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DYNAMIC SIMULATION FOR TIME SERIES MODELING

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ABSTRACT

There has been considerable interest in investigating time series in the fields of economics, business, and engineering. This paper illustrates that dynamic simulation can be used as an instructional tool to introduce time series concepts. We design two simulation models using SIMULINK to represent dynamic models for autoregressive moving average (ARMA) time series data given by a state-space representation. SIMULINK is a program for simulating dynamic systems. It offers a graphical user interface that allows easy development of block diagrams and hierarchical models. This paper presents an overview of how SIMULINK can be used for time series modeling.

1 INTRODUCTION

Time series modeling is widely used in economics, business, and engineering. The topic is often covered in undergraduate courses in statistics or operations research. Many graduate programs in these areas offer an entire course on time series modeling and time series analysis.

Dynamic simulation is a powerful tool for introducing time series concepts. In this paper we will investigate time series modeling using SIMULINK (Simulink 1994). SIMULINK is a program for simulating dynamic systems. A part of its appeal as a learning tool is that it offers a visual (or graphical) user interface that allows the development of block diagrams and hierarchical models. The result of the simulation model can be viewed in real-time as the model progresses.

We consider practical areas of time series modeling application (Section 2), design a simulation model of the time series process (Section 3) and demonstrate it on two illustrative examples (Section 4). The model can be obtained until December 1997 from http://www.eng.auburn.edu/~nembhard.

2 PRACTICAL PROBLEMS

A *time series* is commonly defined as a sequence of observations taken sequentially in time on a variable of interest (*e.g.*, Box, Jenkins, and Reinsel 1994). The variable is usually discrete, observed at equally spaced intervals, and adjacent observations are typically dependent. *Time series modeling* is concerned with representing the behavior of the process by a mathematical model that can be extended into the future. *Time series analysis* is concerned with techniques for analyzing the variable dependence.

Examples of time series in economics and business may include a daily series of corporate stock prices and a monthly series of the level of inventory in a factory. Examples of time series in engineering include an hourly series of uncontrolled chemical concentration and a minutely series of random disturbances. Box, Jenkins, and Reinsel (1994) identify and illustrate the use of time series models in four important practical problems: forecasting time series; estimation of transfer functions; analysis of effects of unusual intervention events to a system; and discrete control systems.

With this broad applicability, there is considerable interest in time series. Recently, there has been more interest in representing time series in the state space form because it lends itself well to simulation analysis as well as to other modeling involving numerical computation. The state space representation provides a convenient way to represent complex mathematical systems.

In Appendix A, we show how the ARMA time series can be represented in the state-space form as a set of equations that describe a dynamic process. We will use this representation in examples of characteristic time series shown in Section 4 of this paper. Several other characteristic patterns are shown in Montgomery, Johnson, and Gardiner (1988) (Chapter 1) and Box, Jenkins, and Reinsel (1994) (Chapter 4).

3 CONSTRUCTING SIMULINK MODELS

SIMULINK is a simulation program designed to model and analyze dynamic systems and can be executed on the Sun, Macintosh, or PC platform. A typical session starts by invoking SIMULINK and defining a model or loading a previous model. To facilitate model definition, SIMULINK has a graphical user interface containing *block diagram* windows that provide libraries of functional blocks. The parameters of each block are evaluated in MATLAB which is an interactive environment for numeric computation. (SIMULINK and MATLAB are written in C by The MathWorks, Inc.)

The standard block library is shown in Figure 1. The SOURCES, SINKS, and CONNECTIONS libraries allow input and output representation. The DISCRETE, LINEAR, and NONLINEAR libraries allow the representation of the system or process. The EXTRAS library provides software demos and other help.

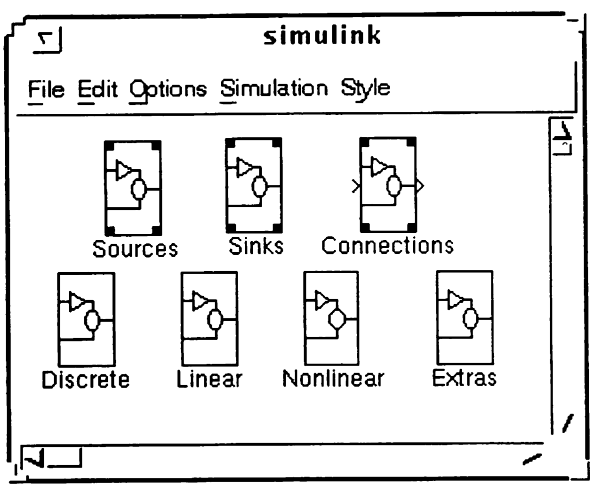


Figure 1: Standard SIMULINK Block Library

Opening any of the libraries shows a subsystem of blocks. For example, the SOURCES library contains the RANDOM NUMBER block (see Figure 2). To define a model, blocks are simply copied from the libraries to draw a block diagram of the system. Such a model is shown for the ARMA Process in Figure 3. Most blocks have an input environment as shown in Figure 4.

Next, parameters of the simulation run may be specified. These parameters include when to start and stop the simulation run, limits on the step size taken during numerical integration, and method of integration (i.e., algorithm) desired.

Once the model has been designed and the parameters specified, ***start*** is chosen from the ***simulation*** pull-down menu to make the model run. The progress of a simulation run can be viewed in real time and the final results made available in the MATLAB workspace. Analysis of the results can be accomplished by selecting built-in options from the SIMULINK menu or by entering commands in the MATLAB command window. In the next section, we construct a SIMULINK simulation model for the ARMA time series.

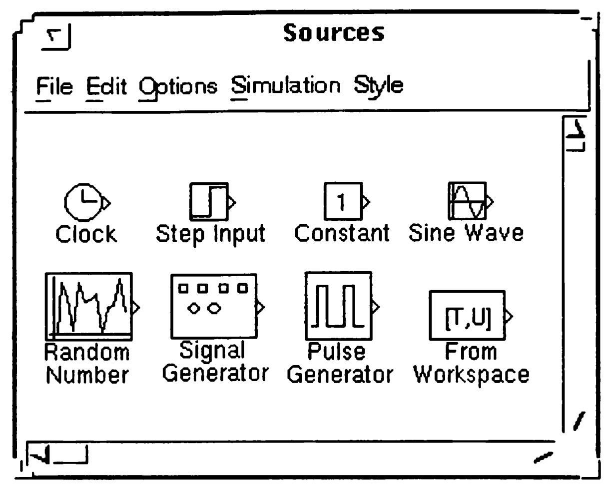


Figure 2: SOURCES Block Library

4 SIMULINK SIMULATION OF THE TIME SERIES

In this section, we present the SIMULINK simulation model that represents the state-space form of the ARMA process. As stated above, Appendix A gives the state-space form of the ARMA time series. Simulation is a powerful tool for solving these equations. Simulation can also approximate solutions to system equations that are intractable by analytic methods.

Figure 3 shows the block diagram of a SIMULINK model that simulates the ARMA process. The STANDARD NORMAL RANDOM NUMBER GENERATOR block provides random shocks from a $N(0,1)$ distribution. At the STANDARD DEVIATION block the output of the random shock is multiplied by the standard deviation. The LINEAR FILTER block provides the discrete state-space form of the linear filter process. The LEVEL block provides the process level (mean). The SUM block combines the linear filter with the process level; the output of the sum block is an ARMA(p,q) process. The ARMA process is plotted during the simulation run via the GRAPH block.

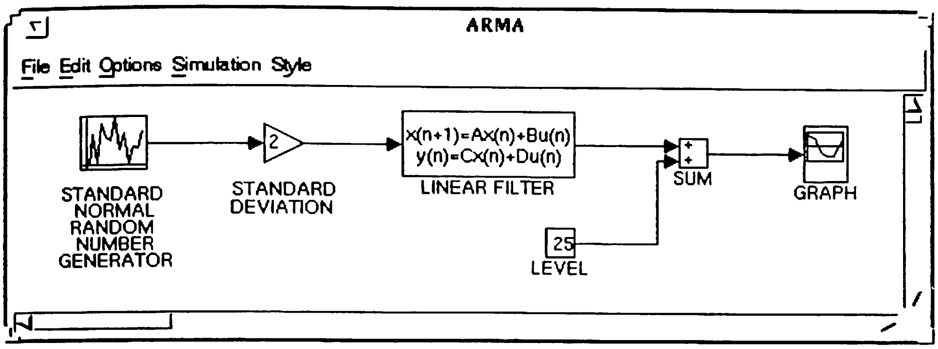


Figure 3: Block Diagram of an ARMA Process Simulation Model

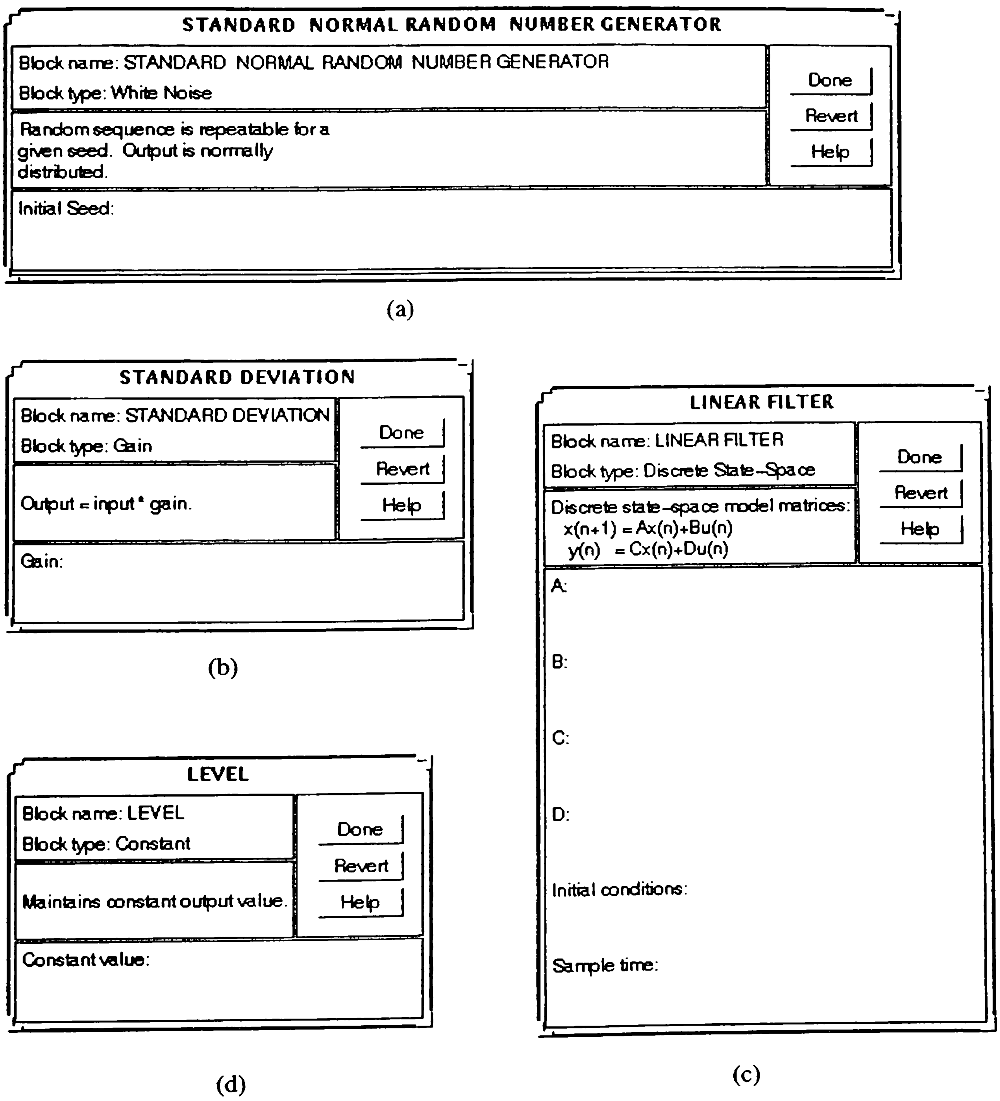


Figure 4: Input Environments for Selected Blocks

Selecting any of these blocks reveals an input environment for the model parameters. For example, Figure 4 shows the input environment for the first four blocks of the model: the STANDARD NORMAL RANDOM NUMBER GENERATOR (Figure 4(a)); the STANDARD DEVIATION (Figure 4(b)); the LINEAR FILTER (Figure 4(c)) and the LEVEL (Figure 4(d)).

4.1 Example 1: ARMA(1, 1) Model

The ARMA (1, 1) process is a special case of the ARMA (p, q) process of Equation (A-1) that has been widely used in practical economic and engineering applications. For comparison with previous results, we will consider the following ARMA (1, 1) process used by Montgomery, Johnson, and Gardiner (1988):

$$y_n = 10 + 0.6y_{n-1} + u_n + 0.9u_{n-1} \qquad (1)$$

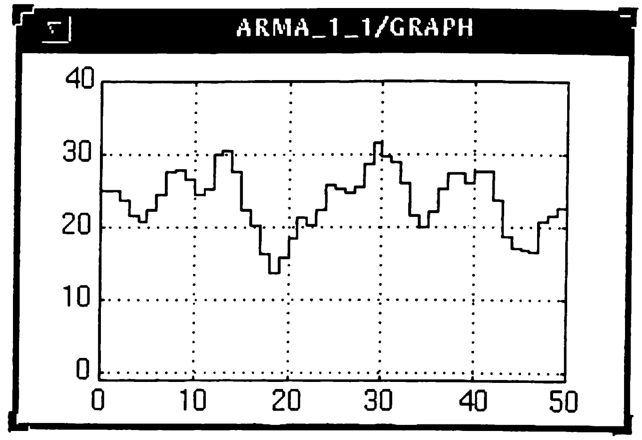
with $\sigma^2 = 4$. By Equation (A-2) the mean (level) of the process is 25. By Equations (A-3) and (A-4), the matrices for the discrete state-space representation of the disturbance are

$$A = \begin{bmatrix} 0.6 & 0.9 \\ 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

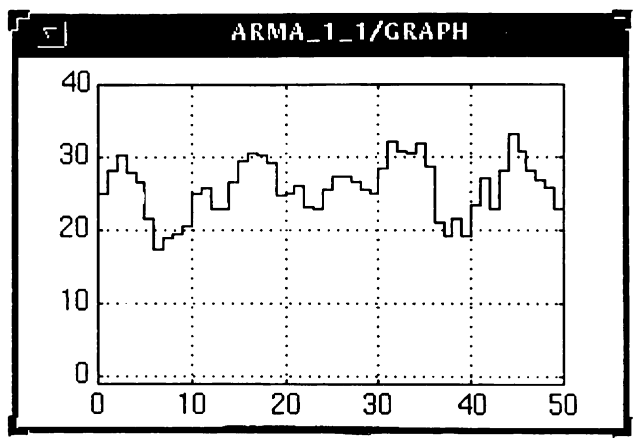
$$C = [1\ 0] \qquad D = [0] \ .$$

The standard deviation is specified in the Figure 4(b) window, the values of the matrices in the Figure 4(c) window and the mean in the Figure 4(d) window.

To keep this example brief, we made only two simulation runs of 50 time periods. The state-space representation of the process was used for each of the runs yielding two different realizations of the ARMA(1, 1) process. Specifying random streams 1, and 2 (in the Figure 4(a) window) result in Figures 5(a) and 5(b), respectively. Of course, to make better probabilistic statements about the process we must make additional replications. To do so, we could change the random number stream manually for each run or use a small MATLAB program to implement them in a batch.



(a)



(b)

Figure 5: Two realizations of the ARMA(1, 1) process $y_n = 10 + 0.6y_{n-1} + u_n + 0.9u_{n-1}$

4.2 Example 2: ARMA(2, 2) Model

Consider two continuous stirred-tank reactors in series used to transform a comonomer (which is a chemical building block used to make film for products like plastic bags and cups). The disturbance in the system can be represented by the ARMA (2, 2) process

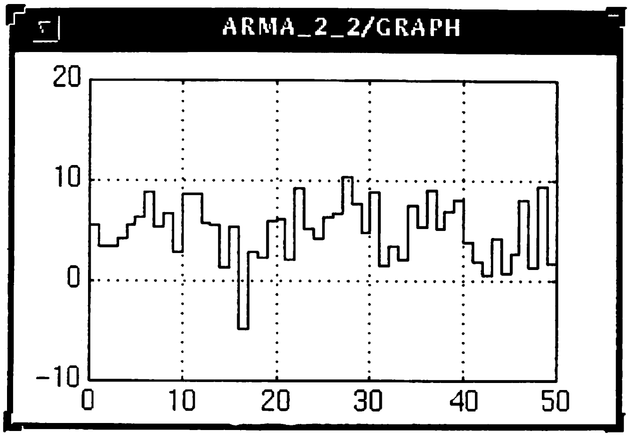
$$\begin{aligned} y_n &= 5 + 0.9y_{n-1} - 0.8y_{n-2} \\ &\quad + u_n + 0.6u_{n-1} + 0.2u_{n-2} \end{aligned} \qquad (2)$$

with $\sigma^2 = 9$. By Equation (A-2) the mean (level) of the process is 5.56. By Equations (A-3) and (A-4),

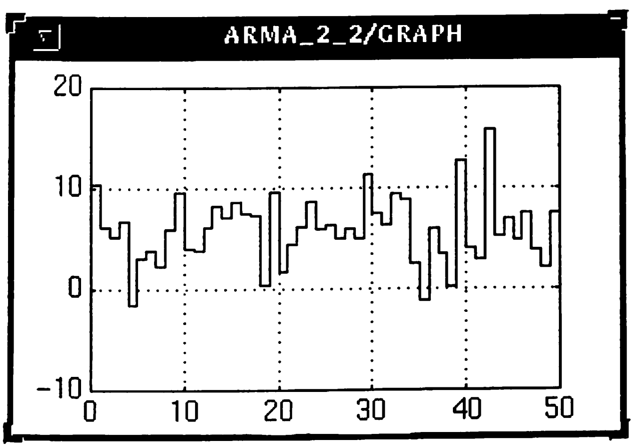
$$A = \begin{bmatrix} 0.9 & -0.8 & 0.2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C = [1\ 0\ 0\ 0] \qquad D = [0]\ .$$

Using the same methods as given in Example 1, two simulation runs were made of this process. The graphical results from these runs are shown in Figure 6.



(a)



(b)

Figure 6: Two realizations of the ARMA(2, 2) process $y_n = 5 + 0.9y_{n-1} - 0.8y_{n-2} + u_n + 0.6u_{n-1} + 0.2u_{n-2}$

5 SUMMARY

Dynamic simulation using SIMULINK is a very powerful tool and has been frequently used in the chemical engineering, electrical engineering, and applied physics disciplines. Part of the motivation for this paper is to increase the exposure of dynamic simulation software as a teaching tool within the industrial engineering and operations management disciplines.

This paper illustrates that dynamic simulation can be used as a learning tool for time series concepts. We developed a SIMULINK simulation model for the state-space representation of a general ARMA time series process. We illustrated the use of the model on examples of an ARMA (1, 1) process and an ARMA (2, 2) process. (They can be down-loaded from the internet address given in Section 1.) The model can, of course, be used to represent other types of systems such as those with cyclic, ramp, and impulse characteristics. It can also be used to illustrate other concepts in the statistics area such as forecasting, prediction, model identification, and maximum likelihood estimation of parameters.

Dynamic simulation has also been used as a research tool to investigate control and monitoring policies for noisy dynamic systems (Nembhard 1996 and Nembhard and Mastrangelo 1996). Future plans include exploring its application in organizational learning and in the economic design of Kalman filtering models.

APPENDIX A: STATE-SPACE REPRESENTATION OF THE TIME SERIES

A state-space representation lends itself well to simulation analysis. It provides a convenient way to represent complex mathematical systems that include multiple inputs or outputs, nonlinearity, or time-varying components. A state-space representation is used in the simulation model to represent a stationary stochastic auto-regressive-moving average (ARMA) model. The ARMA (p, q) process can be represented as a time series (Montgomery, Johnson, and Gardiner 1988) by

$$y_n = \xi + \sum_{i=1}^{p} \phi_i y_{n-i} - u_n - \sum_{j=1}^{q} \theta_j u_{n-j} \qquad \text{(A-1)}$$

where y_n is the current observation and is regressed on previous realizations y_{n-1}, y_{n-2}, ..., y_{n-p}; ϕ_1, ϕ_2, ..., ϕ_p are the unknown process parameters; u_n, u_{n-1}, ..., u_{n-q} are the independent random variables; and θ_1, θ_2, ..., θ_q are a finite set of weights.

The mean of the ARMA (p,q) is

$$\mu = \frac{\xi}{1-\sum_{i=1}^{p}\phi_i} \quad . \qquad \text{(A-2)}$$

The expanded form of Equation (A-1)

$$y_n = \xi + \phi_1 y_{n-1} + \phi_2 y_{n-2} + \cdots + \phi_p y_{n-p} - u_n - \theta_1 u_{n-1} - \theta_2 u_{n-2} - \cdots - \theta_q u_{n-q}$$

allows the state vector to be written as

$$[y_{n-1}\ y_{n-2}\ \cdots\ y_{n-p}\ u_{n-2}\ u_{n-3}\ \cdots\ u_{n-q}]^{\mathrm{T}} \ .$$

We note that the set of state variables is not unique for a given system (see Ogata 1987 for further discussion of this point) so another set could be defined with u_n, u_{n-1}, and u_{n-q-2} if preferred. Hence the discrete state-space representation

$$\begin{aligned} x_{n+1} &= Ax_{n+1} + Bu_n \\ y_n &= Cx_n + Du_n \end{aligned}$$

for the ARMA(p,q) process is given by

$$x_n = \begin{bmatrix} \phi_1 & \cdots & \phi_p & -\theta_2 & \cdots & -\theta_{q-1} & -\theta_q \\ 1 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \cdots \\ \cdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \cdots \\ \cdots \\ 0 & \cdots & 0 & 0 & \cdots & 1 & 0 \end{bmatrix} * $$

$$x_{n-1} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \cdots \\ \cdots \\ -\theta_1 \\ 0 \\ 0 \\ \cdots \\ \cdots \\ 0 \end{bmatrix} u_{n-1} \qquad \text{(A-3)}$$

$$y_n = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} x_n - [\theta_0]\, u_n \ . \qquad \text{(A-4)}$$

There would be little reason to use p or q are greater than 2 in a simulation model because time series model identification methods often show little differentiation between second-order and third-order processes. Further, many industry processes can be represented by first or second-order models.

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