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MULTIVARIATE INPUT MODELING WITH JOHNSON DISTRIBUTIONS

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ABSTRACT

This paper introduces a new method for multivariate simulation input modeling based on the Johnson translation system of probability distributions. This technique matches the first four marginal moments and the correlation structure of a given set of sample data, allowing computationally efficient parameter estimation and random-vector generation. Applications of the technique in ergonomics and production scheduling are discussed. The proposed method is compared to traditional multivariate input-modeling techniques based on the Johnson translation system.

1 INTRODUCTION

In statistical and simulation applications, one is often faced with the task of representing empirical data with a parameterized distribution. Many such distributions exist for modeling univariate data. Few of these distributions are easily extended to model multivariate populations. The most commonly used multivariate distribution, the multivariate normal, has an inflexible shape and is often inappropriate for data modeling.

In this paper we present a more flexible method for multivariate simulation input modeling. The proposed method is based on the Johnson translation system of univariate probability distributions, and it exploits an appropriate affine transformation of independent standardized Johnson variates to generate a multivariate random vector with the desired first four marginal moments and correlation structure. Furthermore, parameter estimation and variate generation are computationally efficient with this procedure.

There are numerous potential applications for this multivariate input-modeling technique. In this paper two applications are discussed in some detail.

Ergonomics Application. We construct a biomechanical model of the forces exerted by the trunk musculature during lifting. Ten trunk muscles are used to describe the trunk's moment generating mechanism. Each lifting task is characterized by the angular kinematics of the torso as well as the total moment generated during the lift. The forces exerted by the ten-muscle system can be modeled as a ten-dimensional random vector. The muscular force is estimated through the use of electromyography (EMG) and is expressed as a percentage of the muscle's capacity. The degree of muscle coactivation (correlation) can be high, so it is inappropriate to use independent random variables to model the joint behavior of these EMG measurements. Additionally, the shape of the marginal distributions can be significantly skewed so it is inappropriate to use a normal multivariate distribution.

Production-Scheduling Application. A simulation model is used to support real-time scheduling of a repair shop. Each job entering the shop for repair must pass through the same series of operations. The operation times are stochastic and nonnormal. In addition, the operation times corresponding to a particular job are stochastically interdependent. Once again, independent univariate and normal multivariate distributions are inappropriate for modeling such a situation. Moreover, component variates are realized sequentially in this system. As a result, it is desired to determine the distribution of remaining operation times conditioned on the completed repair times.

This paper is organized as follows. Section 2 gives a concise introduction to the Johnson translation system of univariate distributions. Section 3 contains a brief survey of multivariate distributions including a previously developed multivariate Johnson distribution. Section 4 outlines the proposed multivariate model, discusses its capabilities and limitations, and gives a method for fitting multivariate distributions to sample data. Section 5 discusses the ergonomics application of the proposed input-modeling procedure. Finally, conclusions and recommendations for future work are given in Section 6.

2 JOHNSON TRANSLATION SYSTEM OF UNIVARIATE DISTRIBUTIONS

Starting from a continuous random variable X whose distribution is unknown and is to be approximated and subsequently sampled, Johnson (1949a) proposed a set of four normalizing translations. These translations have the general form

$$Z = \gamma + \delta \cdot g\left(\frac{X-\xi}{\lambda}\right), \tag{1}$$

where Z is a standard normal random variate, γ and δ are shape parameters, λ is a scale parameter, ξ is a location parameter, and $g(\cdot)$ is a function whose form defines the four distribution families in the Johnson translation system,

$$g(y) = \begin{cases} \ln(y), & \text{for } S_L \text{ (lognormal) family,} \\ \ln\left[y+\sqrt{y^2+1}\right], & \text{for } S_U \text{ (unbounded) family,} \\ \ln[y/(1-y)], & \text{for } S_B \text{ (bounded) family,} \\ y, & \text{for } S_N \text{ normal family.} \end{cases} \tag{2}$$

The translation (1) should approximately transform the continuous random variate X into a standard normal variate. The process of fitting a Johnson distribution to sample data involves first selecting a fitting method and the desired translation function $g(\cdot)$ and then obtaining estimates of the four parameters γ, δ, λ, and ξ. The fitting method utilized in this paper is moment matching. The Johnson translation system of distributions has the flexibility to match any feasible set of sample values for the mean, variance, skewness, and kurtosis. Additionally, the skewness and kurtosis uniquely identify the appropriate translation function $g(\cdot)$. As a result, fitting a data set using moment matching is reduced to the problem of finding the values of γ, δ, λ, and ξ which approximately transform X into a standardized normal variate. Although there are no closed-form expressions for the parameter estimates based on the method of moments, these parameter estimates can be accurately approximated using an iterative procedure of Hill, Hill, and Holder (1976). Moreover, other methods may used to fit each marginal distribution—for example, any of the estimation procedures implemented in the FITTR1 software package (Swain, Venkatraman, and Wilson 1988).

After the data set has been fitted with a Johnson distribution, variate generation is straightforward. First, a standardized normal variate Z should be generated. The corresponding realization of the Johnson variate X is found by applying to Z the inverse translation

$$X = \xi + \lambda \cdot g^{-1}\left(\frac{Z-\gamma}{\delta}\right),$$

where

$$g^{-1}(z) = \begin{cases} e^z, & \text{for } S_L \text{ (lognormal) family,} \\ \left(e^z - e^{-z}\right)/2, & \text{for } S_U \text{ (unbounded) family,} \\ 1/\left(1+e^{-z}\right), & \text{for } S_B \text{ (bounded) family,} \\ z, & \text{for } S_N \text{ (normal) family.} \end{cases} \tag{3}$$

If normal variates are generated by an approximation to the method of inversion, then the generated Johnson variates can enhance the effectiveness of correlation-induction techniques such as common random numbers or antithetic variates; and this can be reflected in improved efficiency of simulation-based performance measures for the target system.

3 MULTIVARIATE DISTRIBUTIONS

3.1 Overview

Many univariate distributions have been generalized to form bivariate distributions. These include numerous discrete (binomial, hypergeometric, Poisson) and continuous (uniform, normal, exponential, beta, and gamma) distributions (Mardia 1970). However, very few of these can be practically extended to higher dimensions. The multivariate normal distribution (Johnson and Kotz 1972) is the most easily manipulated and most frequently used multivariate distribution. However, the component normal distributions have a fixed shape and are often inappropriate for data fitting. In addition to the inflexibility of the multivariate normal, Johnson (1987) cites the following limitations of existing multivariate distributions:

- Some distributions (e.g., the Bessel function distributions) present significant computational problems.
- The support of some distributions (e.g., the beta-Stacy) is too limited to be of interest.
- Some multivariate distributions (e.g., Morgenstern's distribution) are able to represent only weak correlation structures.
- Computational methods for distribution fitting and variate generation have not been developed for some multivariate distributions.

Although the bivariate Bézier distribution family (Wagner and Wilson 1995) seems to have the potential for accurately representing many commonly occurring forms of bivariate dependence, the extension of this family to three or more dimensions appears to be cumbersome and computationally prohibitive. Other approaches to multivariate input modeling can be based on TES (Transform-Expand-Sample) processes (Jagerman and Melamed 1992a, 1992b; Melamed, Hill, and Goldsman 1992) and ARTA (AutoRegressive To Anything) processes (Cario and Nelson 1996). Both methodologies enable the user to specify the autocorrelation function out to an arbitrary lag for a univariate stochastic process with a user-specified marginal distribution, but ARTA processes seem to be substantially easier to use. In the following subsections we consider flexible multivariate distributions that are based on the Johnson family of univariate distributions.

3.2 Multivariate Johnson Distributions

Johnson (1949b) proposed a bivariate distribution based on the univariate Johnson distributions. The parameterized model matches the first four moments for each marginal distribution and then attempts to approximate the correlation between component variates. As detailed below, the technique is easily extended to higher dimensions. Consider a continuous multivariate random vector $\mathbf{X}$ with ν components,

$$\mathbf{X} = (X_1, \ldots, X_\nu)^{\mathrm{T}},$$

which is to be modeled with some parameterized distribution. The Johnson multivariate modeling method determines a normalizing translation such that

$$\mathbf{Z} = \boldsymbol{\gamma} + \boldsymbol{\delta}\mathbf{g}\left[\boldsymbol{\lambda}^{-1}(\mathbf{X} - \boldsymbol{\xi})\right] \sim N_\nu(\mathbf{0}_\nu, \boldsymbol{\Sigma}) . \quad (4)$$

This is accomplished as follows:

1. Identify the transformation
$$\mathbf{g}\left[(y_1, \ldots, y_\nu)^{\mathrm{T}}\right] \equiv [g_1(y_1), \ldots, g_\nu(y_\nu)]^{\mathrm{T}}$$
such that the marginal distribution of X_i is approximated by an appropriate univariate Johnson distribution, where $i = 1, \ldots, \nu$ and $g_i(\cdot)$ is one of the translation functions in (2)
2. Estimate the matrices of shape parameters,
$$\boldsymbol{\gamma} \equiv (\gamma_1, \ldots, \gamma_\nu)^{\mathrm{T}}, \quad \boldsymbol{\delta} \equiv \mathrm{diag}(\delta_1, \ldots, \delta_\nu),$$
and the matrices of the respective location and scale parameters,
$$\boldsymbol{\xi} \equiv (\xi_1, \ldots, \xi_\nu)^{\mathrm{T}}, \quad \boldsymbol{\lambda} \equiv \mathrm{diag}(\lambda_1, \ldots, \lambda_\nu),$$
using the method of moments on each marginal distribution separately.
3. Estimate correlation matrix $\boldsymbol{\Sigma}$ by (a) inserting each sample value $\{\mathbf{X}_j : j = 1, \ldots, n\}$ into the estimated normalizing translation (4) to obtain the corresponding sample $\{\mathbf{Z}_j : j = 1, \ldots, n\}$ of estimated standard normal random vectors; and (b) computing the sample correlation matrix of the $\{\mathbf{Z}_j\}$ as the approximate moment-matching estimator of $\boldsymbol{\Sigma}$.

Random vector generation consists of generating $\mathbf{Z}$ from a ν-dimensional multivariate normal distribution $N_\nu(\mathbf{0}_\nu, \boldsymbol{\Sigma})$ and then applying the inverse translation,

$$\mathbf{X} = \boldsymbol{\xi} + \boldsymbol{\lambda}\mathbf{g}^{-1}\left[\boldsymbol{\delta}^{-1}(\mathbf{Z} - \boldsymbol{\gamma})\right],$$

using the previously determined parameter vectors and the vector-valued inverse translation function

$$\mathbf{g}^{-1}\left[(z_1, \ldots, z_\nu)^{\mathrm{T}}\right] \equiv \left[g^{-1}(z_1), \ldots, g^{-1}(z_\nu)\right]^{\mathrm{T}},$$

where $g_i^{-1}(\cdot)$ is defined by (3) for $i = 1, \ldots, \nu$. This method will generate random vectors with exactly the same marginal moments as the original sample data (at least to the limits of machine accuracy); and if each of the empirical marginal distributions of the original sample data is nearly symmetric about its mean, then the intercomponent correlations of the fitted multivariate Johnson distribution will nearly match the sample correlations of the original sample data. However, if some of the empirical marginal distributions of the original sample data (or the corresponding underlying theoretical marginals) possess marked skewness, then the correlation matrix of the fitted multivariate Johnson distribution will not match the sample correlation matrix of the original data set.

These multivariate Johnson distributions have been shown in several applications to achieve an improved fit when compared to previously used multivariate distributions. Schreuder and Hafley (1977) used the bivariate bounded Johnson distribution (S_{BB}) to fit tree height and diameter data. Due to its flexibility, this distribution produces a consistently better fit than the beta, gamma, Weibull, lognormal, and normal distributions. The implied relationships between the component variates can be interpreted based on the results of a well-known height-diameter regression model. Subsequently, Schreuder, Bhattacharyya, and McClure (1982) successfully used a trivariate Johnson Bounded distribution (S_{BBB}) to fit tree diameter, height, and volume data.

4 A NEW MULTIVARIATE EXTENSION OF THE JOHNSON SYSTEM

An improved method to model multivariate distributions is now presented. Suppose the continuous random vector $\mathbf{X}$ has mean $\boldsymbol{\mu}_\mathbf{X}$ and correlation matrix $\mathbf{C}_\mathbf{X}$. Define the lower triangular matrix

$$\boldsymbol{\Theta}_\mathbf{X} \equiv [\theta_{ij}] = \mathbf{C}_\mathbf{X}^{1/2}$$

based on the Cholesky decomposition $\mathbf{C}_\mathbf{X} = \boldsymbol{\Theta}_\mathbf{X}\boldsymbol{\Theta}_\mathbf{X}^\mathrm{T}$ together with the matrix of standard deviations

$$\boldsymbol{\sigma}_\mathbf{X} \equiv \mathrm{diag}\left[\mathrm{Var}^{1/2}(X_1), \ldots, \mathrm{Var}^{1/2}(X_\nu)\right] .$$

If $\mathbf{Y} = (Y_1, \ldots, Y_\nu)^\mathrm{T}$ consists of *independent* standardized Johnson variates so that each component Y_i has mean zero and variance one for $i = 1, \ldots, \nu$, then

$$\mathbf{W} \equiv \boldsymbol{\mu}_\mathbf{X} + \boldsymbol{\sigma}_\mathbf{X}\boldsymbol{\Theta}_\mathbf{X}\mathbf{Y} \tag{5}$$

has the same mean vector and covariance matrix as $\mathbf{X}$. Notice that $\mathbf{W}$ does not have a multivariate Johnson distribution as defined in Subsection 3.2. The parameters $\gamma_i, \delta_i, \lambda_i$, and ξ_i of Y_i are set so that the ith component W_i of the random vector $\mathbf{W}$ has the same skewness and kurtosis as X_i for $i = 1, \ldots, \nu$.

Let $\mathbf{a}_\mathbf{X}$ and $\mathbf{b}_\mathbf{X}$ be $\nu \times 1$ vectors whose ith elements are the skewness and kurtosis of X_i, respectively. Similarly, let $\mathbf{a}_\mathbf{Y}$ and $\mathbf{b}_\mathbf{Y}$ denote the skewness and kurtosis vectors for $\mathbf{Y}$. Finally, define the k-fold Hadamard product of $\boldsymbol{\Theta}_\mathbf{X}$ as

$$\boldsymbol{\Theta}_\mathbf{X}^{(k)} \equiv \left[\theta_{ij}^k\right] \quad \text{for} \quad k = 3, 4 \tag{6}$$

together with the auxiliary vector

$$\boldsymbol{\Psi}_\mathbf{X} \equiv (\psi_1, \ldots, \psi_\nu)^\mathrm{T} , \tag{7}$$

where

$$\psi_i = 6\sum_{j=1}^{\nu} \sum_{\ell=j+1}^{\nu} \theta_{ij}^2\theta_{i\ell}^2 \quad \text{for} \quad i = 1, \ldots, \nu. \tag{8}$$

Now if the random vector $\mathbf{W}$ is generated according to the affine transformation (5), then it is easily shown that the skewness and kurtosis vectors $\mathbf{a}_\mathbf{W}$ and $\mathbf{b}_\mathbf{W}$ describing the components of $\mathbf{W}$ have the following relationship to the skewness and kurtosis vectors $\mathbf{a}_\mathbf{Y}$ and $\mathbf{b}_\mathbf{Y}$ describing the components of $\mathbf{Y}$,

$$\left.\begin{array}{rcl} \mathbf{a}_\mathbf{W} & = & \boldsymbol{\Theta}_\mathbf{X}^{(3)}\mathbf{a}_\mathbf{Y} \\ \mathbf{b}_\mathbf{W} & = & \boldsymbol{\Theta}_\mathbf{X}^{(4)}\mathbf{b}_\mathbf{Y} + \boldsymbol{\Psi}_\mathbf{X} \end{array}\right\}. \tag{9}$$

Thus, the Johnson parameter matrices $\boldsymbol{\gamma}$, $\boldsymbol{\delta}$, $\boldsymbol{\lambda}$, and $\boldsymbol{\xi}$ that determine the distribution of $\mathbf{Y}$ are selected to satisfy the moment-matching conditions

$$\left.\begin{array}{r} \mathbf{a}_\mathbf{Y} = \left[\boldsymbol{\Theta}_\mathbf{X}^{(3)}\right]^{-1} \mathbf{a}_\mathbf{X} \\ \mathbf{b}_\mathbf{Y} = \left[\boldsymbol{\Theta}_\mathbf{X}^{(4)}\right]^{-1} (\mathbf{b}_\mathbf{X} - \boldsymbol{\Psi}_\mathbf{X}) \end{array}\right\}. \tag{10}$$

It follows from (5), (9), and (10) that the transformed random vector $\mathbf{W}$ has the same second-order moment structure as $\mathbf{X}$; moreover the skewness and kurtosis of W_i match the skewness and kurtosis of X_i for $i = 1, \ldots, \nu$.

This method requires relatively little computational effort. Data fitting requires determining $4 \times \nu$ marginal moments and $\nu(\nu-1)/2$ correlation values, finding the fourth power of $\nu(\nu+1)/2$ numbers, inverting two $\nu \times \nu$ matrices, finding ν ψ_i's and fitting ν univariate Johnson distributions. Random vector generation requires generating ν independent Normal variates, applying ν Johnson inverse translations of the form (3), and two $\nu \times \nu$ matrix multiplication operations.

4.1 Limitations of the Procedure

The ability to match the skewness of $\mathbf{W}$ to that of $\mathbf{X}$ depends only on finding a random vector $\mathbf{Y}$ of standardized Johnson variates with the vector $\mathbf{a}_\mathbf{Y}$ of marginal skewness values specified by (10). A standardized univariate Johnson distribution can always be found whose skewness matches any target value. As a result, the proposed multivariate input-modeling procedure always matches the first three marginal moments and correlation structure of the modeled data.

The ability to match the kurtosis of $\mathbf{W}$ to that of $\mathbf{X}$ depends only on finding a vector $\mathbf{Y}$ of independent standardized Johnson variates with the vector $\mathbf{a_Y}$ of marginal skewness values and the vector $\mathbf{b_Y}$ of marginal kurtosis values specified by (10). Now for every choice of marginal distributions for $\mathbf{Y}$, we must have

$$\mathbf{b_Y} \geq \mathbf{a}_\mathbf{Y}^{(2)} + \mathbf{1}_\nu; \tag{11}$$

see, for example, page 216 of Stuart and Ord (1994). Unfortunately, the requirement (11) is not guaranteed to hold for every solution of (10); and in some applications of the proposed method for multivariate simulation input modeling, we have observed that (11) fails to hold.

To handle the (relatively rare) anomalous situations in which (11) fails to hold for the solution to (10), we match each marginal skewness and "adjust" the corresponding marginal kurtosis values to feasible levels that satisfy (11). Therefore, if $\mathbf{a_Y}$ and $\mathbf{b_Y}$ completely consist of feasible skewness/kurtosis pairs, then the transformed multivariate $\mathbf{W}$ will have the same first four marginal moments and correlation structure as the original variate $\mathbf{X}$. If some infeasible pairs exist, then the corresponding kurtosis is increased to a feasible value. The resulting random vector $\mathbf{W}$ will have the first three marginal moments and correlation structure of $\mathbf{X}$ with some marginal kurtosis values slightly above those of $\mathbf{X}$. It is easy to apply the transformation to the corrected kurtosis values to determine the marginal kurtosis of $\mathbf{W}$ and the deviation from the marginal kurtosis of $\mathbf{X}$. Notice that the order in which the multivariate components are determined has some influence on the adjustment required for infeasible kurtosis pairs.

4.2 Conditional Marginal Distributions

Consider the specific case where the multivariate distribution is used to model operation times for successive jobs passing through a repair shop. Each job requires the same operations to be performed. Additionally, the operation times associated with a single job are correlated. Information about the job repair-time distributions is used in a real-time scheduling system. As a result, it is desired to utilize information from partially complete jobs to improve the estimate of remaining work. For example, we seek to determine the conditional probability distribution of the current job's service time for operation k given the service times of completed operations 1 through $k-1$. Now the density function for the ith independent standardized Johnson variate Y_i is

$$f_{Y_i}(y_i) = \frac{\delta_i}{\lambda_i\sqrt{2\pi}} g_i'\left(\frac{y_i-\xi_i}{\lambda_i}\right) \times \exp\left\{-\frac{1}{2}\left[\gamma_i + \delta_i \cdot g_i\left(\frac{y_i-\xi_i}{\lambda_i}\right)\right]^2\right\}, \tag{12}$$

where $g_i(\cdot)$ is the Johnson translation associated with i and $g_i'(\cdot)$ is the corresponding first derivative. Let

$$\mathbf{Y}(k) \equiv (Y_1, \ldots, Y_k)^{\mathrm{T}} \quad \text{for} \quad k = 1, \ldots, \nu,$$

denote the subvector consisting of the first k components of $\mathbf{Y}$. Since the components of $\mathbf{Y}$ are independent, the joint probability density function (p.d.f.) for $\mathbf{Y}(k)$ is

$$f_{\mathbf{Y}(k)}[\mathbf{y}(k)] = \prod_{i=1}^{k} f_{Y_i}(y_i)\,. \tag{13}$$

Using transformation (5) and the change-of-variables formula, we see that $\mathbf{X}(k)$, the subvector of $\mathbf{X}$ consisting of the first k operation times (components) of $\mathbf{X}$, has joint p.d.f.

$$f_{\mathbf{X}(k)}(\mathbf{x}(k)) = \frac{f_{\mathbf{Y}(k)}\Big([\boldsymbol{\sigma}_{\mathbf{X}(k)}\boldsymbol{\Theta}_{\mathbf{X}(k)}]^{-1}[\mathbf{x}(k) - \boldsymbol{\mu}_{\mathbf{X}(k)}]\Big)}{\det[\boldsymbol{\sigma}_{\mathbf{X}(k)}\boldsymbol{\Theta}_{\mathbf{X}(k)}]}, \tag{14}$$

where $\boldsymbol{\mu}_{\mathbf{X}(k)}$ is a $k \times 1$ subvector consisting of the first k components of $\boldsymbol{\mu}_\mathbf{X}$; and $\boldsymbol{\Theta}_{\mathbf{X}(k)}$ and $\boldsymbol{\sigma}_{\mathbf{X}(k)}$ are the $k \times k$ analogues of $\boldsymbol{\Theta}_\mathbf{X}$ and $\boldsymbol{\sigma}_\mathbf{X}$, respectively. Thus, the conditional p.d.f. of the kth operation time X_k given the vector $\mathbf{X}(k-1) = \mathbf{x}(k-1)$ of the preceding $k-1$ operation times may be computed as follows,

$$f_{X_k|\mathbf{X}(k-1)}[x_k \mid \mathbf{x}(k-1)] = \frac{f_{\mathbf{X}(k)}[\mathbf{x}(k-1), x_k]}{\int_{-\infty}^{\infty} f_{\mathbf{X}(k)}[\mathbf{x}(k-1), w_k]\, dw_k},$$

where the joint p.d.f. $f_{\mathbf{X}(k)}(\cdot)$ is evaluated by combining (12), (13), and (14). This expression may be numerically determined and manipulated. Through similar analysis, nearly any measure of the job service time distribution can be accurately estimated using this procedure.

5 ERGONOMICS APPLICATION

Consider now the ergonomic model of trunk muscle coactivation. The muscle exertion of a random subject on a particular type of lifting task is modeled as follows:

- The sample data set of size $n = 230$ yielded the marginal moments shown in Table 1. The corresponding matrix $\mathbf{C_X}$ of sample correlations given in Table 2.

- Extracting the "square root" $\mathbf{\Theta_X}$ of the correlation matrix yields the results shown in Table 3.

- The skewness vector $\mathbf{a_Y}$ and the kurtosis vector $\mathbf{b_Y}$ are computed from (6)–(10) as shown in Table 4.

- Finally, we fit each Y_i with the appropriate Johnson distributions as shown in Table 5.

- This model can now be used to generate random vectors that drive a biomechanical simulation model.

6 CONCLUSIONS AND RECOMMENDATIONS

In this paper we have developed a method for multivariate simulation input modeling based on the Johnson translation system of probability distributions. The method matches the first four moments and correlation structure of a given set of sample data. The distribution-fitting procedure allows computationally efficient parameter estimation and random-vector generation. An application to the field of ergonomics illustrates the procedure.

Future work should include a comparison of the proposed method for multivariate simulation input modeling versus the approaches based on TES and ARTA processes. Moreover, the approach outlined in this paper could be adapted to Bézier distributions; and the results achieved with such an adaptation should be compared to other techniques for fitting multivariate distributions with Bézier marginals.

In the ergonomics application briefly described in this paper, interest ultimately centers on how the ten-dimensional distribution of trunk-muscle coactivation varies over time. In this context the computational complexity of the parameter-estimation and variate-generation procedures appears to become a dominant consideration.

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Table 1: Marginal Moments of Trunk Muscle Coactivation Data

Component	Mean	Standard Deviation	Skewness	Kurtosis
1	0.00645	0.00431	0.50456	2.62549
2	0.00947	0.00713	0.61072	3.04158
3	0.00831	0.00822	0.45553	2.80806
4	0.01141	0.01151	0.67521	3.01325
5	0.02709	0.02096	0.73188	3.03418
6	0.02938	0.01781	0.49942	2.76829
7	0.07834	0.02973	0.47293	2.71599
8	0.09164	0.03254	0.20482	2.33007
9	0.17535	0.05683	0.42247	2.83503
10	0.20070	0.06729	0.54304	3.20566

Table 2: Sample Correlation Matrix $\mathbf{C}_X$ for Trunk Muscle Coactivation Data

1.0000	0.6157	0.3277	0.4067	0.3611	0.5264	0.3811	0.3122	0.2987	0.2310
0.6156	1.0000	0.2612	0.3960	0.2449	0.4421	0.3138	0.2956	0.3179	0.2179
0.3276	0.2612	1.0000	0.5391	0.1671	0.2779	0.2171	0.2914	0.1923	0.1994
0.4067	0.3960	0.5391	1.0000	0.2152	0.3677	0.2965	0.2682	0.2303	0.1459
0.3611	0.2449	0.1671	0.2152	1.0000	0.6195	0.4547	0.4016	0.3293	0.3736
0.5264	0.4421	0.2779	0.3677	0.6195	1.0000	0.4718	0.4971	0.4833	0.4514
0.3811	0.3138	0.2171	0.2965	0.4547	0.4718	1.0000	0.6490	0.7334	0.7031
0.3122	0.2956	0.2914	0.2682	0.4016	0.4971	0.6490	1.0000	0.6928	0.7582
0.2987	0.3179	0.1923	0.2303	0.3293	0.4833	0.7335	0.6928	1.0000	0.7499
0.2310	0.2179	0.1994	0.1459	0.3736	0.4514	0.7031	0.7582	0.7499	1.0000

Table 3: Sample Estimate of $\mathbf{\Theta}_X$ for Trunk Muscle Coactivation Data

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.6156	0.7880	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.3276	0.0754	0.9417	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.4067	0.1848	0.4161	0.7919	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.3611	0.0287	0.0495	0.0535	0.9292	0.0000	0.0000	0.0000	0.0000	0.0000
0.5264	0.1497	0.1000	0.1065	0.4460	0.6929	0.0000	0.0000	0.0000	0.0000
0.3811	0.1005	0.0899	0.1080	0.3271	0.1295	0.8372	0.0000	0.0000	0.0000
0.3122	0.1312	0.1902	0.0477	0.2940	0.2278	0.4406	0.7171	0.0000	0.0000
0.2987	0.1700	0.0867	0.0521	0.2254	0.2681	0.5741	0.2480	0.5980	0.0000
0.2310	0.0960	0.1237	0.0218	0.3039	0.2449	0.5559	0.3637	0.1862	0.5414

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Table 4: Skewness and Kurtosis Vectors $\mathbf{a}_Y$ and $\mathbf{b}_Y$ for Trunk Muscle Coactivation Data

Component	Skewness	Kurtosis
1	0.004244	2.087854
2	1.097259	4.179445
3	-0.287684	1.702515
4	-0.204774	7.474299
5	0.658706	3.054686
6	0.033781	6.183929
7	1.503758	3.289309
8	1.032861	2.103015
9	-0.787371	3.155087
10	-0.499475	3.865285

Table 5: Parameters of Fitted Marginal Johnson Distributions for Trunk Muscle Coactivation Data

Muscle i	Type	γ_i	δ_i	ξ_i	λ_i
1	S_B	0.808	1.139	−2.004	5.692
2	S_B	1.019	0.727	−1.220	4.685
3	S_B	0.966	1.283	−2.155	6.350
4	S_B	0.818	0.340	−0.794	3.290
5	S_B	0.994	0.819	−1.375	4.914
6	S_B	0.633	0.159	−0.707	2.606
7	S_B	0.670	0.654	−1.305	3.992
8	S_B	0.161	0.205	−1.065	2.421
9	S_B	0.677	0.152	−0.678	2.639
10	S_B	1.189	0.161	−0.453	3.568

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