

## DERIVATION OF THE INITIAL STOCK OF BAGS – A MONTE-CARLO APPROACH

Zubin Dowlaty

United Parcel Service  
55 Glenlakes Parkway NE  
Atlanta, GA 30328

Chong Loo

United Parcel Service  
55 Glenlakes Parkway NE  
Atlanta, GA 30328

### ABSTRACT

The objective of this paper is to calculate the number of bags needed to successfully supply a large package delivery operation. These bags are used to consolidate many small packages into an easier to transport large container -- thus decreasing the handling cost associated with them. Presently, disposable plastic bags are being used. The hypothesis is that permanent bags may make economical sense if the net present value of this modification is positive. The major determinant in the financial success of this proposition is the number of bags needed to feed the operation. Through monte-carlo simulation techniques, we were able to accurately model the inherent dynamic uncertainty in this calculation. Thereby enabling the financial calculation to be performed.

Keywords: Monte-Carlo Simulation, Transportation Systems, Selection Procedures

### 1 INTRODUCTION

Plastic disposable bags are presently used at United Parcel Service (UPS) to consolidate small packages into an easier to transport large container. Plastic bags have a very limited life and contribute to a large disposal expense. A durable bag having more permanent characteristics than the disposable bag may make economical sense. To calculate the net present value of this capital budgeting project the initial outlay must be estimated. The overwhelming determinant of the initial outlay is the amount of these durable bags that are needed to feed the operation. If the operation needs an excessive amount of durable bags, the investment will not be profitable. Given the magnitudes of dollars involved and the importance of the estimated initial

stock of bags on the capital budgeting decision, the calculation of bags needed must approach reality as accurately as possible. The complexity of the UPS delivery system, that is capable of delivering a package to any address in the US, is enormous; building a mathematical model to estimate the stocks and flows of every hub would be an extremely time consuming task. Compounding the problem is the everyday potential of random shocks to the system, weather, breakdowns, etc.. Another scenario would be to examine the UPS system at the Macro level, abstracting away from examining every hub's interrelationship and viewing the system as one large stock. This results in loss of accuracy, however the gain is in simplicity and significantly less time in model construction. The proposed solution is in using a dynamic multiple run monte-carlo simulation of the initial stock of bags viewing the UPS system at the Macro level. This simulation approach has the benefit of having the capability of being an empirically based model, further the inherent random shocks to the system are naturally modeled. The combination of using empirical data in the simulation while controlling for the uncertainty in everyday operations makes this approach extremely appealing. This paper is organized into the following sections: Section 2 describes the variables that influence the flow of bags. Section 3 develops the simulation method. Section 4 discusses the implementation of the simulation. Section 5 concludes the paper with a discussion of the method used and its appeal.

### 2 STOCK-FLOW VARIABLES

To determine the number of bags needed to support the package operation, we must first derive the equations that will enable us to intuitively understand the variables that influence the calculation. This problem can be understood in a stock-flow framework, see figure one.

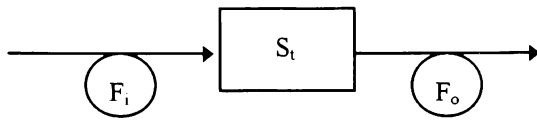


Figure 1: Bag Inflow, Initial Stock of Bags, Bag Outflow. Stock-Flow framework

BagOutflow:  $F_o = \frac{V_{to}}{C}$ ; where  $V_{to}$  = Outflow of Packages,  $C$  = Bag Package Capacity

BagInflow:  $F_i = \frac{V_{ti}}{C}$ ; where  $V_{ti}$  = Inflow of Packages

Let  $Z = T_l + T_r$ ; Lag Time = Transit Time Lag + Restocking Lag

Transit Time lag is defined as the time the bags are in transit. Restocking lag is the time to actually restock the bags for use after arrival at the hub.

Lets assume that the inflow of bags equals the outflow of bags with a lag of  $Z$ . This assumption is valid for the operation as a whole and yields:

$$F_{i(t)} = F_{o(t-z)}$$

Therefore:

$$S_T = S_{T-1} + (F_i)_T - (F_o)_T$$

Substitute for  $F_i$ ,

$$S_T = S_{T-1} + (F_o)_{T-Z} - (F_o)_T$$

Discrete aggregation of stock over time yields:

$$\sum S_T - S_{T-1} = \sum (F_o)_{T-Z} - \sum (F_o)_T$$

Evaluate at  $T = 1$  to  $Z$

$$= \sum_{T=1}^Z (F_o)_{(T-Z)} - \sum_{T=1}^Z (F_o)_T$$

$$= 0 - \sum_{T=1}^Z (F_o)_T \tag{1}$$

Evaluate at  $T = Z + 1$  to Infinity:

$$= \sum_{T=Z+1}^{\infty} (F_o)_{T-Z} - \sum_{T=Z+1}^{\infty} (F_o)_T \tag{2}$$

Result (1) states that the total stock of bags needed on or before time period  $Z$  is equal to the bag outflow summed up over time until time period  $Z$ . In other words the number of bags needed before the inflow of bags is received is simply the sum of outflows.

Result (2) presents three different interpretations given the basic trend of bag outflow:

1. If the slope of the trend of bag outflow over time is zero or bag outflow is constant then result (2) reduces to zero.
2. If the slope of the trend of bag outflow over time is negative then result (2) implies a surplus of bags will accumulate, therefore reducing the number of bags needed from result (1).
3. If the slope of the trend of bag outflow over time is positive then result (2) implies a deficit of bags will accumulate, therefore more bags are needed than is stated in result (1).

### 3 SIMULATION METHOD

United Parcel Service(UPS) volume can be assumed to grow in the future therefore it follows from result (2) that a deficit of bags would occur. More bags are needed than simply the sum of outflows until time  $Z$ . Because the objective is to calculate the total number of bags that will be required over a span of time a projection of future exogenous values are required. This introduces additional uncertainty into the analysis. The identified variables that drive the initial bag calculation and impart uncertainty into the analysis are:

1. Package Volume : Number of Packages
2. Physical Package Size
3. Bag Shrinkage: Damages, loss, etc..
4. Transit Time Lag
5. Restocking Lag

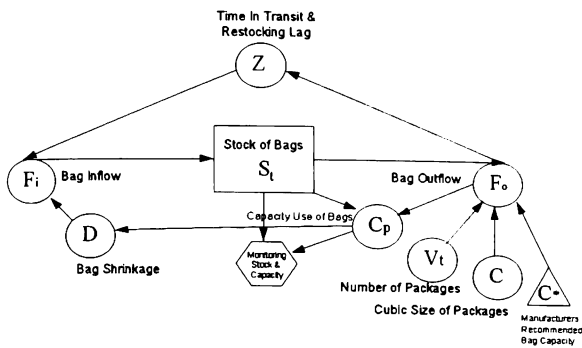


Diagram 1: Simulation Structure

The simulation design chosen is a monte-carlo simulation. Monte-carlo is used because we can place precise probability distributions around the uncertain variables, therefore accounting for all possible scenarios. The structure of the simulation is displayed graphically in Diagram 1.

The variables in Diagram 1 not described in Section 2 are as follows:

$$C_p = \begin{cases} S_t \geq F_o, C_p = 100\% \\ : \text{Capacity Use of Bags} \\ S_t < F_o, C_p = F_o / S_t \end{cases}$$

$$D = \begin{cases} C_p = 100\%, D = \text{Random Variable} \\ : \text{Bag Shrinkage, reduction in bag inflow} \\ C_p > 100\%, D = \text{Random Variable} \\ \text{with an increase in the mean.} \end{cases}$$

#### 4 SIMULATION IMPLEMENTATION

Using the model described in the Simulation Methodology section, the simulation of the initial stock of bags can be performed. The simulation model is designed to analyze only one initial stock at a time, set the initial stock and monitor performance throughout the simulation. The time period of the simulation is 254 working days (approximately 1 year). Each working day is simulated. The initial stock of bags is fixed and the simulation is run for thousands of iterations during that year; the model stops when the standard error of the mean deficient days reaches a preset limit (The software used to build this model is Excel 5.0 with Visual Basic Implementation, Crystal Ball 4.0 which is a monte-carlo simulation package, and Eviews [econometric views] for the forecasting of the exogenous variables).

The exogenous variables have been calibrated for the most empirical accuracy as possible. The following are the exogenous variables and explanation of how the values for the simulation were determined:

1. Package Volume = Projected UPS package volume using econometric models. A Normal Probability Distribution with mean zero and standard deviation of the residuals derived from the econometric model are used and added to the forecast per simulation day.
2. Cube Size of Packages = Projected UPS, system average package cube size derived by econometric models. A Normal Probability Distribution with mean zero and standard deviation of the residuals derived from the econometric model are used and added to the forecast per simulation day.
3. Bag Shrinkage = Manufacturers recommendation of bag failure, with an added Probability distribution accounting for unforeseen damages. This Probability distribution is designed to be dynamic through the simulation – the mean increases when the Capacity Utilization of the bags exceeds 100%.
4. Time in Transit and Restocking Lag = Projected UPS average time in transit derived by econometric models. A Discrete Normal Probability Distribution with mean zero and standard deviation of the residuals derived from the econometric model are used and added to the forecast per simulation day. Estimated mean restocking lag with a Discrete Normal Probability Distribution with standard deviation equal to the standard error of the estimated restocking lag. A seasonality adjustment is integrated into the restocking lag estimate.

The variables throughout the simulation being monitored are the mean number of days the bags were unavailable during the year and the mean number of deficient bags on those days. A *successful* scenario would be when the mean number of days and bags during those deficient days are within acceptable bounds. For example: A simulation of an initial stock of 10,000 bags yielded a mean of 2 days out of 254 where there was not enough bags available, with a mean of 500 bags deficient. The acceptable bound could be set allowing for zero days of deficiency, or X days deficient. This model provides the ability to quantify

the jump from less than 100% success to 100% success, and assess the magnitude of the number of bags needed to realize this increased coverage.

The first step in identifying the minimum number of bags required was to find the minimum value of the initial stock that meets the requirements of acceptability. This was accomplished by running multiple iterations of the monte-carlo model, capturing mean deficient days and deficient bags given the initial stock, then incrementing the initial stock and rerunning the model. The advantage of this is to thoroughly test each initial stock for performance. Given the nature of monte-carlo simulation all realistic scenarios are probed. The start value of the initial stock was derived from result (1). The data is then plotted (see Chart 1, for a typical plot using fictitious data). This plot has the initial stock on the X axis and the number of deficient days on the Y. One can also plot initial stock on the X axis and mean number of bags deficient on the Y axis. The final minimum number of bags would be determined by selecting the minimum initial stock that achieves an acceptable range.

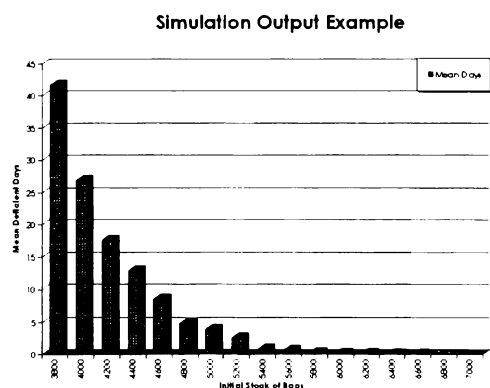


Chart 1: Plot of Mean Deficient Days for multiple initial stocks.

## 5 CONCLUSION

A multi-run monte-carlo model was successfully developed to estimate the initial stock of bags to support an entire package delivery operation. Through simulation a realistic representation of the operation was derived. The model incorporates empirical exogenous variables and adds the uncertainty that accompanies future projections. Using dynamic probability parameters (e.g. in the case of mean bag shrinkage changing when bag capacity exceeds 100%), real life dynamics are modeled. Furthermore, it provides users with modeling flexibility, ranging from

assumptions regarding uncertainty, the time span of the simulation, and quantifying the level of confidence in determining the initial stock of bags.

## ACKNOWLEDGEMENTS

The authors would like to thank United Parcel Service for supporting this project.

## REFERENCES

- Crystal Ball Users Manual*. 1996. Decisioneering Inc.  
 Hilderbrand and Ott. 1987. *Statistical Thinking For Managers*. Duxbery Press.  
 Law, Averell. 1991. *Simulation Modeling and Analysis*. McGraw Hill.  
 Pindyck & Rubinfeld. 1981. *Econometric Models and Economic Forecasts*. McGraw Hill.  
 Richmond, Barry. 1987. *An Academic Users Guide to Stella*. High Performance Systems.  
 Roberts, Nancy. 1994. *Introduction to Computer Simulation*. Productivity Press.

## AUTHOR BIOGRAPHIES

**ZUBIN DOWLATY** is a quantitative methods supervisor at the UPS corporate office. He has a B.S. degree in Finance from the University of Florida and a M.A. in Economics with an emphasis on Econometrics from the University of South Carolina. Research interests are in simulation of dynamic systems, cybernetic principles in simulation, and multivariate statistics. He is a member of the ASA.

**CHONG LOO** is a financial controller at the UPS corporate office. He has a B.S. in Electrical Engineering and a M.S. in Industrial Engineering from Louisiana Tech. His research interests are in cost models and applications of activity based costing in transportation systems. He has previously published papers in the field of Ergonomics. He is a member of I.I.E.