*Proceedings of the 1994 Winter Simulation Conference*
*ed. J. D. Tew, S. Manivannan, D. A. Sadowski, and A. F. Seila*

# FITTING A MIXTURE-BASED RESPONSE SURFACE USING COMPUTER SIMULATION

Lotfi K. Gaafar and Tariq A. Aldowaisan

Mechanical and Industrial Engineering Department
College of Engineering and Petroleum
Kuwait University
P. O. Box 5969, Safat 13060, Kuwait

## ABSTRACT

In this paper, an augmented simplex-lattice mixture design is used to fit a mathematical metamodel for predicting work in process in a flexible manufacturing cell. The average work in process is modeled as a response variable by varying the incoming percentages of different products. Computer simulation is utilized to run different experiments to obtain the required data for fitting the metamodel. The accuracy of the metamodel predictions warrant its use in place of the simulation model, thus eliminating the need for lengthy simulation runs and complicated output analysis. This work provides an example of the utility of mixture experiments in a discrete manufacturing environment.

## 1 INTRODUCTION

A flexible manufacturing cell (FMC) consists of a group of machines that are arranged to produce a family of similar products in small batches. These machines are typically linked through an automated material handling system, and controlled via a central computer.

Computer simulation has been extensively used to model different aspects of FMC production to support decision making at the planning and controlling levels (for examples see Mannivannan and Banks 1991 and Jain et al. 1990). However, applying computer simulation at the operational level has received less attention in the literature. This might be due to the fact that lengthy computer simulation runs and complicated output analysis are usually not suitable for the real-time requirement of operational decisions. A common approach to address the real-time requirement is to fit a mathematical model (metamodel) to simulation results, and then utilize the metamodel for operational decisions. The response surface methodology (as described by Box and Draper 1987) is widely utilized to develop the metamodel.

In this paper, the relationship between the average work in process (WIP) and production mix in a FMC is investigated. The response (WIP) is assumed to be a function of the proportions (which add up to 1) of the different product types entering the cell. This formulation falls in a special class of experimental designs known as mixture designs. These designs are briefly introduced in Section 2. Section 3 describes the manufacturing system and defines the research problem. The approach used to solve this problem is introduced in Section 4. Section 5 presents the results and addresses model validation. Finally, Section 6 summarizes the research findings and presents the final conclusions.

## 2 MIXTURE EXPERIMENTS

As Cornell (1990) defines it, a mixture experiment is an experiment in which a response of interest is assumed to depend only on the relative proportions of the ingredients of a mixture, but not the amount of the mixture itself. In mixture experiments, factors of interest are ingredients of a mixture; so if $x_i$ denotes the proportion of the $i^{th}$ ingredient in the mixture, then:

$$\sum_{i=1}^{q} x_i = 1,$$ where $q$ is the number of ingredients in the mixture (1)

In general, polynomial functions are used to approximate the mathematical relationship (or response surface) between the response η and the mixture ingredients. The first-degree polynomial

$$\eta = \sum_{i=1}^{q} \beta_i x_i \qquad (2)$$

or the second-degree polynomial

$$\eta = \sum_{i=1}^{q} \beta_i x_i + \sum_{i<j}^{q}\sum \beta_{ij} x_i x_j \qquad (3)$$

are usually sufficient to model the response surface (Cornell 1990). These polynomials are similar to those used in regular factorial designs except for the intercept term which is dropped to address the linear dependency introduced by the constraint in Eq. (1). Usually, the *method of least squares* is used to fit the parameters of Eq. (2 and 3) after collecting data from a designed experiment. The designed experiment will consist of a number of runs performed at different combinations of the mixture's ingredients. These combinations are selected within the possible factor space which, based on Eq. (1), is a $(q-1)$-dimension simplex. With three components ($q = 3$), the factor space is an equilateral triangle (see Figure 1). The following discussion will focus on the three-component case.

Several designs have been devised for experimenting with mixtures. In this paper, we will utilize a class of designs known as the simplex-lattice designs. These designs spread the experimentation points evenly over the whole simplex factor space. Figure 1 shows the simplex-lattice arrangement for fitting a second-degree polynomial.

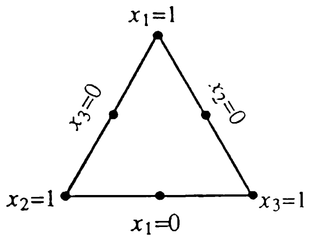


Figure 1. Three-component simplex region with simplex-lattice design points.

The arrangement in Figure 1 only provides the minimum number of points to estimate the terms of the second-degree response surface and all experimentation points are on the peripheral of the factor space. To alleviate these problems, the design in Figure 1 is augmented with points inside the simplex including the overall center. The resulting design is called the augmented simplex-lattice design and is shown in Figure 2.

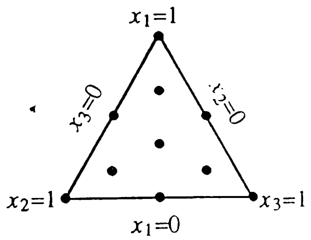


Figure 2. An augmented simplex-lattice design.

The simplex coordinates of the 10 design points, in Figure 2, are (1, 0, 0), (0, 1, 0), (0, 0, 1),(1/2, 1/2, 0), (1/2, 0, 1/2), (0, 1/2, 1/2), (1/3, 1/3, 1/3), (4/6, 1/6, 1/6), (1/6, 4/6, 1/6), and (1/6, 1/6, 4/6). Of course, these coordinates correspond to the proportions of each ingredient in the mixture. This design will be utilized in Section 4 to investigate the relationship between the production mix and the average work in process in a FMC.

## 3 SYSTEM DESCRIPTION

The system investigated in this paper is a FMC consisting of three different machining centers linked with an automated guided vehicle (AGV). The FMC is part of a large factory that produces a variety of parts. The majority of these parts pass through the cell as part of their production process. However, based on their production steps, parts machined in the FMC are grouped into three families, and an aggregate part will represent each family in this paper. Table 1 summarizes the routings and machining times for the three product types. This information and other data throughout the paper have been modified for confidentiality considerations. Parts arrive to the FMC on pallets which carry 4 parts at a time according to an exponential distribution with a mean of 20 minutes. Parts on the same pallet travel together throughout the FMC.

A study was conducted to investigate the relationship between the percentages of arriving parts and many responses. This paper will focus on the average work in process as the response of interest. The goal is to develop a mathematical model to allow management to predict the average WIP for any product mix. The following section describes how this task was accomplished.

## 4 RESEARCH APPROACH

As indicated in Section 1, an augmented simplex-lattice mixture experiment was utilized to investigate the relationship between the average WIP and the product mix. This design was described in Section 2. However, to allow for error estimation, all peripheral points of the simplex in Figure 2 were replicated twice. This resulted in a 16-run design. Computer simulation was utilized to run the different experiments and obtain the desired results. A simulation model for the system, as described in Section 3, was developed using the SIMAN simulation language (Pegden et al. 1991). The following section shows the results of the simulation runs and the fitted model.

| Product Type | Sequence | Processing time on each machine (minutes) | | |
|---|---|---|---|---|
| | | Machine Center 1 (M1), 2 machines | Machine Center 2 (M2), 1 machine | Machine Center 3 (M3), 1 machine |
| 1 | M3, M1, M2 | TR (8,10,12) | UN (12, 14) | NR (18, 2) |
| 2 | M1, M3, M1, M2 | NR (17, 2)/(18, 2) | TR (8, 10, 11) | NR (10, 1) |
| 3 | M2, M1, M3 | UN (11, 12) | NR (17, 2) | UN (6, 8) |

TR = Triangular UN = Uniform NR = Normal

Table 1. Routings and machining times for the three product types.

## 5 RESULTS AND ANALYSIS

As indicated in Section 4, 16 simulation experiments were run at the points indicated in Figure 2 with peripheral points replicated. Table 2 summarizes the results of these 16 simulation runs (Table 2 includes other information that will be covered later). A second-order model was fitted to the data using DESIGN-EXPERT (Stat-Ease 1992), a statistical package for fitting response surfaces. The fitted model is:

$y = 15.24x_1 + 17.89x_2 + 12.79x_3 - 43.86x_1x_2 - 27.86x_1x_3 - 39.15\,x_2x_3$, where $y$ is the predicted response. (4)

Eq. (4) shows that type 2 products ($x_2$) have the biggest effect on the response since pure and quadratic coefficients of terms involving $x_2$ are the largest. This is consistent with the modeled system since type 2 products need four operations and require the longest production time. It is important to mention that the maximum effect of a quadratic coefficient is only 25% of its value. This is because the maximum value of the term $x_ix_j$ is (1/4). Therefore, even though all quadratic coefficients in Eq. (4) are larger than the largest pure coefficient, all pure effects are still larger than the quadratic ones. According to Eq. (4), the maximum quadratic effect is about 11 (43.86/4), while the minimum pure effect is 12.79.

Table 2 includes many parameters that are automatically calculated by DESIGN-EXPERT and are used to check the statistical validity of the model. Residuals are the differences between the actual observations and the corresponding predicted values. A standardized residuals is the residual divided by the estimated standard deviation of the residuals. A standardized residual far outside the range (-2, +2) usually indicates an outlier (Hines and Montgomery 1990). As Table 2 shows, none of the standardized residuals is outside this range. Cook's distance is a measure of how much the regression equation would change if the corresponding point is deleted. Large values indicate an inadequate model. Such is not the case in Table 2. Finally, the outlier t is a t-statistic which is calculated by leaving the corresponding observation out and predicting its value from the remaining observations. It is a measure of how consistent is the corresponding observation with the other observations. A t value above 3.5 should be considered an outlier. No such value appears in Table 2. Therefore, all statistical tests show no reason to suspect the statistical validity of the model. In addition, a normal probability plot of the residuals (not shown here because of space limitations) showed no sign of any obvious problem.

Figure 3a shows a three dimensional plot of the response surface as modeled by Eq. (3). The plot shows that WIP decreases as we move toward a more homogeneous mixture. The contour plot in Figure 3b further confirms this observation. This adds to the validity of our metamodel since such a performance is expected. As we move toward pure mixtures (where only one product type is included), demand on particular machines become more concentrated and WIP accumulates. It is worth mentioning that the real system is a constrained mixture where all product proportions must be greater than zero and less than one (i.e. pure and binary blends are not allowed). This results in an irregular factor space that could not be addressed with simplex-lattice designs. A D-optimal design was used to experiment with the constrained system. This paper reports on the initial studies of the system in which the production constraints were not considered in order to simplify the presentation.

| Mixture | Actual resp. ($\eta$) | Predicted value ($y$) | Residual | Standard. residual | Cook's Distance | Outlier t |
|---|---|---|---|---|---|---|
| 1, 0, 0 | 16 | 15.24 | 0.76 | 0.996 | 0.153 | 0.995 |
| 1, 0, 0 | 14.5 | 15.24 | -0.74 | -0.958 | 0.142 | -0.954 |
| 1/2, 1/2, 0 | 5.2 | 5.6 | -0.4 | -0.484 | 0.027 | -0.464 |
| 1/2, 1/2, 0 | 5.7 | 5.6 | 0.1 | 0.127 | 0.002 | 0.121 |
| 1/2, 0, 1/2 | 6.4 | 7.05 | -0.65 | -0.792 | 0.073 | -0.776 |
| 1/2, 0, 1/2 | 6.8 | 7.05 | -0.25 | -0.304 | 0.011 | -0.289 |
| 0, 1, 0 | 17.1 | 17.89 | -0.79 | -1.027 | 0.163 | -1.03 |
| 0, 1, 0 | 19.2 | 17.89 | 1.31 | 1.709 | 0.451 | 1.927 |
| 0, 1/2, 1/2 | 5.6 | 5.55 | 0.05 | 0.06 | 0 | 0.057 |
| 0, 1/2, 1/2 | 5.1 | 5.55 | -0.45 | -0.551 | 0.035 | -0.531 |
| 0, 0, 1 | 14 | 12.79 | 1.21 | 1.575 | 0.383 | 1.724 |
| 0, 0, 1 | 11.5 | 12.79 | -1.29 | -1.681 | 0.436 | -1.883 |
| 4/6, 1/6, 1/6 | 6.5 | 6.21 | 0.29 | 0.293 | 0.003 | 0.28 |
| 1/6, 4/6, 1/6 | 5.4 | 6.6 | -1.2 | -1.227 | 0.048 | -1.263 |
| 1/6, 1/6, 4/6 | 6 | 5.38 | 0.62 | 0.631 | 0.013 | 0.611 |
| 1/3, 1/3, 1/3 | 4.4 | 2.99 | 1.41 | 1.464 | 0.077 | 1.567 |

Table 2. Simulation results and statistical analysis.

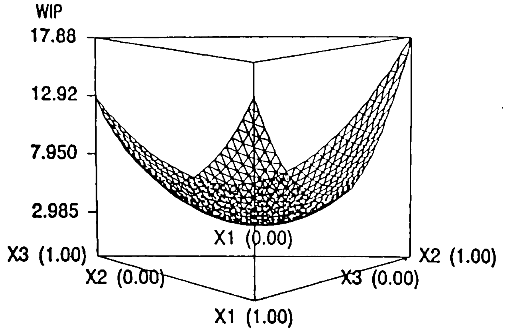


Figure 3a. Three-dimensional plot of WIP.

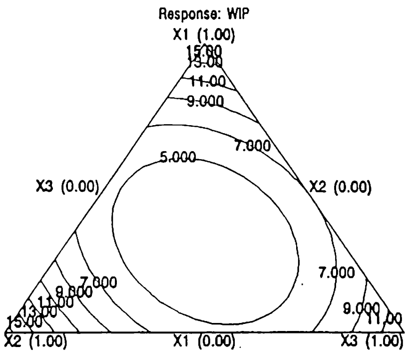


Figure 3b. Contours plot of WIP.

Figure 3. Graphical representation of WIP as a function of the product mix.

The graphical displays in Figure 3 offer a great help in understanding the system's dynamics and can be utilized for predictions, where applicable, in place of Eq. (4).

## 6 CONCLUSIONS

An augmented simplex-lattice mixture design was utilized to fit a mathematical model between ingredients of a product mix and average WIP. The required data was obtained using a simulation model of the system. With a constrained mixture, the accuracy of the metamodel increases since it covers a smaller space. As Figure 3 showed, graphical displays could be used to understand the system behavior, as well as, predict responses. It is important to mention that in our simulations, the assumption of a "pure" mixture experiment were slightly violated since the total number of parts produced during simulation runs were slightly different. When the total amount of the ingredients vary, the experiment is known as a mixture-amount experiment. Nevertheless, the changes in the product amounts were very small and their effect was neglected.

## REFERENCES

Box, G. E. and Draper, N. R. 1987. *Empirical model-building and response surfaces*. John Wiley & Sons, Inc.

Cornell, J. A. 1990. *Experiments with mixtures, 2nd. Edition.* John Wiley & Sons, Inc.

Hines , W. W. and Montgomery, D. C. 1990. *Probability and Statistics for Engineering and Management Science. 3rd. Edition.* John Wiley & Sons, Inc.

Jain, S., Barber, K., and Osterfeld, D. 1990. "Expert Simulation for On-Line Scheduling." *Communications of the ACM*, Vol. 33, No. 10.

Mannivannan, S. and Banks, J. 1991. "Real-Time Control of a Manufacturing Cell Using Knowledge-Based Simulation." *Proceedings of the 1991 Winter Simulation Conference.* pp. 251-260, Phoenix, AZ.

Montgomery, D. C. 1991. *Design and analysis of experiments, 3rd. Edition.* John Wiley & Sons, Inc.

Pegden, C. D., Shannon, R. E., and Sadowski, R. P. 1990. Introduction to simulation using SIMAN. McGraw-Hill, Inc.

Stat-Ease 1992. *DESIGN-EXPERT Version 3.0 User's Guide.* Stat-Ease, Inc. 2021 E. Hennepin, #191, Minneapolis, MN 55413.

## AUTHORS BIOGRAPHIES

**LOTFI K. GAAFAR** is an Assistant Professor of Industrial Engineering at Kuwait University. He received the MSIE and Ph.D. degrees from Arizona State University. He has an active research program in computer simulation which includes aspects of real-time simulation, the use of experimental design concepts in simulation, and developing intelligent simulation environments. He is a senior member of SCS, IIE, and SME.

**TARIQ A. ALDOWAISAN** is an Assistant Professor of Industrial Engineering at Kuwait University. He received his B.S., M.S., and Ph.D. degrees from Arizona State University. His current interests are in the areas of design of experiments, quality engineering, simulation, and facility planning. He is a senior member of SCS and IIE.