18	15	-1.02 -1.02 -1.02	-0.75 -0.75 -0.75	-0.57 -0.57 -0.57
Livestock production Herbage production (pound/acre)	mean min max	7	15	5	2	5.6	6	8	433.1 -82.7 1042.3	416.7 315.7 514.7	94 -32 303
Grass (pound/acre)	mean min max	7	10	5	2	5.5	5	6.5	168.4 -90.3 473.6	76.1 43.1 107.5	96.1 34.3 132.3
Carrying capacity (AUM/acre)	mean min max	6	10	5	1	5.5	5	6	0.08 -0.05 0.22	0.0355 0.0201 0.0511	0.02 -0.03 0.08
Management costs (dollar/acre)	mean min max	5	5	5	5	16.6	6	6.5	-135.91 -135.91 -135.91	-63.189 -63.189 -63.189	-75.79 -75.79 -75.79
Wood production Growth (sell as wood) (board feet/acre)	mean min max	10	10	20	5	8.3	10	9.5	-41.1 -37.7 -47.5	-52.3 -64.9 -38.7	-47.7 -63.9 -15
Growth (total wood) (cubic feet/acre)	mean min max	5	5	15	5	8.3	7	6.5	-33.1 -39.3 -29.9	-26.1 -35.2 -19.1	-19.3 -24.5 -8

Table 2: Comparison of Numerical Results

Type of Uncertainty	Method	Case	Control	Clear Cut	Uniform Thinning 75%	Strip Cut and Thinning
Mean Value Analysis	A	1	42.155	70.42	46.386	42.287
		2	45.806	65.617	44.282	43.099
	C	1	6.200	8.334	5.444	5.114
		2	6.510	8.049	5.249	5.196
	E	1	12.000	18.000	13.235	10.059
		2	12.500	15.000	11.029	8.602
	D	1	7.358	5.350	6.072	6.449
		2	7.125	5.792	6.244	6.388
	F	1	18.000	12.000	9.437	10.933
		2	15.000	12.500	10.181	11.927
Triangular Distribution	Method	Case	Control	Clear Cut	Uniform Thinning 75%	Strip Cut and Thinning
	A	1	0.96693	0.00000	0.00020	0.03287
		2	0.84915	0.00000	0.02159	0.12926
	C	1	0.26961	0.00000	0.06528	0.66511
		2	0.11192	0.00000	0.15429	0.73379
	E	1	0.65814	0.00000	0.00000	0.34186
		2	0.52496	0.00000	0.20927	0.26577
	D	1	0.99919	0.00000	0.00000	0.00081
		2	0.99548	0.00000	0.00000	0.00452
	F	1	1.00000	0.00000	0.00000	0.00000
		2	1.00000	0.00000	0.00000	0.00000
Uniform Distribution	Method	Case	Control	Clear Cut	Uniform Thinning 75%	Strip Cut and Thinning
	A	1	0.98471	0.00000	0.00034	0.01495
		2	0.90582	0.00000	0.02120	0.07298
	C	1	0.50251	0.00000	0.05692	0.44057
		2	0.26848	0.00000	0.15016	0.58136
	E	1	0.72100	0.00000	0.00000	0.27900
		2	0.56925	0.00000	0.21355	0.21720
	D	1	0.99916	0.00000	0.00000	0.00084
		2	0.99520	0.00000	0.00028	0.00452
	F	1	1.00000	0.00000	0.00000	0.00000
		2	1.00000	0.00000	0.00000	0.00000