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IMPROVEMENTS AND EXTENSIONS TO SIMULATION INTERVAL PROCEDURES

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ABSTRACT

This paper presents improvements to two common confidence interval procedures: the two-stage Student's t intervals and the paired t intervals. Further, we provide a conservative extension to the paired t intervals to deal with any finite number of alternatives. Also, we include an approximate procedure that performs better with large numbers of alternatives. These procedures have been presented in the literature previously, but special elementary cases, revised notation, and comparison to common methods should make these procedures accessible to a broader audience. Numerical examples are included.

1 INTRODUCTION

This paper presents a number of procedures for finding confidence intervals for discrete event simulations. The procedures listed are revisions and convenience extensions of those in Matejcik and Nelson (1993). Each of the procedures (except the sphericity procedure) are written so that the repetitive calculations may be done with spreadsheet functions. These repetitive calculations may be done using *average* commands for $\bar{x}$, $\bar{d}$, and $\bar{d}_{ij}$; using *standard deviation* commands for $s(\ )$; and copying cells of the form "=b3-c3" down columns for $d_{ij,\ell}$. So, these procedures are convenient for many users, and the ease of computation may comfort potential users. Also, comments and advice are included in the procedures to aid in understanding and implementation.

Each of the multi-system interval procedures is robust to unequal variances between systems and exploits common random numbers to achieve more narrow confidence intervals. Also, the multi-systems confidence intervals exploit Hsu's (1984) Multiple Comparison with Best (MCB) interval scheme to obtain better inferences. Explicit statement of Hsu's MCB for paired observations may be new in this paper; it provides more powerful inference than the commonly used paired-t procedure. Also, Hsu's MCB intervals are explicitly stated for finding minimums as well as maximums, thus saving readers the work of deriving the minimum format. Additionally, the k-systems and the paired-t procedures have the intervals using fewer elements in the maximization operators and no use of $\lceil\ \rceil$, $(\ )^+$, and $-(\ )^-$.

2 TWO-STAGE INTERVAL PROCEDURE

Pegden, Shannon, and Sadowski (1995) and other elementary simulation texts include an approximate multi-stage procedure for finding confidence intervals for a simulated mean performance parameter, μ. I call this procedure, the common procedure. In light of Stein's methods (see e.g. Wetherhill and Glazebrook 1986) this may be shortened to the following *conservative* two-stage method.

Two-stage mean interval procedure

1. Choose a Confidence Level $1-\alpha$.
2. Generate n_1 independent sample observations X_i from the system simulation model.
3. Compute the common summary statistics:
$$\bar{x} = \sum_i^{n_1} \frac{X_i}{n_1}$$
$$s(x) = \sqrt{\sum_i \frac{(X_i-\bar{x})^2}{n_1-1}}$$
4. Compute a first-stage half width.
$$h_1 = t_{1-\alpha/2,n_1-1}s(x)/\sqrt{n_1}$$
Here t is the usual tabled Student's t value.

5. If h_1 is sufficiently small assign $h = h_1$, and proceed to the last step, forming the confidence intervals. Otherwise, go to the next step.

6. Select a target value $h < h_1$, which would provide a sufficiently small value of h.

7. Compute the total required sample size N and second stage sample size n_2.

$$N = \text{INT}\left[n_1 \cdot \left(\frac{h_1}{h}\right)^2\right] + 1$$

$$n_2 = N - n_1$$

Although not strictly necessary at this point, it is wise to consider the run time required to generate n_2 independent sample observations. If the run time is too long, go back to the previous step and select a larger value of h.

8. Generate n_2 independent sample observations X_i from the system simulation model.

9. Recompute $\bar{x}$ using the total sample.

$$\bar{x} = \sum_i^N \frac{X_i}{N}$$

10. Finally, form the confidence interval:

$$1 - \alpha \leq \Pr\{\bar{x} - h < \mu < \bar{x} + h\}.$$

The above procedure has advantages over the common procedure. The above procedure is conservative (probability statement for the confidence interval can be proven correct when the observations X_i are normally distributed), and the common procedure is only approximately correct. Secondly, this procedure will terminate in two-stages, while the common procedure may take more stages. Also the common and the above procedures have the same first and second stage sample sizes. And, finally the above procedure requires that only one standard deviation be computed, while the common procedure requires at least two be computed.

3 PAIRED-T INTERVAL PROCEDURES

Paired-t procedure provide interval estimates for the difference of two means $\mu_X - \mu_Y$, when obtaining samples of X and Y using common random numbers. If the confidence interval excludes zero, we may determine with probability $1-\alpha$ which mean is larger. Elementary statistics books and introductory simulation texts include paired-t procedure (see e.g. Pegdon, Shannon, and Sadowski 1995). One-stage procedures are most often presented in texts. Also, a two-stage procedure analogous to the one the previous section is also available. The following procedures use unbalanced paired-t intervals to better determine which mean is larger. The procedures may be proved correct by observing that they are special cases of procedures from Matejcik and Nelson (1993).

One-stage paired-t procedure

1. Choose a Confidence Level $1 - \alpha$.

2. Generate $X_1, X_2, \ldots, X_n$, and $Y_1, Y_2, \ldots, Y_n$ using common random numbers across systems.

3. Compute $d_i = X_i - Y_i$ for all $i = 1, 2, \ldots, n$.

4. Compute the common summary statistics:

$$\bar{d} = \sum_i^n \frac{d_i}{n}$$

$$s(d) = \sqrt{\sum_i \frac{(d_i - \bar{d})^2}{n-1}}$$

5. Compute the "half width".

$$h = t_{1-\alpha,n-1} s(d)/\sqrt{n}$$

Here t is the usual tabled Student's t value. Notice, the subscript $1 - \alpha$ is indeed correct; our unbalanced intervals allow us to use a smaller t value than the common procedures.

6. Finally, form the confidence interval:

$$1-\alpha \leq \Pr\{\min(0, |\bar{d}|-h) < |\mu_X - \mu_Y| < |\bar{d}|+h\}.$$

Two-stage paired-t procedure

1. Choose a Confidence Level $1 - \alpha$.

2. Generate $X_1, \ldots, X_{n_1}$, and $Y_1, \ldots, Y_{n_1}$ using common random numbers across systems.

3. Compute $d_i = X_i - Y_i$ for all $i = 1, 2, \ldots, n_1$.

4. Compute the common summary statistics:

$$\bar{d} = \sum_i^{n_1} \frac{d_i}{n_1}$$

$$s(d) = \sqrt{\sum_i \frac{(d_i - \bar{d})^2}{n_1 - 1}}$$

5. Compute the first stage "half width".

$$h_1 = t_{1-\alpha,n_1-1}s(d)/\sqrt{n_1}$$

Here t is the usual tabled Student's t value.

6. If h_1 is sufficiently small assign $h = h_1$, and proceed to the last step, forming the confidence intervals. Otherwise, go to the next step. Note, we must not compare the means of the systems in judging if h_1 is sufficiently small.

7. Select a target value $h < h_1$, which would provide a sufficiently small value of h.

8. Compute the total required sample size N and second stage sample size n_2.

$$N = \text{INT}\left[n_1 \times \left(\frac{h_1}{h}\right)^2\right] + 1$$

$$n_2 = N - n_1$$

Although not strictly necessary at this point, it would be wise to consider the run time required to generate n_2 independent sample observations. If the run time is too long, go back to the previous step and select a larger value of h.

9. Generate $X_{n_1+1}, X_{n_1+2}, \ldots, X_N$, and $Y_{n_1+1}, Y_{n_1+2}, \ldots, Y_N$ using common random numbers across systems.

10. Compute $d_i = X_i - Y_i$ for all $i = n_1+1, n_1+2, \ldots, N$.

11. Recompute $\bar{d}$ using the total sample.

$$\bar{d} = \sum_i^N \frac{d_i}{N}$$

12. Finally, form the confidence interval:

$$1-\alpha \leq \Pr\{\min(0, |\bar{d}|-h) < |\mu_X - \mu_Y| < |\bar{d}|+h\}.$$

With both of the above procedures if the confidence interval excludes zero, we may determine with probability $1-\alpha$ which mean of the two means is larger.

4 K SYSTEM PAIRED PROCEDURES

Extending the paired-t procedure to more than two systems can be done by the all pairwise procedure as shown in Law and Kelton (1991). Alternatively, we extend the procedure by forming Hsu's (1984) Multiple Comparison with the Best (MCB) intervals. Our procedure is closely related to the Selection procedure developed by Clark and Yang (1986). MCB provides interval estimates of $\mu_i - \max_{j \neq i} \mu_j$ for $i = 1, 2, \ldots, k$, when we desire to select the largest system. If system i is the largest $\mu_i - \max_{j\neq i}\mu_j > 0$ and if system i is not the largest $\mu_i - \max_{j\neq i}\mu_j < 0$. Similarly, if we find confidence intervals for $\mu_i - \max_{j\neq i}\mu_j$ covering only positive numbers we may declare system i the largest with the same confidence level with which we formed the intervals. Also, if we find confidence intervals for $\mu_i - \max_{j\neq i}\mu_j$ covering only negative numbers we may reject system i from consideration as being the largest with the same confidence level with which we formed the intervals. Analogous reasoning allows to make similar statements when seeking the smallest system and using $\mu_i - \min_{j\neq i}\mu_j$ for $i = 1, 2, \ldots, k$.

One-stage k system procedure

1. Choose a Confidence Level $1-\alpha$.

2. Generate independent and identically distributed sample observations $X_{1,1}, X_{1,2}, \ldots, X_{1,n}, X_{2,1}, X_{2,2} \ldots, X_{2,n}, \ldots, X_{k,1}, X_{k,2} \ldots, X_{k,n}$, using common random numbers across systems.

3. Compute the differences between the systems $d_{ij,\ell} = X_{i,\ell} - X_{j,\ell}$ for all $i, j = 1, 2, \ldots, k$ and for all $\ell = 1, 2, \ldots n$. Observe that $d_{ij,\ell} = -d_{ji,\ell}$ and $d_{ii,\ell} = 0$ to save calculations.

4. For all $i, j = 1, 2, \ldots k$ obtain the following summary statistics:

$$\bar{d}_{ij} = \sum_\ell^n \frac{d_{ij,\ell}}{n}$$

$$s(d_{ij}) = \sqrt{\sum_i \frac{(d_{ij,\ell} - \bar{d}_{ij})^2}{n-1}}$$

5. Compute the "half width".

$$h = \frac{t_{1-\frac{\alpha}{k-1},n-1}}{\sqrt{n}} \max_{i,j} s(d_{ij})$$

Here t is the usual tabled Student's t function. However, unusual probability values are used, so I provide a short table for $\alpha = 0.05$ as an appendix.

6. Finally, if we are interested in selecting the largest system mean form the following set of confidence intervals:

$$1-\alpha \leq$$
$$\Pr\{\min(0, -h + \min_{j\neq i} d_{ij}) < \mu_i - \max_{j\neq i}\mu_j < \max(0, h + \min_{j\neq i} d_{ij})\} \text{for } i = 1, 2, \ldots k$$

Alternatively, if we are interested in selecting the the smallest system mean form the following set of confidence intervals:

$$1 - \alpha \leq \Pr\{\min(0, -h + \max_{j \neq i} d_{ij}) < \mu_i - \min_{j \neq i} \mu_j < \max(0, h + \max_{j \neq i} d_{ij})\} \text{for } i = 1, 2, \ldots k$$

Two-stage k system procedure

1. Choose a Confidence Level $1 - \alpha$.
2. Generate independent and identically distributed sample observations $X_{1,1}, X_{1,2}, \ldots, X_{1,n_1}, X_{2,1}, X_{2,2} \ldots, X_{2,n_1}, \ldots, X_{k,1}, X_{k,2} \ldots, X_{k,n_1}$, using common random numbers across systems.
3. Compute the differences between the systems $d_{ij,\ell} = X_{i,\ell} - X_{j,\ell}$ for all $i, j = 1, 2, \ldots, k$ and for all $\ell = 1, 2, \ldots n_1$. Observe that $d_{ij,\ell} = -dji, \ell$ and $d_{ii,\ell} = 0$ to save calculations.
4. For all $i, j = 1, 2, \ldots k$ obtain the common summary statistics:
$$\bar{d}_{ij} = \sum_{\ell}^{n} \frac{d_{ij,\ell}}{n}$$
$$s(d_{ij}) = \sqrt{\sum_{i} \frac{(d_{ij,\ell} - \bar{d}_{ij})^2}{n-1}}$$
5. Compute the first stage "half width".
$$h = \frac{t_{1-\frac{\alpha}{k-1}, n_1 - 1}}{\sqrt{n_1}} \max_{i,j} s(d_{ij})$$
Here t is the usual tabled Student's t function. However, unusual probability values are used, so I provide a short table for $\alpha = 0.05$ as an appendix.
6. If h_1 is sufficiently small assign $h = h_1$, and proceed to the last step, forming the confidence intervals. Otherwise, go to the next step. Note, we must not compare the means of our groups in judging if h_1 is sufficiently small.
7. Select a sufficiently small target value $h < h_1$.
8. Compute the total required sample size N and second stage sample size n_2.
$$N = \text{INT}\left[n_1 \times \left(\frac{h_1}{h}\right)^2\right] + 1$$
$$n_2 = N - n_1$$
Although not strictly necessary at this point, it would be wise to consider the run time required to generate n_2 independent sample observations. If the run time is too long, go back to the previous step and select a larger value of h, or consider another technique such as the two-stage procedure from the next section.
9. Generate independent and identically distributed sample observations $X_{1,n_1+1}, X_{1,n_1+2}, \ldots, X_{1,N}, X_{2,n_1+1}, X_{2,n_1+2}, \ldots, X_{2,N} \ldots X_{k,n_1+1}, X_{k,n_1+2}, \ldots, X_{k,N}$, using common random numbers across systems.
10. Compute for all $d_{ij,\ell} = X_{i,\ell} - X_{j,\ell}$ for all $i, j = 1, 2, \ldots, k$ $\ell = n_1 + 1, n_1 + 2, \ldots, N$.
11. Recompute $\bar{d_{ij}}$ using the total sample.
$$\bar{d_{ij}} = \sum_{\ell}^{N} \frac{d_{ij,\ell}}{N} \quad \text{for all} \quad i, j = 1, 2, \ldots k$$
12. Finally, if we are interested in selecting the largest system mean form the following set of confidence intervals:
$$1 - \alpha \leq \Pr\{\min(0, -h + \min_{j \neq i} d_{ij}) < \mu_i - \max_{j \neq i} \mu_j < \max(0, h + \min_{j \neq i} d_{ij})\} \text{ for } i = 1, 2, \ldots k$$
Alternatively, if we are interested in selecting the smallest system mean form the following set of confidence intervals:
$$1 - \alpha \leq \Pr\{\min(0, -h + \max_{j \neq i} d_{i,j}) < \mu_i - \min_{j \neq i} \mu_j < \max(0, h + \max_{j \neq i} d_{i,j})\} \text{ for } i = 1, 2, \ldots k$$

5 LARGE K INTERVAL PROCEDURE

This final procedure, which also allows the systems to be simulated under common random numbers and is fairly robust to unequal variances across systems, is an extension of Nelson's (1993) robust MCB procedure. The analogous one stage procedure is available in Nelson (1993). Although the procedures of the previous section are correct for any finite number of systems, this final procedure is better when a large number of systems are studied. The point at which this occurs depends on many factors, so I suggest that the final procedure be used only when the procedures of the previous section are inadequate. This procedure assumes that covariance matrix between systems, $\mathbf{\Sigma}$, has a particular structure known as *sphericity*, specifically

$$\Sigma = \begin{pmatrix} 2\psi_1 + \tau^2 & \psi_1 + \psi_2 & \cdots & \psi_1 + \psi_r \\ \psi_2 + \psi_1 & 2\psi_2 + \tau^2 & \cdots & \psi_2 + \psi_r \\ & & \ddots & \\ \psi_r + \psi_1 & \psi_r + \psi_2 & \cdots & 2\psi_r + \tau^2 \end{pmatrix}.$$

Sphericity implies that

$$\mathrm{Var}[X_{ij} - X_{\ell j}] = 2\tau^2$$

for all $i \neq \ell$. In other words, the variances of all pairwise differences across systems are equal, even though the marginal variances and covariance may be unequal. Sphericity generalizes *compound symmetry*, which is

$$\Sigma = \sigma^2 \begin{pmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \rho \\ & & \ddots & \\ \rho & \rho & \cdots & 1 \end{pmatrix}.$$

Compound symmetry has been assumed by many researchers to account for the effect of common random numbers.

The procedure below is exact when Σ satisfies sphericity. Nelson (1993) showed that an MCB procedure based on the assumption of sphericity is remarkably robust to departures from sphericity provided that the covariances $\sigma_{ij} \geq 0$, which is commonly assumed and easily verified when common random numbers are used. In the procedure, $g = T^{(1-\alpha)}_{k-1,(k-1)(n_0-1),\frac{1}{2}}$ is the $(1-\alpha)$-quantile of the maximum of a multivariate t random variable of dimension $k-1$ with $(k-1)(n_0-1)$ degrees of freedom and common correlation 1/2; see, for instance, Table 4 in Hochberg and Tamhane (1987).

Two stage large k sample procedure

1. Specify h, α and n_1. Let $g = T^{(1-\alpha)}_{k-1,(k-1)(n_0-1),\frac{1}{2}}$.

2. Take i.i.d. sample $X_{i1}, X_{i2}, \ldots, X_{in_0}$ from each of the k systems using common random numbers across systems.

3. Compute the approximate sample variance of the difference

$$S^2 = \frac{2\sum_{i=1}^{k}\sum_{j=1}^{n_1}\left(X_{ij} - \bar{X}_{i\cdot} - \bar{X}_{\cdot j} + \bar{X}_{\cdot\cdot}\right)^2}{(k-1)(n_1-1)}.$$

4. Compute the second-stage sample size

$$N = \max\left\{n_1, \mathrm{INT}\left((gS/h)^2\right) + 1\right\}.$$

5. Take $N - n_1$ additional i.i.d. observations from each system, using common random numbers across systems.

6. Compute the overall sample means

$$\bar{\bar{X}}_{i\cdot} = \frac{1}{N}\sum_{j=1}^{N} X_{ij}$$

for $i = 1, 2, \ldots, k$.

7. When interested in the largest system, simultaneously form the MCB confidence intervals

$$\mu_i - \max_{j\neq i}\mu_j \in$$

$$\Big[\min(0, \bar{\bar{X}}_{i\cdot} - \max_{j\neq i}\bar{\bar{X}}_{j\cdot} - h),$$

$$\max(0, \bar{\bar{X}}_{i\cdot} - \max_{j\neq i}\bar{\bar{X}}_{j\cdot} + h)\Big]$$

for $i = 1, 2, \ldots, k$.

Alternatively, when interested in the smallest system, simultaneously form the MCB confidence intervals

$$\mu_i - \min_{j\neq i}\mu_j \in$$

$$\Big[\min(0, \bar{\bar{X}}_{i\cdot} - \min_{j\neq i}\bar{\bar{X}}_{j\cdot} - h),$$

$$\max(0, \bar{\bar{X}}_{i\cdot} - \min_{j\neq i}\bar{\bar{X}}_{j\cdot} + h)\Big]$$

for $i = 1, 2, \ldots, k$.

6 EXAMPLES

Pegdon, Shannon, and Sadowski (1995) provide a comparison of two material handling priority systems. I added a third set of numbers to show k systems comparisons methods. The following table includes the raw data: FIFO, and SPT observations from Pegdon, Shannon, and Sadowski (1995) and a newly added EDD column. The final three columns are the differences $d_{ij,\ell}$ between the observations, and below are the summary statistics. This table was prepared using a spreadsheet.

Table 1: Material Handling Priority Systems

FIFO	SPT	EDD	d_{FS}	d_{FL}	d_{SL}
8.87	9.19	9.853	-0.32	-0.983	-0.663
31.52	19.18	30.97	12.34	0.55	-11.79
14.14	12.3	15.002	1.84	-0.862	-2.702
14.11	13.04	15.2	1.07	-1.09	-2.16
16.72	17.79	18.713	-1.07	-1.993	-0.923
27.78	11.49	25.671	16.29	2.109	-14.181
34.05	21.61	33.723	12.44	0.327	-12.113
22.96	10.5	21.518	12.46	1.442	-11.018
10.98	13.27	12.765	-2.29	-1.785	0.505
11.66	8.5	11.878	3.16	-0.218	-3.378
		$\bar{d}_{ij}$	5.592	-0.2503	-5.8423
		$s(d_{ij})$	6.960	1.350	5.6943

We may find a paired-t interval for the difference between FIFO and SPT. Using our one stage procedure we have

$$0.95 \leq \Pr\{0 < |\mu_{FIFO} - \mu_{SPT}| < 9.620\},$$

which indicates that μ_{FIFO} is larger than μ_{SPT}. We have also found the MCB intervals using the one stage k system procedure for selecting the largest or the smallest and displayed them in tables 2 and 3 below.

Table 2: 95% CI $\mu_i - \max_{j \neq i} \mu_j$

i	$\min_{j \neq i} \bar{d}_{ij}$	lower bound	upper bound
FIFO	-0.2503	-5.22	4.71
SPT	-5.8423	-10.81	0
EDD	0.2503	-4.71	5.22

Table 3: 95% CI for $\mu_i - \min_{j \neq i} \mu_j$

i	$\max_{j \neq i} \bar{d}_{ij}$	lower bound	upper bound
FIFO	5.592	0	10.56
SPT	-5.592	-10.56	0
EDD	5.8423	0	10.81

From the table 2 we declare SPT as not the largest, but we cannot state which of the other two is the largest. From the table 3 above we may declare SPT to be the smallest.

7 SUMMARY

Six procedures were listed starting with a two-stage form of the Student's t procedure, and concluding with a sphericity procedure. The procedures interest simulators because they generalize and improve basic interval procedures, and are accessible to many users. The improvements for the new procedures include narrower confidence intervals, fewer stages, exploitation of common random numbers, robustness to unequal variances, and the extension to k systems.

APPENDIX

Table 4: k System Paired Procedures $\alpha = 0.05$

k	2	3	4	5	6	7
F(t)	.95	.975	.983	.988	.99	.992
ν						
1	6.31	12.7	19.1	25.5	31.8	38.2
2	2.92	4.30	5.34	6.21	6.96	7.65
3	2.35	3.18	3.74	4.18	4.54	4.86
4	2.13	2.78	3.19	3.50	3.75	3.96
5	2.02	2.57	2.91	3.16	3.36	3.53
6	1.94	2.45	2.75	2.97	3.14	3.29
7	1.89	2.36	2.64	2.84	3.00	3.13
8	1.86	2.31	2.57	2.75	2.90	3.02
9	1.83	2.26	2.51	2.69	2.82	2.93
10	1.81	2.23	2.47	2.63	2.76	2.87
11	1.80	2.20	2.43	2.59	2.72	2.82
12	1.78	2.18	2.40	2.56	2.68	2.78
13	1.77	2.16	2.38	2.53	2.65	2.75
14	1.76	2.14	2.36	2.51	2.62	2.72
15	1.75	2.13	2.34	2.49	2.60	2.69
16	1.75	2.12	2.33	2.47	2.58	2.67
17	1.74	2.11	2.32	2.46	2.57	2.65
18	1.73	2.10	2.30	2.45	2.55	2.64
19	1.73	2.09	2.29	2.43	2.54	2.63
20	1.72	2.09	2.29	2.42	2.53	2.61
25	1.71	2.06	2.25	2.38	2.49	2.57
30	1.70	2.04	2.23	2.36	2.46	2.54
60	1.67	2.00	2.18	2.30	2.39	2.46
120	1.66	1.98	2.15	2.27	2.36	2.43
∞	1.64	1.96	2.13	2.24	2.33	2.39

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