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A NEW OUTPUT ANALYSIS APPROACH FOR SYSTEMS EXPERIENCING DISRUPTIONS

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ABSTRACT

The authors present a new method of performing simulation output analysis on systems experiencing random disruptions. The new method, alternating regenerative, outperforms the batch means method in coverage. The basis for the new method lies in the variance reduction technique, the cyclic regenerative method of simulation.

1 INTRODUCTION

The alternating regenerative method is presented as a simulation output analysis technique for analyzing simulation models that incorporate random disruptions. The foundation for the technique lies in a variance reduction technique, the cyclic regenerative method of simulation (Sargent and Shanthikumar 1982). The cyclic regenerative method of simulation takes advantage of regenerative processes exhibiting more than one regeneration point. By ignoring *weak regeneration points* and utilizing only the *strong regeneration points*, Sargent and Shanthikumar (1982) showed that cyclic regeneration reduced the variance of the estimate over the classical regenerative method of simulation. Glynn (1983) proved that the cyclic regenerative confidence intervals are asymptotically valid and identified a sufficient condition under which the variance reduction technique produces a variance lower than that of the classical regenerative method of simulation.

The development of output analysis techniques for analyzing systems subjected to random disruptions is necessary. Closed form solutions exist only for single-server queueing systems experiencing one type of exponentially distributed disruption (Sengupta 1990). It is important to note that the disruptions do not have to be limited to breakdowns, they can represent any type of interruption to the system, such as scheduling changes, the arrival of a new class of customers, etc. Since most systems do operate with some type of disruption, whether planned or unplanned, a new method will prove to be a valuable tool in practice.

The alternating regenerative method is compared to the batch means method. The comparison is based on coverage tests performed on various simulated queueing systems where the theoretical mean time in queue exists.

The authors assume that the reader is already familiar with the batch means method and the classical regenerative method of simulation. For a detailed description of the methods, the reader is referred to Crane and Iglehart (1973), Crane and Lemoine (1977), and Law and Kelton (1990).

The alternating regenerative method is presented in Section 2. The queueing system used to compare the alternating regenerative and batch means method is presented in Section 3. The results of the coverage tests are in Section 4. Our conclusions and directions for future research are presented in Section 5.

2 ALTERNATING REGENERATIVE METHOD

The graph in Figure 1, displays the effect on the output response when introducing a random disruption to a simulation model. The system is a M/M/1 queue subjected to an exponentially distributed disruption.

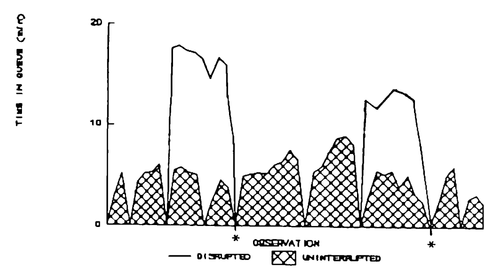

Figure 1: Weak versus Strong Regeneration

The *uninterrupted cycles* are produced when the server is operating, and the *disrupted cycles* are produced when the server is in repair. The renewal epoch of the random disruption and the repair time distribution, determine the distribution of the time the system is *on*. Thus, the system oscillates between two states *on* and *off* (Cox 1962). As a result of this oscillation, two types of regeneration points are produced: *weak regeneration points* and *strong regeneration points* (Sargent and Shanthikumar 1982). A strong regeneration point is when the system returns to the empty and idle state, after a disruption has occurred. Figure 1 depicts the strong regeneration points by asterisks (*).

The *alternating regenerative method* makes use of the distinction between the two types of regeneration points, where an *alternating regenerative cycle* is defined only by strong regeneration points (Court 1993). In contrast, the classical regenerative method does not distinguish between the two types of regeneration points. The net effect of ignoring the weak regeneration points, is a reduction in the variance of the point estimate (Sargent and Shanthikumar 1982).

Intuitively, the alternating regenerative cycles are independent by construction and provide a more homogeneous grouping of the output data. When employing a stopping rule based on the total number of disrupted cycles, the classical and alternating regenerative methods' estimate of the mean is the same. Thus, the new method does not introduce any more bias in its estimate of the mean than the classical regenerative method (Court 1993).

3 THEORETICAL MODEL

Performance of the alternating regenerative and batch means methods was investigated by conducting coverage tests, using an M/M/1 queue subjected to exponentially distributed breakdowns and repair times. The queueing discipline is pre-preemptive resume with (i) breakdowns being allowed to occur while a customer is in service, (ii) interrupted customers continuing with their service when the server is next available, (iii) arrivals to the system during a breakdown, continuing with their queueing discipline of first-come-first-serve, and, (iv) no balking. The parameter of interest is the mean time in queue.

The single-server queue operates in a random environment defined by the state of the system: State 1, when the server is working, and State 2, when the server is in repair. The distribution, $F_i(t)$ for i=1,2, of the time spent in both states is exponential, where $F_2(t)$ is the exponential distribution of the repair time with first and second moments, $f_2^{(1)}$ and $f_2^{(2)}$, respectively; and $F_1(t)$ is the exponential distribution of the busy period with first and second moments, $f_1^{(1)}$ and $f_1^{(2)}$, respectively.

The arrival rates and service time distributions are the same for both states. The arrivals occur according to a Poisson process at a mean rate of $\lambda = \lambda_1 = \lambda_2$. The service time distribution is exponential $B_i(t)$ for i=1,2, with first and second moments, $b^{(1)}=b_1^{(1)}=b_2^{(1)}$ and $b^{(2)}=b_1^{(2)}=b_2^{(2)}$, respectively.

When the renewal epoch, $F_1(t)$, follows an exponential distribution, a closed form solution is available for the mean time in queue (Sengupta 1990). With μ denoting the mean of the renewal epoch, the theoretical mean time in queue is calculated as follows:

$$E(\overline{Wq}) = \frac{\bar{\lambda}\left\{\left[b^{(2)}/(c_1)^2\right] + \mu b^{(1)} f_2^{(2)}\right\}}{2\left[1-(\bar{\lambda} b^{(1)}/c_1)\right]} + \frac{c^2 f_2^{(2)}}{f_2^{(1)}} \quad (1)$$

where,

$$c_1 = \frac{f_1}{f_1^{(1)} + f_2^{(1)}} \quad \text{and } c_2 = 1 - c_1$$

4 COVERAGE TEST RESULTS

Coverage tests were performed on the queueing system described above. The goal was to obtain 90% coverage while varying (i) the mean of the renewal epoch, μ, and (ii) the mean time spent in repair, $f_2^{(1)}$. The arrival rate, $\bar{\lambda} = 1$, remained constant, as well as the mean service time, $b^{(1)}=0.75$.

Table 1 contains the 90% coverage results for the methods. Each test was performed on 100 simulations of the model, via the SIMAN simulation language, (Pegden 1990), with a stopping rule of 200 alternating regenerative cycles. Columns 12-13 contain the average batch size, $\bar{l}$, and the average number of batches, $\bar{m}$, respectively. Columns 4-5 contain the coverage and precision of the alternating regenerative and batch means methods, respectively. Precision is based on $\hat{p}$, the proportion of method's confidence intervals covering the true mean waiting time, (Column 3, calculated via Equation 1). The parameters for the model were chosen to yield catastrophic failures, i.e., the probability of a breakdown occurring is small, (Column 1), but when a breakdown does occur the repair time, (Column 2), is over twenty times the service time. For all but one case, (Case 11), the alternating regenerative method's coverage was closer to the claimed statement of confidence than the batch means method.

Table 1: Alternating Regenerative Method (AR) versus Batch Means Method (BM)

MODEL PARAMETERS			90% COVERAGE +/- PRECISION $\hat{p} \pm z_{0.90}\sqrt{\frac{\hat{p}(1-\hat{p})}{100}}$		AVE. HALF-WIDTH		AVE. MEAN		AVE %BIAS		BM	
μ	$f_2(1)$	$\overline{W}_{q_{theor}}$	AR	BM	AR	BM	AR	BM	AR	BM	$\bar{l}$	$\overline{m}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
$\frac{1}{720}$	15	3.81	.910+/- .047	.920+/- .045	0.43	0.41	3.81	3.81	0.00	0.00	692	254
$\frac{1}{720}$	30	8.26	.870+/- .055	.840+/- .060	1.70	1.62	8.44	8.43	2.18	2.06	1272	166
$\frac{1}{720}$	45	16.16	.900+/- .049	.850+/- .059	3.89	3.62	16.29	16.31	0.80	0.93	1652	131
$\frac{1}{960}$	15	3.40	.890+/- .051	.850+/- .059	0.33	0.32	3.43	3.43	0.88	0.88	800	297
$\frac{1}{960}$	30	6.65	.900+/- .049	.880+/- .053	1.22	1.17	6.75	6.74	1.50	1.35	1352	183
$\frac{1}{960}$	45	12.25	.860+/- .057	.860+/- .057	2.78	2.60	12.43	12.36	1.47	0.90	1792	152
$\frac{1}{1200}$	15	3.17	.900+/- .049	.860+/- .057	0.27	0.27	3.19	3.19	0.63	0.63	2080	282
$\frac{1}{1200}$	30	5.72	.890+/- .051	.860+/- .057	0.96	0.92	5.80	5.78	1.40	1.05	1925	193
$\frac{1}{1200}$	45	10.06	.920+/- .045	.840+/- .060	2.14	2.00	10.21	10.21	0.39	0.39	2248	131
$\frac{1}{2400}$	30	3.94	.890+/- .051	.890+/- .051	0.47	0.46	3.97	3.96	0.76	0.51	1715	197
$\frac{1}{2400}$	45	5.98	.920+/- .045	.910+/- .047	1.03	1.01	6.08	6.07	1.67	1.51	3024	277
$\frac{1}{2400}$	60	8.88	.880+/- .053	.840+/- .060	1.85	1.76	9.06	9.04	2.03	1.80	3040	251

The bias of the batch means method in estimating the variance is the reason for the inability of that method to the match the coverage of the alternating regenerative method. This is evident when examining the average mean and the average half-width of the confidence intervals, (Columns 6-9). For all but one case, (Case 3), the batch means method produces a point estimate for the mean time in queue with less bias than that of the alternating regenerative method. Bias is calculated as the ratio of difference between the average mean and the theoretical mean, to the theoretical mean. Thus, the shorter half-widths of the batch means method's confidence intervals, indicate bias in its variance estimate.

5 CONCLUSIONS AND FUTURE RESEARCH

This paper presents an extensive study of the alternating regenerative and the batch means methods' performance, in the context of coverage. In particular, when analyzing output data generated by single-server queues subjected to random disruptions.

In addition to achieving closer coverage to the claimed statement of confidence, employing the alternating regenerative method is computationally more efficient and easier than the batch means method. The batch means method requires analysis for (i) steady-state behavior, (ii) the batch size, and (iii) testing the independence of the batch means. Thus, this method is not a *true* single replication method, in the sense that (i) the steps for its analysis are reiterative, and (ii) depending on the method used for determining steady-state behavior, may require more than one run of the simulation model.

The alternating regenerative methods does not require analysis for steady state, data collection can begin immediately, and no manipulation of the data is required. However, as with the classical regenerative method, the alternating regenerative method requires the frequent return to the empty and idle state, but only after a disruption has occurred. For many systems the time between the occurrences of the empty and idle state is quite long. Future research will be directed at developing an approach for (i) analyzing a more complex simulation model, and (ii) when the number of alternating regenerative points are too few for efficient parameter estimation.

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