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AN ORDINAL OPTIMIZATION APPROACH TO A TOKEN PARTITION PROBLEM FOR STOCHASTIC TIMED EVENT GRAPHS

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ABSTRACT

The paper addresses the optimal partition of tokens in stochastic timed event graphs. The transition firing times are random variables with general distribution. The problem consists in choosing an initial marking among a large set of candidate initial markings such that a criterion function (linear or nonlinear) of the initial marking and the average cycle time is minimized. We propose a simulation-based ordinal optimization algorithm for solving this problem. The algorithm simultaneously simulates a set of event graphs, each has a candidate initial marking as its initial marking. The most important feature of this approach is the capability of identifying with very short simulation run the candidate initial markings which can hardly be optimal solutions. Significant computation time saving is realized by stopping the simulation of the related nets at appropriate time.

1. INTRODUCTION

Optimization in discrete solution space becomes more and more important for discrete event dynamic systems. There are numerous potential applications such as production capacity and buffer capacity dimensioning in manufacturing systems. The only general enough tool for evaluating such systems is the simulation. Due to the lack of viable optimization approaches, empirical and sometime blind solution search approaches are used.

The purpose of this paper is to propose an ordinal optimization approach to a token partition problem for stochastic timed event graphs. In such a system, the average cycle time (1/throughput rate) is function of the token partition. The problem consists in choosing one among a set of candidate initial markings in order to minimize a criterion which is a (linear or non-linear) function of the initial marking and the average cycle time. A new ordinal optimization approach is proposed. This approach simultaneously simulates different candidate initial markings. Distributions of their average cycle times are estimated along the simulation and are used to evaluate the probability for a candidate initial marking to be optimal. The candidate initial markings whose probability to be optimal are low are discarded at appropriate points of the simulation.

This approach is applied to the kanban distribution determination for a four-machines serial systems. Optimal solutions are obtained in all test examples which contain 36 candidate initial markings.

Closely related to this work are works on the marking optimization problems for deterministic event graphs (Laftit, Proth, Xie 1992) and stochastic timed event graphs (Proth, Sauer, Wardi, and Xie 1993). The marking optimization problem consists in finding an initial marking such that a given cycle time is obtained while a linear p-invariant criterion is minimized. This problem can be considered as a special case of the new token partition problem.

This paper is organized as follows. Section 2 contains notations about the stochastic timed event graphs and some preliminary results. Section 3 defines the new token partition problem and proposes a parallel simulation algorithm. Based on this algorithm, Section 4 proposes an ordinal optimization approach. Section 5 applies the ordinal optimization approach to the optimal kanban distribution determination of a four-machines serial systems. Section 6 is a conclusion.

2. STOCHASTIC TIMED EVENT GRAPHS

Let $N = (\mathcal{P}, T, F)$ be the strongly connected event graph considered. $\mathcal{P}$ is the set of places, T is the set of transitions, and $F \subseteq (\mathcal{P} \times T) \cup (T \times \mathcal{P})$ is the set of directed arcs connecting places to transitions and transitions to places. We denote by $\mathcal{M}$ the initial marking of N.

We assume that no transition can be fired by more than one token at any time. This implies that there is a self loop place with one token related to each transition, i.e. $(t, t) \in \mathcal{P}$ and $\mathcal{M}((t,t)) = 1$, $\forall\, t \in T$ where (t, s) indicates the place connecting transition t to transition s. As a result, the set of places $\mathcal{P}$ can be written as $\mathcal{P} = P \cup Pt$ where Pt denotes the set of self loop places and P the other places. Furthermore, since $\mathcal{M}((t,t)) = 1$, $\forall\, t \in T$, only the marking of the places belonging to P will be considered.

Since N is an event graph, each place has exactly one input transition and one output transition. Without loss

of generality, we assume that there exists at most one place between any two transitions. The following notations will be used :

in(t): set of transitions which immediately precede transition t, i.e. in(t) = $^\bullet(^\bullet t)$
(t, s): place connecting transition t to s
M: initial marking of the places in P
$X_t(k)$: time required for the k-th firing of t
$S_t(k)$: starting time of the k-th firing of t
$D_t(k)$: termination time of the k-th firing of t

By convention, $X_t(k) = 0, \forall\, k \le 0$ and $S_t(k) = 0, \forall\, k \le 0$. As shown in Chretienne (1983), $S_t(k)$ can be determined by the following recursive equations:

$$S_t(k) = \max_{\tau \in in(t)} \left\{ D_\tau \left(k - \mathcal{M}((\tau, t)) \right) \right\} \qquad (1)$$

with

$$D_t(k) = S_t(k) + X_t(k), \quad \forall t \in T, \forall k \ge 0$$

We assume that $\{X_t(k)\}_{k=1}^{\infty}$ for all $t \in T$ are mutually independent sequences of i.i.d. integrable r.v.s. It was proven by Baccelli (1992) that:

$$\lim_{n\to\infty} \frac{S_t(n)}{n} = \lim_{n\to\infty} \frac{E[S_t(n)]}{n} = \pi(M), \quad \text{a.s.} \qquad (2)$$

for all transition t where $\pi(M)$ is the average cycle time of the event graph.

Since any stochastic timed event graph is completely characterized by its net structure, its initial marking and the firing time sequences, it can be denoted by the triplet SPN = (N, M, $\{X_t(k)\}$).

Let us consider now the simulation of a stochastic timed event graph. Thanks to the ergodicity relation (2), we can use the evolution equation (1) to simulate a stochastic timed event graph instead of the classical discrete event simulation technique. This approach turns out to be more efficient than the discrete simulation technique.

ALGORITHM A1 (Simulation of an event graph)
1. Choose a simulation cycle K.
2. Compute a sequence of transitions $(\sigma_1, \ldots, \sigma_{|T|})$ firable from the initial marking M in which each transition appears exactly once.
3. Set $S_t(k) := 0$, for all $t \in T$ and for all $k \le 0$.
4. For k = 1 to K do
   4.1. Generate the r.v.s $X_t(k)$ for all $t \in T$
   4.2. For $t = \sigma_1$ to $\sigma_{|T|}$, compute $S_t(k)$ using equation (1)
5. $\pi(M_0) = S_{\sigma 1}(K)/K$

The existence of the sequence $(\sigma_1, \ldots, \sigma_{|T|})$ was proved by Commoner, Holt, Even, and Pnueli (1971). Since $(\sigma_1, \ldots, \sigma_{|T|})$ is firable from M, the k-th firing of any transition $t = \sigma_i$ is independent of the k-th firings of the transitions $\sigma_{i+1}, \ldots, \sigma_{|T|}$. This guarantees the computability of $S_t(k)$ at step 4.2.

3. PROBLEM SETTING AND A PARALLEL SIMULATION APPROACH

The token partition problem consists in choosing an initial marking among a set of candidate initial markings in order to minimize a criterion function of the average cycle time and the initial marking. More precisely,

$$\min_{M \in Q} \left\{ F(M, \pi(M)) \right\} \qquad (3)$$

where $Q = \{M_1, M_2, \ldots, M_n\}$ is a set of n candidate initial markings and $F(M,\pi(M))$ is the criterion function.

Let us consider the motivations of this token partition problem. When designing a system, a very popular approach consists in first elaborate a set of alternative solutions under some technical and economical considerations, then quantify the performance of these solutions and finally choose the one with the best performance.

Our token partition problem applies to the resources dimensioning of a manufacturing system. In the stochastic timed event graph model of such a system, a potential solution can be represented by the initial marking. The set of the alternative solutions corresponds to the set Q. The value $1/\pi(M)$ can be considered as the throughput value of the related system or productivity rate when manufacturing is concerned. The tokens appeared in the net usually model the resources employed. The criterion function $F(M,\pi(M))$ quantifies the balance between the total cost of the resources employed and the average cycle time obtained. The optimal token partition allows to achieve a good balance between two contradictory factors.

For problem (3), the lack of closed-form solutions of the average cycle time leaves the simulation the only acceptable tool. A natural approach consists in repeating algorithm A1 for each element of the candidate solution set Q to obtain an estimate of its average cycle time and then choosing the one which minimizes the criterion function.

The major drawback of this approach is clearly the computation burden as the set of candidate initial markings is usually large.

To reduce the computational burden, we first propose a parallel simulation algorithm A2 which takes advantage of the structure of the event graphs. In this algorithm, SPN_i denotes the stochastic timed event graph $(N, M_i, \{X_t(k)\})$. $S_{i,t}(k)$ is the starting time of the

k-th firing of transition t of SPN_i. $\pi(M_i)$ is its average cycle time.

Algorithm A2 (Parallel simulation of n event graphs)
1. Choose a simulation cycle K.
2. For each net SPN_i, compute a sequence of transitions $(\sigma_{i,1}, \ldots, \sigma_{i,|T|})$ firable from the initial marking M_i in which each transition appears exactly once.
3. Set $S_{i,t}(k) := 0$, for all $t \in T$, $k \leq 0$ and $1 \leq i \leq n$.
4. For k = 1 to K do
   4.1. Generate the r.v.s $X_t(k)$ for all $t \in T$
   4.2. For i = 1 to n and for $t = \sigma_{i,1}$ to $\sigma_{i,|T|}$, compute $S_{i,t}(k)$ using equation (1)
5. For i = 1 to n do
   5.1. Compute $\pi(M_i) = S_{i,t^*}(K)/K$ where t* is a given transition
   5.2. Compute $F(M_i, \pi(M_i))$
6. Determine the optimal solution M*.

As it can be remarked, the different SPNs share the same random variable generation in the parallel simulation approach while each SPN needs its own random variable generation in the traditional approach. Clearly, this advantage disappears if each SPN uses the same pre-generated firing times in traditional approach. However, we show in the next section that the computational burden can be significantly reduced by combining this parallel simulation algorithm and an ordinal optimization approach.

4. AN ORDINAL OPTIMIZATION APPROACH

4.1. Introduction to Ordinal Optimization

Let us come back to the algorithm A2. The major drawback of algorithm A2 is an implicit self-imposed requirement of accurate enough estimation of the criterion value for all candidate initial markings. As a result, long simulation run (i.e. large K) is necessary for the convergence of criterion value estimates. The ordinal optimization approaches, first proposed by Ho Sreenivas, and Vakili (1992), reduce the computation burden by appropriately relaxing this requirement.

The primary concern of the ordinal optimization is the rankings of the candidate solutions instead of their exact criterion values. Numerous simulations conducted by different authors for a wide range of problems have shown that the candidate solution rankings stabilize before the convergence of the criterion value estimates.

Ordinal optimization approaches generally simulate simultaneously different candidate solutions using common random variable generation. It has been shown by Deng, Ho, and Hu (1992) that the resulted correlations between the criterion value estimation errors can only help and increase the chance of identifying good solutions very early in the simulation.

To significantly reduce the simulation time, ordinal optimization approaches typically relax the goal of simulation to the isolation of a set of good candidate solutions. The observations of numerous simulation experiments indicate that it is possible to determine whether a candidate solution is good or bad very early in the simulation with high probability. Typical relaxing goal of existing ordinal optimization approaches is to identify a small subset of candidate solutions containing at least one top-r solution with high probability. Estimates of this probability has been proposed by Chen (1993) for a class of discrete event systems.

Another principle of ordinal optimization approaches is the use of different simulation length for different candidate solutions. The idea is to discard solutions which can hardly be optimal ones whenever we are confident enough. Figure 1 is a typical simulation time distribution when using an ordinal optimization approach.

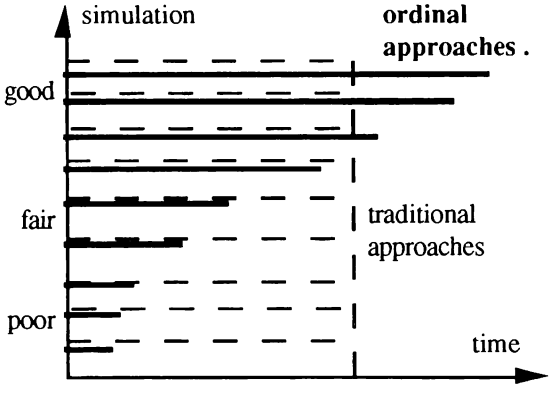

Figure 1: Simulation Length Profile (from Ho and Cassandras (1993))

The main question regarding the ordinal approaches is how to determine that a subset of candidate solutions can hardly be optimal ones with high probability at a given point of the simulation. This confidence probability problem has been addressed by Chen (1993) for a class of discrete event systems. This approach does not apply to our case as it cannot handle criteria which are non-linear functions of performance measures.

4.2. Proposed Ordinal Optimization Approach

4.2.1. Average Cycle Time Estimation

The starting point of our ordinal optimization approach is the estimation of the average cycle time at different point of the simulation. The approach of Chen (1993) is used and is summarized in the following.

We first define the following quantities :

$$Y_i(j) = (S_{i,t^*}(j.L) - S_{i,t^*}((j-1).L))/L \quad (4)$$

and assume that $Y_i(j)$ for $j \geq 1$ are independent samples from a common random variables Y_i. This is a well-known "batched means" technique in simulation to obtain roughly independent observations (see Mitrani 1982).

At any point k = J.L of the simulation, the following sample estimate can be derived:

$$\frac{1}{J}\sum_{j=1}^{J} Y_i(j) = \frac{S_{i,t^*}(J.L)}{J.L} \quad (5)$$

From the ergodicity property (2), it converges to the exact average cycle time with probability 1, i.e.

$$E[Y_i] = \lim_{J\to\infty} \frac{1}{J}\sum_{j=1}^{J} Y_i(j) = \pi(M_i), \text{a.s.} \quad (6)$$

However, at any point of the simulation, the sample estimate is only an approximation of the exact average cycle time. The Bayesian approach is used to characterize the exact average cycle time $\pi(M_i)$. This approach treats as a random variable $\pi(M_i)$ and derives its posterior distribution based the sample path $\{Y_i(1), \ldots, Y_i(J)\}$.

Let $\hat{\pi}_i(J)$ be a random variable whose distribution is the posterior distribution of $\pi(M_i)$ conditioned on the samples $\{Y_i(1), \ldots, Y_i(J)\}$.

Assume that the distribution of $\hat{\pi}_i(0)$ is $N(0, v^2)$ for some very large v which implies that there is no prior knowledge about the average cycle time. Furthermore, we assume that the distribution of Y_i is $N(\pi(M_i), \sigma_i^2)$.

It can be easily shown that the distribution of $\hat{\pi}_i(J)$ is also a normal distribution with:

$$E[\hat{\pi}_i(J)] = \frac{1}{J}\sum_{j=1}^{J} Y_i(j), \quad Var[\hat{\pi}_i(J)] = \frac{\sigma_i^2}{J} \quad (7)$$

The last obstacle is that the variance σ_i^2 is unknown. We replace it by its sample estimate:

$$\frac{1}{J-1}\sum_{j=1}^{J}\left(Y_i(j) - \frac{1}{J}\sum_{j=1}^{J} Y_i(j)\right)^2 \quad (8)$$

4.2.2 Proposed Ordinal Optimization Approach

Let us first consider a trivial example of 7 candidate solutions. Assume that their criterion value estimates follow normal distributions at any point of the simulation. Table 1 contains the mean and the standard deviations obtained at a given point of the simulation.

Table 1. A Trivial Example

No.	1	2	3	4	5	6	7
$E[F_i]$	1	2	2.1	2.2	3	4	5
σ_i	0.1	0.2	0.2	0.2	2	2	1

The top-3 solutions are candidates 1, 2 and 3. However, the probability that candidate 2 or 3 provides better criterion value candidate 1, i.e. $F_2 < F_1$ or $F_3 < F_1$, is very small while the probability that $F_5 < F_1$ or $F_6 < F_1$, is not insignificant. As a result, we can discard candidates 2, 3, and 7 at this point and further simulate candidates 1, 5 and 6.

This is what differs our approach and the existing ordinal optimization approaches which always keep all the top-r candidates. Instead, the central idea of our approach is to determine a subset of candidate solutions which outperforms all other candidate solutions with high enough probability.

The ordinal optimization algorithm for the token partition problem can be summarized as follows.

Algorithm A3 (Ordinal optimization)
1. Choose a simulation cycle K.
2. For each net SPN_i, compute $(\sigma_{i,1}, \ldots, \sigma_{i,|T|})$.
3. Set $S_{i,t}(k) := 0$, for all $t \in T$, $k \leq 0$ and $1 \leq i \leq n$.
4. Set $Q = \{M_1, \ldots, M_n\}$, $\rho^* = 1$.
4. For k = 1 to K do
   4.1. Generate the r.v.s $X_t(k)$ for all $t \in T$
   4.2. For $M_i \in Q$ and for $t = \sigma_{i,1}$ to $\sigma_{i,|T|}$, compute $S_{i,t}(k)$ using equation (1)
   4.3. If k = J . L, update the posterior distribution of $\hat{\pi}_i(J)$ for all $M_i \in Q$ by using (7) and (8)
   4.4. If $|Q| > r$ and $J = J' . L_1$ and $J \geq J_0$, select a subset of candidate initial markings G such that $\rho(G) \geq P_0$ where $\rho(G)$ is the probability that an optimal solution in G is also optimal in Q.
   4.5. If $G \neq Q$, set Q = G and set $\rho^* = \rho^* . \rho(G)$
5. For all $M_i \in Q$ do
   5.1. Compute $\pi(M_i) = S_{i,t^*}(K)/K$ where t* is a given transition
   5.2. Compute $F(M_i, \pi(M_i))$
6. Determine the optimal solution M*.

In this algorithm, at step 4.3, the posterior distribution of the average cycle time is updated according to the method presented in section 4.2.1. L is the batch size of the raw data to be grouped together. At some regular intervals (i.e. L_1 * L), we determine a subset of candidate initial markings G such that initial markings in G outperform initial markings in Q - G with probability greater than P_0, i.e. $\rho(G) \geq P_0$. Candidate initial markings in Q - G which correspond to not good enough solutions are discarded at step 4.4. The integer r is the number of candidate initial markings that can be simulated until K. J_0 is the number of samples necessary for the variance estimates (8) to be accurate enough. ρ^* is the confidence probability for the candidate initial marking M* to be the real optimal.

The main question regarding this algorithm is the determination of the subset G and $\rho(G)$. A Monte Carlo method is used to solve this problem. For this purpose, a sequence of m samples $\{Z_{i,l}$ for $1 \leq l \leq m\}$ of the random variable is generated for each candidate $M_i \in Q$. We then determine $\rho(M_i)$, the number of times that M_i is optimal over all candidates in Q, i.e. $F(M_i, Z_{i,l}) = \text{Min}\{F(M_a, Z_{a,l})$ for all $M_a \in Q\}$. We then add the candidate initial markings M_i to G in decreasing order of $\rho(M_i)$ until $\rho(G) = (1/m) \sum_{Mi \in G} \rho(M_i) \geq P_0$.

The computation burden for the generation of $\{Z_{i,l}$ for $1 \leq l \leq m\}$ can be reduced by generating at the initialization phase a sequence of samples $\{z_{i,l}$ for $1 \leq l \leq m\}$ of a r.v. with distribution N(0, 1). The sequence $\{Z_{i,l}$ for $1 \leq l \leq m\}$ can then be obtained as follows:

$$Z_{i,l} = E\left[\hat{\pi}_i(J)\right] + z_{i,l} \cdot \sqrt{\text{Var}\left[\hat{\pi}_i(J)\right]} \qquad (9)$$

5. OPTIMAL KANBAN DISTRIBUTION

This section applies the proposed ordinal optimization approach to the optimal kanban distribution determination for a four-machines serial system. Its Petri net model (see Xie (1993) for more detail) is given in Figure 2.

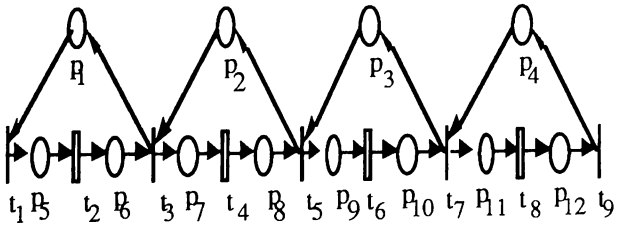

Fig. 2: A 4-machine Serial System

The work in process is controlled by kanbans. The manufacturing process is decomposed into stages and we assume that each stage corresponds to one operation (or one machine). Each stage is assigned a given number of kanbans. A kanban is attached to each part within a stage and the kanban is detached from the part when it leaves the stage. Let $K_j > 0$ be the number of kanbans assigned to the stage j. We assume that the transportation times between stages are small enough and can be neglected.

To illustrate the functioning, let us consider the second stage modeled by the elementary circuit (p_2, t_3, p_7, t_4, p_8, t_5, p_2). Place p_2 contains as many tokens as free kanbans, the place p_7 contains as many tokens as kanbans attached to parts waiting to be serviced (including the one being serviced) in stage j. Place p_8 contains as many tokens as completed parts waiting to be moved to the next stage. Transition t_4 represents the operation performed in stage 2. It is a timed and recycled transition which means that at most one firing can be initiated at any time. The completed parts will be transfered to the next stage 3 if free kanbans are available in stage 3, i.e. place p_3 contains tokens. The transfer of these parts are represented by transition t_5. Firings of transition t_5 remove tokens in places p_8 and add them to place p_2 which detaches the kanbans attached to the completed parts. The kanbans detached from the completed parts become free kanbans. If there are completed parts waiting in stage 1, transition t_3 fires which attaches one free kanban to each of these parts and moves them to stage 2.

The total numbers of tokens in the four elementary circuits are invariant under the transition firings and are equal to K_1, K_2, K_3 and K_4 respectively.

The criterion to be minimized is as follows:

$$F(M, \pi(M)) = \sum_{i=1}^{12} M(p_i) + \alpha.(C - \pi(M))^+ \qquad (10)$$

Let us consider the set of candidate initial markings to be taken into account. For this serial kanban system, Property 4 of Xie (1993) implies that the average cycle time depends on the number of kanbans associated to each elementary circuit, i.e. K_1, K_2, K_3 and K_4 but it does not depend on the exact token distribution. Since $\Sigma_i M(p_i) = K_1+K_2+K_3+K_4$, we only need to consider the set of initial markings M satisfying $M(p_i) = 0$, for $5 \leq i \leq 12$.

Furthermore, it has been proved by Tayur (1992) that for a 4-stage serial system with a given number of kanbans, the average cycle time is minimized when $K_1 = K_4 = 1$. As a result, we can further limit ourselves to the set of candidate initial markings M satisfying $M(p_1) = M(p_4) = 1$ and $M(p_i) = 0$, for $5 \leq i \leq 12$. In the following, we consider the following 36 initial markings which contain at most 11 tokens:

M_{00} : 1 1 1 1 0 0 0 0 0 0 0 0, M_{01} : 1 2 1 1 0 0 0 0 0 0 0 0,
M_{02} : 1 1 2 1 0 0 0 0 0 0 0 0, M_{03} : 1 3 1 1 0 0 0 0 0 0 0 0,
M_{04} : 1 2 2 1 0 0 0 0 0 0 0 0, M_{05} : 1 1 3 1 0 0 0 0 0 0 0 0,
M_{06} : 1 4 1 1 0 0 0 0 0 0 0 0, M_{07} : 1 3 2 1 0 0 0 0 0 0 0 0,
M_{08} : 1 2 3 1 0 0 0 0 0 0 0 0, M_{09} : 1 1 4 1 0 0 0 0 0 0 0 0,
M_{10} : 1 5 1 1 0 0 0 0 0 0 0 0, M_{11} : 1 4 2 1 0 0 0 0 0 0 0 0,
M_{12} : 1 3 3 1 0 0 0 0 0 0 0 0, M_{13} : 1 2 4 1 0 0 0 0 0 0 0 0,
M_{14} : 1 1 5 1 0 0 0 0 0 0 0 0, M_{15} : 1 6 1 1 0 0 0 0 0 0 0 0,
M_{16} : 1 5 2 1 0 0 0 0 0 0 0 0, M_{17} : 1 4 3 1 0 0 0 0 0 0 0 0,
M_{18} : 1 3 4 1 0 0 0 0 0 0 0 0, M_{19} : 1 2 5 1 0 0 0 0 0 0 0 0,
M_{20} : 1 1 6 1 0 0 0 0 0 0 0 0, M_{21} : 1 7 1 1 0 0 0 0 0 0 0 0,
M_{22} : 1 6 2 1 0 0 0 0 0 0 0 0, M_{23} : 1 5 3 1 0 0 0 0 0 0 0 0,
M_{24} : 1 4 4 1 0 0 0 0 0 0 0 0, M_{25} : 1 3 5 1 0 0 0 0 0 0 0 0,
M_{26} : 1 2 6 1 0 0 0 0 0 0 0 0, M_{27} : 1 1 7 1 0 0 0 0 0 0 0 0,
M_{28} : 1 8 1 1 0 0 0 0 0 0 0 0, M_{29} : 1 7 2 1 0 0 0 0 0 0 0 0,
M_{30} : 1 6 3 1 0 0 0 0 0 0 0 0, M_{31} : 1 5 4 1 0 0 0 0 0 0 0 0,
M_{32} : 1 4 5 1 0 0 0 0 0 0 0 0, M_{33} : 1 3 6 1 0 0 0 0 0 0 0 0,
M_{34} : 1 2 7 1 0 0 0 0 0 0 0 0, M_{35} : 1 1 8 1 0 0 0 0 0 0 0 0.

The control parameters needed in the algorithm A3 are as follows : K = 20000, L = 100, r = 5, J_0 = 9, L_1 = 5 and P_0 = 0.98.

In the remainder of this section, we reported numerical results for four different cases.

Case 1. The firing time distributions are as follows. X_{t2} = 10, X_{t4} is exponentially distributed with mean equal to 10, X_{t6} is uniformly distributed on [5, 15] and X_{t8} is a two stage Erlang distributed r.v. with mean equal to 8. C = 10.5 and α = 20.

The exact criterion values are obtained by algorithm A2 by simulating all candidate solutions until K = 20000. The criterion values of these candidate initial markings are given in the following in non-increasing order of the criterion value:

(30 20.82)	(31 21.76)	(23 22.59)	(29 23.23)
(22 24.22)	(24 24.91)	(32 25.09)	(17 25.80)
(16 26.07)	(11 29.32)	(18 30.44)	(25 30.73)
(12 31.26)	(33 31.45)	(7 34.90)	(28 38.98)
(21 39.21)	(1540.01)	(10 41.57)	(13 41.60)
(19 42.13)	(8 42.21)	(26 42.97)	(34 43.92)
(6 44.66)	(4 46.08)	(3 50.88)	(1 63.48)
(5 71.09)	(9 71.34)	(14 72.14)	(20 73.11)
(2 73.97)	(27 74.11)	(35 75.11)	(0 95.16)

where the first number in any couple indicates the candidate marking and the second one indicates the criterion value.

Let us report the result of the ordinal optimization algorithm A3. At iteration k = 600 and l=5, the following criterion values are obtained:

(30 15.96)*	(31 16.39)*	(23 17.56)*	(29 19.28)*
(32 19.51)	(24 19.51)*	(22 19.98)	(17 20.64)*
(16 21.21)*	(18 24.27)	(25 24.52)	(11 24.83)
(12 25.23)*	(33 25.30)*	(7 29.66)*	(13 34.78)
(19 35.32)	(8 35.78)	(26 36.27)	(34 37.27)
(4 40.51)	(28 40.84)	(21 40.91)	(15 41.05)
(10 41.64)	(6 44.76)	(3 49.60)*	(1 60.51)*
(5 65.99)*	(9 66.77)*	(14 67.77)*	(2 68.51)
(20 68.77)	(27 69.77)*	(35 70.77)*	(0 93.71)*.

Surprisingly, a large number of candidate markings (see marked candidate markings) are ranked correctly although the criterion values are far from stable. The same observation is made for other cases. This confirms observations made by other authors.

The criterion values obtained at iteration k = 1100 and l=11 are as follows:

(30 17.98)	(31 18.99)	(23 20.05)	(29 21.52)
(32 22.22)	(24 22.33)	(22 22.53)	(17 23.46)
(16 24.28)	(25 27.31)	(18 27.48)	(11 27.77)
(33 27.83)	(12 28.44)	(7 32.61)	(13 37.49)
(19 37.84)	(26 38.45)	(8 38.56)	(34 39.45)
(28 41.52)	(21 41.59)	(15 42.13)	(10 42.97)
(4 43.05)	(6 45.85)	(3 50.89)	(1 62.01)
(5 66.23)	(9 66.73)	(14 67.54)	(20 68.54)
(27 69.54)	(2 69.65)	(35 70.54)	(0 94.00).

The probabilities for candidate markings to be optimal are as follows : (30, 0.382), (31, 0.231), (23, 0.145), (29, 0.062), (32, 0.058), (24, 0.036), (22, 0.047), (17, 0.023), (16, 0.015), (11, 0.001), and 0 for the other candidate markings. At this point of the simulation, the candidate markings (16 11 0 1 2 3 4 5 6 7 8 9 10 12 13 14 15 18 19 20 21 25 26 27 28 33 34 35) are rejected. The confidence probability ρ^* = 0.984.

At iteration k = 1600 and l=16, the criterion values become:

(30 18.59)	(31 19.01)	(23 20.32)	(32 22.08)
(24 22.15)	(29 22.20)	(22 23.10)	(17 23.49).

The probabilities for candidate markings to be optimal are as follows : (30, 0.392), (31, 0.296), (23, 0.145), (32, 0.053), (24, 0.036), (29, 0.040), (22, 0.020), (17, 0.018). At this point of the simulation, the candidate marking 17 is rejected. The confidence probability ρ^* = 0.966288.

Candidate marking 22 is rejected at iteration k = 2100, candidate marking 29 is rejected at iteration 2600. The 5 remainder candidate markings are simulated until k =20000. The correct optimal candidate initial marking M_{30} is obtained with confidence probability ρ^* = 0.946091.

Case 2. The firing times distributions are the same as in the first case. C = 11.5 and α = 50.

For this case, the exact criterion values are as follows:

(16 9.00)	(17 9.00)	(22 10.00)	(23 10.00)
(24 10.00)	(29 11.00)	(30 11.00)	(31 11.00)
(32 11.00)	(11 11.30)	(25 11.84)	(33 12.12)
(18 12.61)	(12 16.14)	(7 26.75)	(28 30.96)

(21 33.02)	(15 36.52)	(19 41.82)	(10 41.92)
(13 42.01)	(26 42.42)	(34 43.30)	(8 45.02)
(6 51.14)	(4 56.19)	(3 68.21)	(1 101.21)
(9 117.85)	(14 118.34)	(5 118.73)	(20 119.27)
(27 120.26)	(35 121.26)	(2 127.42)	(0 181.90).

Let us report the result of the ordinal optimization algorithm A3. The criterion values obtained at iteration k = 1100 and l=11 are as follows:

(11 8.00)	(16 9.00)	(17 9.00)	(18 9.00)
(12 9.11)	(22 10.00)	(23 10.00)	(24 10.00)
(25 10.00)	(29 11.00)	(30 11.00)	(31 11.00)
(32 11.00)	(33 11.00)	(7 21.03)	(19 31.10)
(26 31.13)	(13 31.72)	(34 32.13)	(8 35.90)
(28 37.30)	(21 38.97)	(15 41.83)	(10 45.42)
(4 48.64)	(6 54.12)	(3 68.22)	(1 97.53)
(9 106.33)	(5 106.57)	(14 106.86)	(20 107.86)
(27 108.86)	(35 109.86)	(2 116.62)	(0 178.99).

The probabilities for candidate markings to be optimal are as follows : (11, 0.464), (16, 0.201), (17, 0.014), (18, 0.002), (12, 0.245), (7, 0.073), and 0 for the other candidate markings. At this point of the simulation, the candidate markings (17 18 13 0 1 2 3 4 5 6 8 9 10 14 15 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35) are rejected. The confidence probability $\rho^* = 0.983$.

Let us notice that the candidate markings 17 and 18 ranked No. 3 and No. 4 are rejected as they can hardly outperform the set of retained candidate markings while the candidate marking 7 ranked No. 15 is kept.

The 4 remainder candidate markings (11, 12, 16, 7) are simulated until k =20000. The correct optimal candidate initial marking M_{16} is obtained with confidence probability $\rho^* = 0.983$.

Case 3. The firing time distributions are as follows. $X_{t2} = 10$, X_{t4} is a three-stages Erlang distributed r.v. with mean equal to 9, X_{t6} is uniformly distributed on [5, 15] and X_{t8} is a two stage Erlang distributed r.v. with mean equal to 8. C = 10.5 and $\alpha = 20$.

For this case, the exact criterion values are the following ones:

(12 8.00)	(17 9.00)	(18 9.00)	(23 10.00)
(24 10.00)	(25 10.00)	(19 10.40)	(16 10.54)
(13 10.57)	(11 10.66)	(26 10.76)	(22 10.91)
(30 11.00)	(31 11.00)	(32 11.00)	(33 11.00)
(34 11.48)	(29 11.58)	(8 11.83)	(7 11.90)
(4 16.32)	(9 30.31)	(6 30.46)	(5 30.56)
(14 30.86)	(3 30.99)	(10 30.99)	(20 31.74)
(15 31.85)	(27 32.74)	(21 32.84)	(35 33.74)
(28 33.83)	(2 34.31)	(1 35.19)	(0 57.01).

Algorithm A3 proceeds as follows. The criterion values obtained at iteration at k = 1100 and l = 11 are as follows:

(12 8.00)	(13 8.56)	(17 9.00)	(18 9.00)
(19 9.00)	(11 9.78)	(23 10.00)	(24 10.00)
(25 10.00)	(26 10.00)	(8 10.17)	(16 10.38)
(7 10.62)	(30 11.00)	(31 11.00)	(32 11.00)
(33 11.00)	(34 11.00)	(22 11.04)	(29 11.86)
(4 14.85)	(9 26.68)	(14 26.84)	(5 27.59)
(20 27.66)	(27 28.65)	(35 29.65)	(6 31.58)
(3 31.93)	(10 32.34)	(2 32.66)	(15 33.08)
(21 33.87)	(28 34.69)	(1 35.83)	(0 56.36).

The probabilities for candidate markings to be optimal are as follows : (12, 0.882), (13, 0.030), (17, 0.000), (18, 0.001), (11, 0.010), (8, 0.054), (7, 0.023), and 0 for the others. At this point of the simulation, the candidate markings (11 18 0 1 2 3 4 5 6 9 10 14 15 16 17 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35) are rejected. The confidence probability $\rho^* = 0.989$.

The 4 remainder candidate markings (12, 13, 8, 7) are simulated until k =20000. The correct optimal candidate initial marking M_{12} is obtained with confidence probability $\rho^* = 0.989$.

Case 4. The firing time distributions are as follows. X_{t2} is uniformly distributed on [0, 18], X_{t4} is a three-stages Erlang distributed r.v. with mean equal to 9, X_{t6} is uniformly distributed on [5, 15] and X_{t8} is a two stage Erlang distributed r.v. with mean equal to 8. C = 10.5 and $\alpha = 20$.

In this case, the exact criterion values are:

(17 9.00)	(18 9.00)	(12 9.37)	(23 10.00)
(24 10.00)	(25 10.00)	(30 11.00)	(31 11.00)
(32 11.00)	(33 11.00)	(22 11.29)	(16 11.29)
(29 11.85)	(11 12.22)	(26 12.54)	(19 12.61)
(34 12.91)	(13 13.41)	(7 14.77)	(8 15.57)
(4 21.20)	(6 32.12)	(10 32.15)	(15 32.82)
(21 33.73)	(3 33.85)	(28 34.70)	(9 37.87)
(14 38.17)	(5 38.75)	(20 38.94)	(27 39.86)
(1 40.64)	(35 40.84)	(2 43.81)	(0 67.05).

Let us report the result of the ordinal optimization algorithm A3. The results obtained at iteration k = 1100 and l=11 are given in the following:

(17 9.00)	(18 9.00)	(23 10.00)	(24 10.00)
(25 10.00)	(12 10.36)	(26 10.41)	(30 11.00)
(31 11.00)	(32 11.00)	(33 11.00)	(34 11.00)
(19 11.05)	(13 12.54)	(16 13.49)	(22 13.80)
(11 14.07)	(29 14.56)	(8 15.57)	(7 16.25)
(4 21.80)	(6 35.81)	(10 36.16)	(14 36.24)
(9 36.60)	(20 36.66)	(15 37.00)	(27 37.23)
(3 37.59)	(21 37.75)	(35 38.22)	(5 38.28)
(28 38.58)	(1 44.00)	(2 44.79)	(0 69.59).

The probabilities for a candidate marking to be optimal is as follows: (17, 0.418), (18, 0.270), (23, 0.006), (24, 0.002), (25, 0.001), (12, 0.240), (19, 0.020), (13, 0.030), (11, 0.010), (8, 0.003), and 0 for the others. At this point of the simulation, the candidate markings (23 8 24 25 0 1 2 3 4 5 6 7 9 10 14 15

16 20 21 22 26 27 28 29 30 31 32 33 34 35) are rejected. The confidence probability $\rho^* = 0.988$.

The candidate markings 13 and 19 are rejected at iteration k = 1600. The 4 remainder candidate markings (17, 18, 12, 11) are simulated until k =20000. The correct optimal marking M_{17} is obtained with confidence probability $\rho^* = 0.988$.

6. CONCLUSION

In this paper, we have proposed an ordinal optimization approach to a token partition problem for stochastic timed event graphs. This approach simultaneously simulates all candidate initial markings. The distributions of the average cycle times are characterized along with the simulation. The distributions are used to calculate the probability for a candidate initial marking to be optimal. Candidate initial markings whose probabilities to be optimal are low are rejected along the simulation. Numerical results show the power of this approach in identifying good solutions.

We believe that this approach is general enough and can be extended to general discrete event systems such as general Petri nets. Several problems remain open. First, the definition of the candidate solutions is crucial due to the combinatorial feature of such problems. Theoretical results such as ergodicity, monotonicity and deadlock freeness are needed to restrict the number of candidate solutions. Another open problem is the confidence interval of the cycle time distribution estimates.

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