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A HYBRID APPROACH OF THE STANDARD CLOCK METHOD AND EVENT SCHEDULING APPROACH FOR GENERAL DISCRETE EVENT SIMULATION

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ABSTRACT

The Standard Clock method is an efficient approach for discrete event simulation. Its basic ideas are quite different from traditional approaches. SC has neither an event list nor event lifetimes. However, its applicability is limited to exponential distributions and a class of non-exponential distributions. In this paper we provide an effective approach to extend SC to all general distributions, while preserving the efficiency advantage of SC. Numerical testing shows that our approach can effectively accomplish this goal.

1 INTRODUCTION

The Standard Clock (SC) method (Vakili 1991 and Vakili, Mollamustafaoglu, Ho 1992) is an efficient technique for Discrete Event System (DES) simulation, particularly for simulating a set of parametrically different but structurally similar DES's. The basic ideas of SC are quite different from traditional approaches (event-scheduling simulation, ESS). ESS builds an active event list based on the current state, determines lifetimes for each event in the list, and chooses the event with minimum lifetime to be the next triggering event for state transition. By contrast, SC has neither an event list nor event lifetimes. Events for all experiments/simulations are derived from a global event stream. However, SC is limited by Markovian assumption, i.e., all lifetimes must be exponentially distributed.

Chen and Ho (1995) provides an effective approaches for extending the applicability of SC to a class of non-exponential distributions. Although their approach performs very well in most cases, it becomes inefficient when dealing with deterministic problems or distributions having very small variance. How to overcome the difficulty and preserve the efficiency advantage of SC is the main issue of this paper.

Our approach is a hybrid approach of ESS and SC. For ease of explanation, we will give a brief description of ESS and SC in Sections 2 and 3. The difficulty of SC for general distributions will be discussed in Section 3. Section 4 presents our hybrid approach. Numerical testing in Section 5 demonstrates that this hybrid can effectively deal with the difficulties that the existing SC suffers, and accomplish our efficiency goal.

2 EVENT SCHEDULING SIMULATION

Event-scheduling simulation approach has been well used. It can be found in many simulation books. In particular, Cassandras (1993) and Nelson (1995) provide very good introduction to this approach. ESS builds an active event list based on the current state, determines lifetimes for each event in the list, and chooses the event with minimum lifetime to be the next triggering event for state transition. The cycle repeats with event time determination and state transition interacting continuously. The major components are: simulation clock t, system state x, feasible event set $\Gamma(x)$, event types e_k, event times t_k, and event list $\{(e_k, t_k)\}$, where k=1, 2, 3, ... The ESS algorithm is as follows:

Step 1. Initialization.

Step 2. Event Triggering. $e \leftarrow \arg\min_{k \in \Gamma(x)} \{ t_k \}$.

Step 3. Time Advancing. $t \leftarrow t_e$.

Step 4. State Transition. $x \leftarrow f(x, e)$.

Step 5. Feasible Event Set and Event List Updating.

Step 6. Go to Step 2.

3 STANDARD CLOCK METHOD

The basic ideas of SC are quite different from ESS. SC has neither an event list nor event lifetimes. Events for all experiments/simulations are derived from a global event stream. For simplicity, we explain the SC approach by using an M/M/1 queue simulation example with arrival rate 0.5 and service rate 1.0.

Instead of generating the two types of events (arrival and departure) from separate exponential distributions, we consider a single stream of events that occur at the (faster) rate 0.5 + 1.0 = 1.5. Namely, the interval time between two events is exponentially distributed with rate 1.5. In Figure 1, a straight line denotes an event.

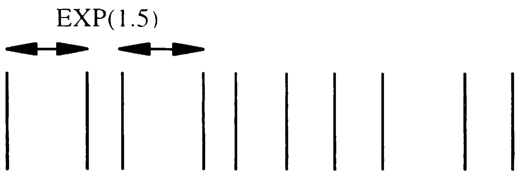

Figure 1: An Example of An Event Stream Before The Determination of Event Types

Because of the properties of Poisson processes, we would expect 0.5 / (0.5+1.0) = 1/3 of the events to be arrivals, and 2/3 of the events to be departures. We determine the event type according to the outcome of a Unif[0,1) random number r placed onto a ratio yardstick.

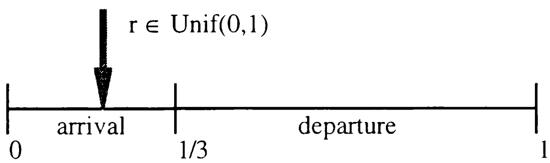

Figure 2: An Example of Ratio Yardstick for Determining Event Types

If r < 1/3, this particular event is an arrival event. Otherwise, this event could be a departure event. A Unif[0,1) random number is generated for each event in Figure 1 and the event types are determined shown in Figure 3. A down arrow denotes an arrival event and an up arrow denotes a departure event.

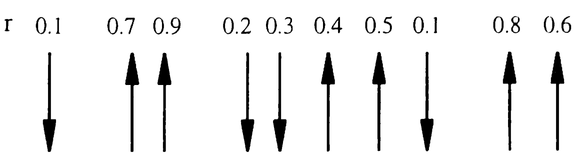

Figure 3: An Example of An Event Stream After The Determination of Event Types.

Statistically, this process is equivalent to generating two separate Poisson event streams at rates 0.5 and 1.0. These two event streams, representing the maximal rates of arrival and departure events, are further **thinned** (deleted) according to the state of the DES. We ignore departure events whenever the queue is empty, since the events are infeasible. A sample path based on the event stream in Figure 3 is constructed as follows.

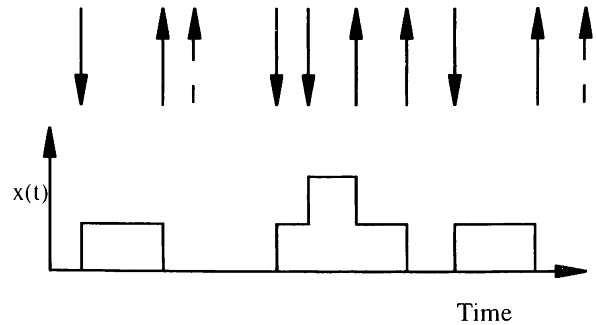

Figure 4: The Sample Path Constructed From The Event Stream in Figure 3

Because of exponential distribution's memoryless property, the sample path constructed in this way is statistically indistinguishable from the path constructed by ESS. The idea of thinning a Poisson event stream can be applied to all networks subject to Markovian assumptions (i.e., the interarrival and service times must be exponentially distributed). In general, if we want to simulate a DES with n types of exponential events at rates λ_i, i=1,..,n, let $\Lambda = \sum_{i=1}^{n} \lambda_i$.

SC Algorithm

Step 1. Initialization.

Step 2. Time Advancing. $t \leftarrow t + \delta$, where δ is exponentially distributed with rate Λ.

Step 3. Event Type Determination.
The event type $e = 1$, if $0 \le r < \lambda_1/\Lambda$
$e = 2$, if $\lambda_1/\Lambda \le r < (\lambda_1+\lambda_2)/\Lambda$
$\vdots$
$e = n$, if $(\lambda_1+\lambda_2+ ..+\lambda_{n-1})/\Lambda \le r < 1$
where r is a Unif[0,1) random number.

Step 4. Event Feasibility Checking and State Transition.
If feasible, State Transition: $x \leftarrow f(x, e)$.
Else, ignore this event.

Step 5. Go to Step 2.

Note that the generation of event streams (Steps 2 and 3) is independent of system states and, therefore, can be done off-line. Given an event stream, we only need to continually check event feasibility (Step 4) during simulation. This significantly reduces on-line simulation cost. Simplicity and ease of implementation are additional advantages of the SC method. When SC is applied to a set of parametrically different but structurally similar (pdss) simulation experiments (e.g., the testing example in Section 5), the superiority of SC to ESS is more significant. These pdss experiments are individually "thinned" from the **same** global event stream using the **same** set of simulation program instructions. Other statistical advantages of common random numbers (Glasserman and Yao 1992), coupling (Glasserman and Vakili 1992), and correlation (Deng, Ho, and Hu 1992) further accrue to such a simulation approach.

As previously discussed, when event lifetime distributions are exponentially distributed, SC is not only an efficient simulation approach, but also can be easily implemented on computers. For non-exponential distributions, Chen and Ho (1995) provides efficient approaches for extending the applicability of SC to a class of general distributions. They approximate non-exponential distributions using either shifted exponential distributions (when the coefficients of variation are smaller than 1) or hyperexponential distributions (when the coefficients of variation are bigger than 1). They also show that good approximation property can be obtained by only matching the first two moments of non-exponential distributions.

The main difficulty emerges when applying to distributions having very small variance. A shifted exponential distribution can be represented by K+T, where K is a constant and T is an exponentially distributed random variable with rate μ. To use K+T to approximate a non-exponential random variable S, we choose K and T such that

$$E(K + T) = E(S) \text{ and } Var(K + T) = Var(S).$$

Solving the above two equations,

$$\mu = \frac{1}{\sqrt{Var(S)}} \quad \text{and} \quad K = E(S) - \sqrt{Var(S)}$$

μ is the rate which SC has to generate for this event. When $\sqrt{Var(S)}$ is very small, μ can be very large. In this case, SC may become inefficient.

4 A HYBRID APPROACH

We propose a new approach to overcome the difficulty of SC for general distributions, while preserving the efficiency advantage of SC. The basic idea is as follows. SC can efficiently simulate a DES in which the distributions' variances are not too small. We want to take such an advantage of SC. On the other hand, we use ESS to deal with those distributions which are not suitable for SC. Under this structure, our approach is a hybrid approach of SC and ESS.

First of all, we identify which events are suitable for SC and which events are not. Then the event set is decomposed into two subsets: E_{SC} and E_{ESS}. E_{SC} contains those events which are suitable for SC and E_{ESS} contains the remaining events. The events in E_{SC} will be simulated using SC and the events in E_{ESS} will be simulated using ESS. Thus, we can avoid the difficulty of simulating those **tough** events when using SC. The remaining question is how to aggregate the two approaches into one simulation structure and to assure that the correct statistical property is maintained.

SC has neither an event list nor event lifetimes. Simulation clock proceeds based an exponential distribution with rate Λ. Event type is not determined until the event epoch. From the ESS point of view, before event type determination, all the events from SC can be treated as one single special type of events, say e_0, which is scheduled to happen at some special event time, say t_0. Thus, we can extend the event list by merge the original event list of ESS and this special event from SC. The whole structure of this approach is like an ESS approach with the following additional definitions:

* Hybrid Event List: $\{(e_0, t_0)\} \cup \{(e_k, t_k)\}$, where $k \in E_{ESS}$.
* ESS Feasible Event Subset: $\Gamma'(x) \equiv \Gamma(x) \cap E_{ESS}$.
* n': the number of elements in E_{SC}.
* Function g: I $\rightarrow$ I. g(i) denotes the event type of the i-th event in E_{SC}, i.e., events g(1), g(2),.., g(n') are included in E_{SC}.
* $\Lambda' \equiv \sum_{i=1}^{n'} \lambda_{g(i)}$.

Figure 5 illustrates the basic idea of our hybrid simulation structure.

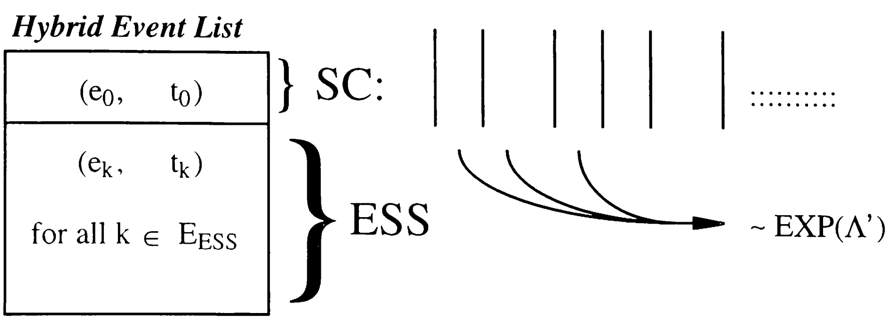

Figure 5: A Hybrid Event List

When the lifetimes of all events in E_{SC} are exponentially distributed, the algorithm is as follows:

Hybrid Algorithm

Step 1. Initialization.

Step 2. Triggering Event $e = \arg \min_{k \in \{0\} \cup \Gamma(x)} \{ t_k \}$.

Step 3. Time Advancing. $t \leftarrow t_e$.

Step 4. If $e \neq 0$ (i.e., $e \in E_{ESS}$), Go to Step 9.

Else (i.e., next event $\in E_{SC}$), Go to Step 5.

Step 5. Event Type Determination. The event type e

$= g(1)$, if $0 \leq r < \lambda_{g(1)}/\Lambda'$

$= g(2)$, if $\lambda_{g(1)}/\Lambda' \leq r < (\lambda_{g(1)}+\lambda_{g(2)})/\Lambda'$

$\vdots$

$= g(n')$, if $(\lambda_{g(1)}+\lambda_{g(2)}+ \ldots+\lambda_{g(n'-1)})/\Lambda' \leq r < 1$

where r is a Unif[0,1) random number.

Step 6. Event Feasibility Checking and State Transition.
If feasible, State Transition: $x \leftarrow f(x, e)$.
Else, ignore this event.

Step 7. $t_0 \leftarrow t + \delta$, where δ is exponentially distributed with rate Λ'.

Step 8. Go to Step 10.

Step 9. State Transition. $x \leftarrow f(x, e)$.

Step 10. ESS Feasible Event Subset and Hybrid Event List Updating

Step 11. Go to Step 2.

When the lifetimes of all events are exponentially distributed, the sample path obtained by SC is statistically indistinguishable from the sample path by ESS. Due to memoryless property of exponential distribution, the sample path obtained by our hybrid approach is also statistically indistinguishable from the sample path by ESS. In general, the sample path by the hybrid approach is statistically equivalent to the sample path by ESS whenever the lifetimes of all events in E_{SC} are exponentially distributed.

If the lifetimes of some event types in E_{SC} are not exponentially distributed, we may apply the Chen and Ho (1995)'s extension approach and revise Steps 5 and 6 to deal with non-exponential distributions. The details are omitted here.

5 NUMERICAL TESTING

The following examples are used to demonstrate that our approach can effectively deal with distribution having very small variance, while preserving the efficient advantage of SC.

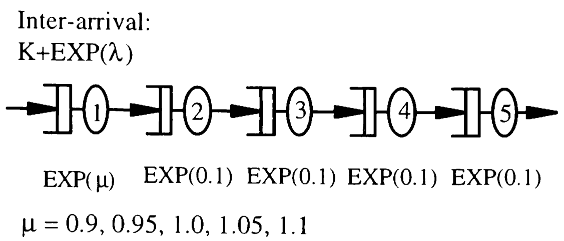

Figure 6: A Five-Node Tandem Network

There are 5 nodes serially connected in this network. Jobs will be sent to the node i+1 after the service at node i is completed. The queue discipline is FIFO. The service times are exponentially distributed with rate 1.0 at Nodes 2 ~ 5 and with rate μ at Node 1. Given an job-arrival distribution, we want to simulate the systems with five different μ's from 0.9, 0.95, 1.0, 1.05, to 1.10. The distribution of the inter-arrival times of jobs are shifted exponential distribution, which is represented by K + EXP(λ). By varying K and λ, we can obtain different coefficients of variation to compare the performance of different simulation approaches. We test ESS, SC, and our new hybrid approach for different coefficients of variation of inter-arrival times.

In this testing example, E_{SC} contains the departure events at all nodes and E_{ESS} contains the arrival events. Figure 7 on the following page shows the testing results.

In this numerical testing, SC beats the others when coefficient of variation is not too small. However, SC becomes inefficient when coefficient of variation is close to 0. Our hybrid approach and ESS are not sensitive to coefficient of variation. The hybrid approach is more efficient than ESS throughout the testing.

6 CONCLUDING REMARKS

In this paper, we presented a hybrid approach for extending SC to non-exponential distributions. This is done by aggregating SC and ESS. Numerical testing shows that the hybrid approach can effectively deal with distribution having very small coefficient of variation (or variance), while preserving the efficient advantage of SC.

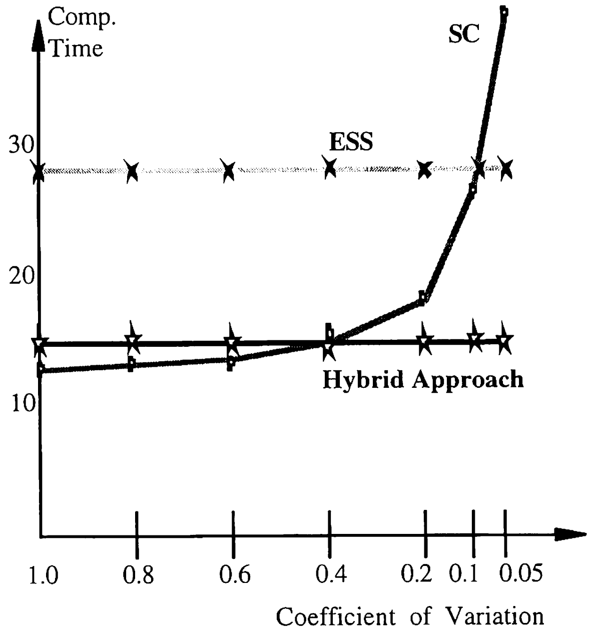

Figure 7: Numerical Testing Results

In the numerical testing, when the coefficient of variation is not too small, SC is much more efficient than ESS. The performance of our hybrid approach is in between. When E_{ESS} has only one event, the hybrid approach is slightly slower than SC. If the number of events in E_{ESS} increases, we believe that the hybrid approach will behave more like an ESS and the performance will get closer to ESS. As a result, there is a tradeoff in determining the event subset E_{ESS} for the hybrid approach. If we put more events in E_{ESS}, the hybrid approach is less sensitive to variances, but may suffer from lower efficiency. On the other hand, if we put less events in E_{ESS}, the hybrid approach will behave more like a SC and is more efficient in some cases, but may suffer from being sensitive to variances.

Which events should be included in E_{ESS} to guarantee highest efficiency is remained to be investigated.

ACKNOWLEDGMENT

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