A GRAPHICAL TEST BED (SIMTBED) FOR ANALYZING AND REPORTING THE RESULTS OF A STATISTICAL EXPERIMENT

Peter A. W. Lewis
Department of Operations Research
Naval Postgraduate School
Monterey, CA 93940.

ABSTRACT

It is unfortunately true that statisticians seldom apply the tools of their own trade to the analysis of simulation experiments. This refers to the experimental design for the input, and the data presentation and summary and analysis for the output. A graphical test bed in which the output of a simulation experiment can be reported and analyzed is described. From the graphics and the associated numerics the experimenter can summarize and see simultaneously relative properties—such as bias, normality and spread—of several estimators of a characteristic of the population for up to sixteen sample sizes. The evolution of these properties with sample size is also displayed and analyzed using regression and asymptotic expansions. The graphics is supported on a line printer to make it and the program portable. The use of the program is illustrated by a study of estimates of serial correlation in normal time series.

The latest version of SIMTBED incorporates a restart facility which makes the program ideal for efficient utilization of microcomputers. A microcomputer version which uses color in the output to differentiate factors in the simulation is described, as well as improvements which are being incorporated into the latest version. This latest version will run on micros, minis or mainframes.

1. INTRODUCTION

The SIMTBED program is a graphical test bed for analyzing the results of a statistical simulation experiment. An earlier version of the program was reported in Lewis, et al (1985) and the latest version, which was used extensively for illustrative purposes in Lewis and Orav (1989), is described quite extensively in Lewis, Orav and Uribe (1989).

The purpose of this report is to summarize further enhancements to the package, some of which have been implemented, and some of which are still experimental.

The main innovation in the version in Lewis, Orav and Uribe (1989) —referred to as SMTBPC12 in microcomputer versions and SIMTBED12 in the mainframe version—is the ability to restart the simulation after a certain number of super-replications have been performed. This would be the case if not enough precision had been attained in the simulation, and this is easy to judge since the program has a built in facility for assessing the variability of estimates of characteristics of the distribution of the statistic or statistics which are under investigation. This facility is particularly useful for microcomputers since it allows the experimenter to use his microcomputer for simulations when he is not using it, for example, overnight.

2. FURTHER ENHANCEMENTS

The original intent in designing SIMTBED was to make it as portable as possible, and we therefore used a line-printer plotter package for graphical output. It is clear now, however, that better graphics are necessary for better visual discrimination of the output. For example, using cheap color dot-matrix printers we have been able to coalesce three of the basic output figures, such as those in figures 1, 2 and 3 so that the effects of both sample size and different methods of bootstrapping can be seen simultaneously. While it is usable, this enhancement still suffers from the lack of resolution of line printer graphics.

As another example, on the mainframe we have used the DISSPLA package to obtain high resolution graphics—these plots are shown in Figures 1 to 6 and should be compared to the identical line plotter figures in Lewis and Orav (1989, Figures 9.6.1 to 9.6.6). The improvement is quite clear and the problem right now is to find a suitable package on the microcomputer to refine the graphics.

Joint properties of estimators are also of interest and the issue here is the usual one of compactification. In Figure 7 we show an attempt to incorporate the factors

of sample size and different statistics into a single plot. Each cluster of scatter plots is a draftsman's plot of the scatter plots showing the relationship between the statistics at a given sample size. The effect of sample size is seen, either singly or jointly, by looking along the diagonal.

3. THE BOOTSTRAP SIMULATION

Bootstrapping is an important new idea for assessing variability in statistical situations and in simulations. Figures 1 to 6 are equivalent to those in Lewis and Orav (1989), as mentioned above, and that book should be referenced for details. The simulation experiment is an extension, with graphics, of results given in Efron and Gong (1983).

Briefly one can see (Figure 2) that the Jackknife estimate of the standard deviation of an estimate of the correlation coefficient in a bivariate normal sample is more variable—and has bigger outliers—than the normal theory estimate (Figure 1). The bootstrap estimator (Figure 3) is better than the jackknife estimator and the bootstrap estimator improves as B, the number of replications in the bootstrap simulation, increases from 128 (Figure 3) to 512 (Figure 4), although the improvement is slight. Again, the Normal-smoothed bootstrap gives even more improvement (Figures 5 and 6).

4. SUMMARY AND CONCLUSIONS

Other possible enhancements and extensions of the simulation test bed will be discussed in the talk.

SAMPLE SIZE (N): 100   NO. OF REPLICATIONS (M): 1   DEGREE OF REGRESSION (D): 0   NO. OF SECTIONS: 8

SUBSAMPLE SIZE	8	14	20	26	32	38	44	50
MEAN	0.3043	0.2125	0.1771	0.1434	0.1431	0.1311	0.1215	0.1110
STD	0.1115	0.6053E-01	0.3916E-01	0.2980E-01	0.2322E-01	0.2000E-01	0.2299E-01	0.1525E-01
STD MEAN	0.8427E-02	0.6053E-02	0.4680E-02	0.4094E-02	0.3540E-02	0.3333E-02	0.4130E-02	0.2993E-02
SKEWNESS	-0.6386	-0.3095	-0.5838	-0.8180E-01	-0.6368	-0.2697	-0.6997	-1.570
KURTOSIS	-0.5284	-0.9571	0.3779	-0.8300	0.5218	-0.8155	0.9886E-01	0.7954

ESTIMATOR: NORMAL THEORY ESTIMATE OF STD OF CORR. OF BIVARIATE NORMALS
VERTICAL SCALE: YMIN = 0.0179
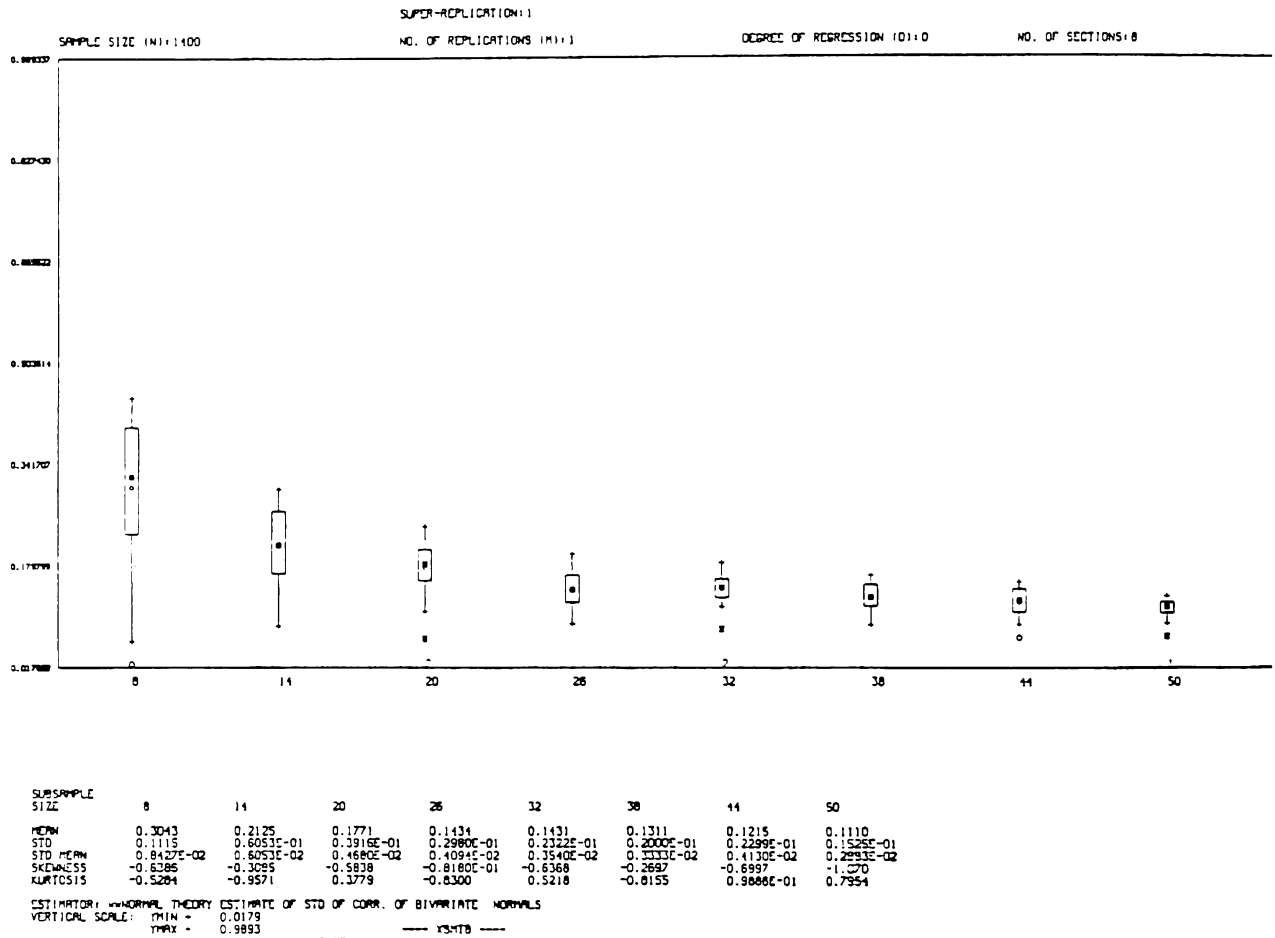   YMAX = 0.9893   ——— Y3-TB ———

Figure 1: SIMTBED runs to investigate the sample properties of the Normal theory estimate of the standard deviation of the correlation estimate $\hat{\rho}$ in a bivariate Normal sample. The estimate is $(1 - \hat{\rho}^2)/(m - 3)^{1/2}$. The true value of ρ is 0.5.
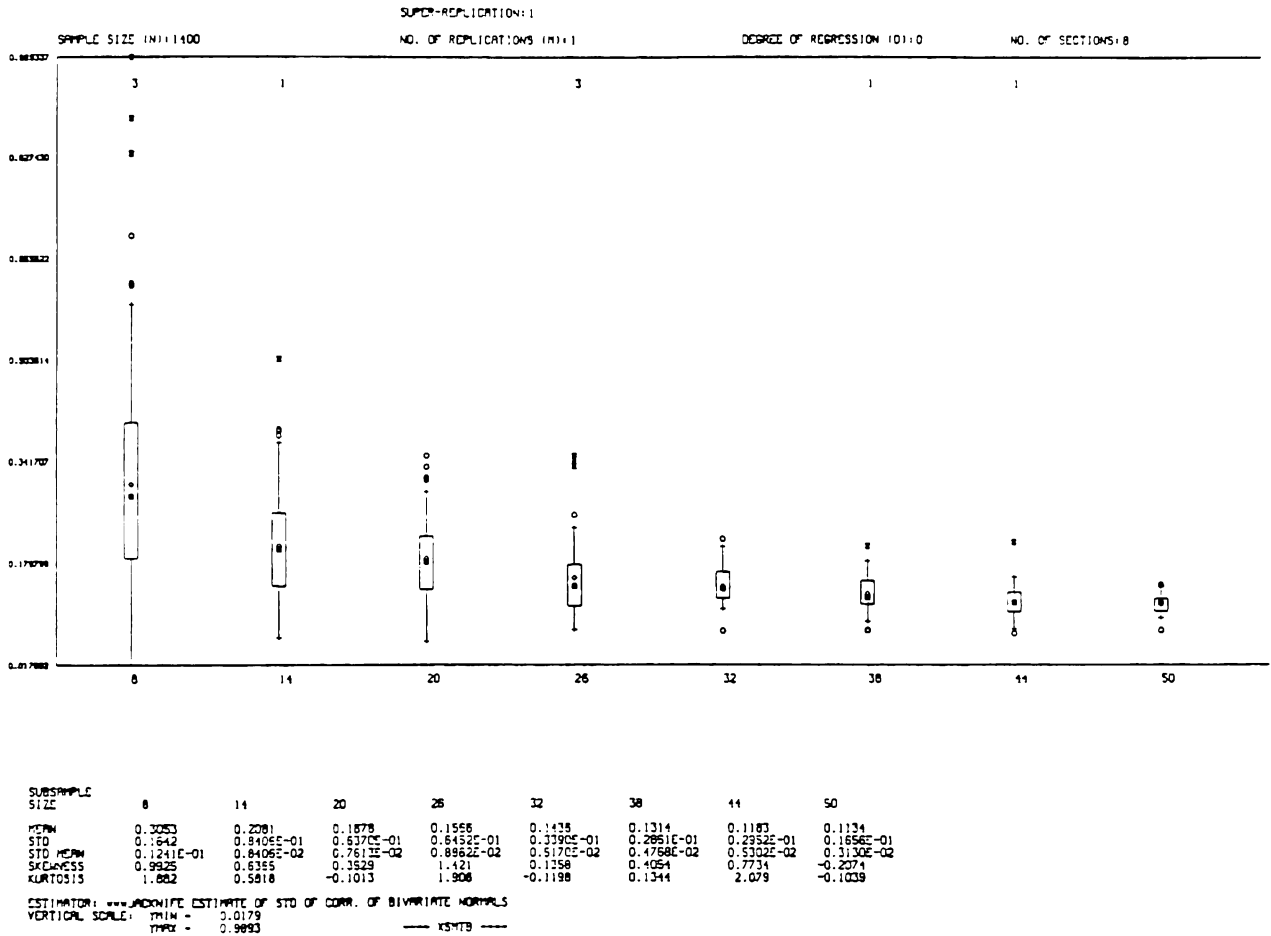
| SAMPLE SIZE (N): 1400 | NO. OF REPLICATIONS (M): 1 | DEGREE OF REGRESSION (D): 0 | NO. OF SECTIONS: 8 |

0.8837

| 3 | 1 | 3 | 1 | 1 |

0.627

0.5

0.4614

0.3417

0.1757

0.0178

| 8 | 14 | 20 | 26 | 32 | 38 | 44 | 50 |

SUBSAMPLE SIZE	8	14	20	26	32	38	44	50
MEAN	0.3053	0.2081	0.1878	0.1556	0.1435	0.1314	0.1183	0.1134
STD	0.1642	0.8405E-01	0.637CE-01	0.6452E-01	0.339CE-01	0.2851E-01	0.2952E-01	0.1656E-01
STD MEAN	0.1241E-01	0.8406E-02	0.7613E-02	0.8962E-02	0.5170E-02	0.4768E-02	0.5302E-02	0.3130E-02
SKEWNESS	0.9925	0.6355	0.3529	1.421	0.1258	0.4054	0.7734	-0.2074
KURTOSIS	1.882	0.5818	-0.1013	1.908	-0.1198	0.1344	2.079	-0.1039

ESTIMATOR: ***JACKKNIFE ESTIMATE OF STD OF CORR. OF BIVARIATE NORMALS
VERTICAL SCALE: MIN = 0.0179
MAX = 0.9893   ——— KSMTB ———

Figure 2: SIMTBED runs to investigate the properties of the jackknife estimate of the standard deviation of an estimate of the correlation coefficient in a bivariate Normal sample ($\rho = 0.5$). The estimate whose standard deviation is being investigated is actually the estimate, $\hat{\rho}$, after complete jackknifing, as at Equation 9.5.1 in Lewis and Orav (1989), to remove the bias term of order $1/m$.
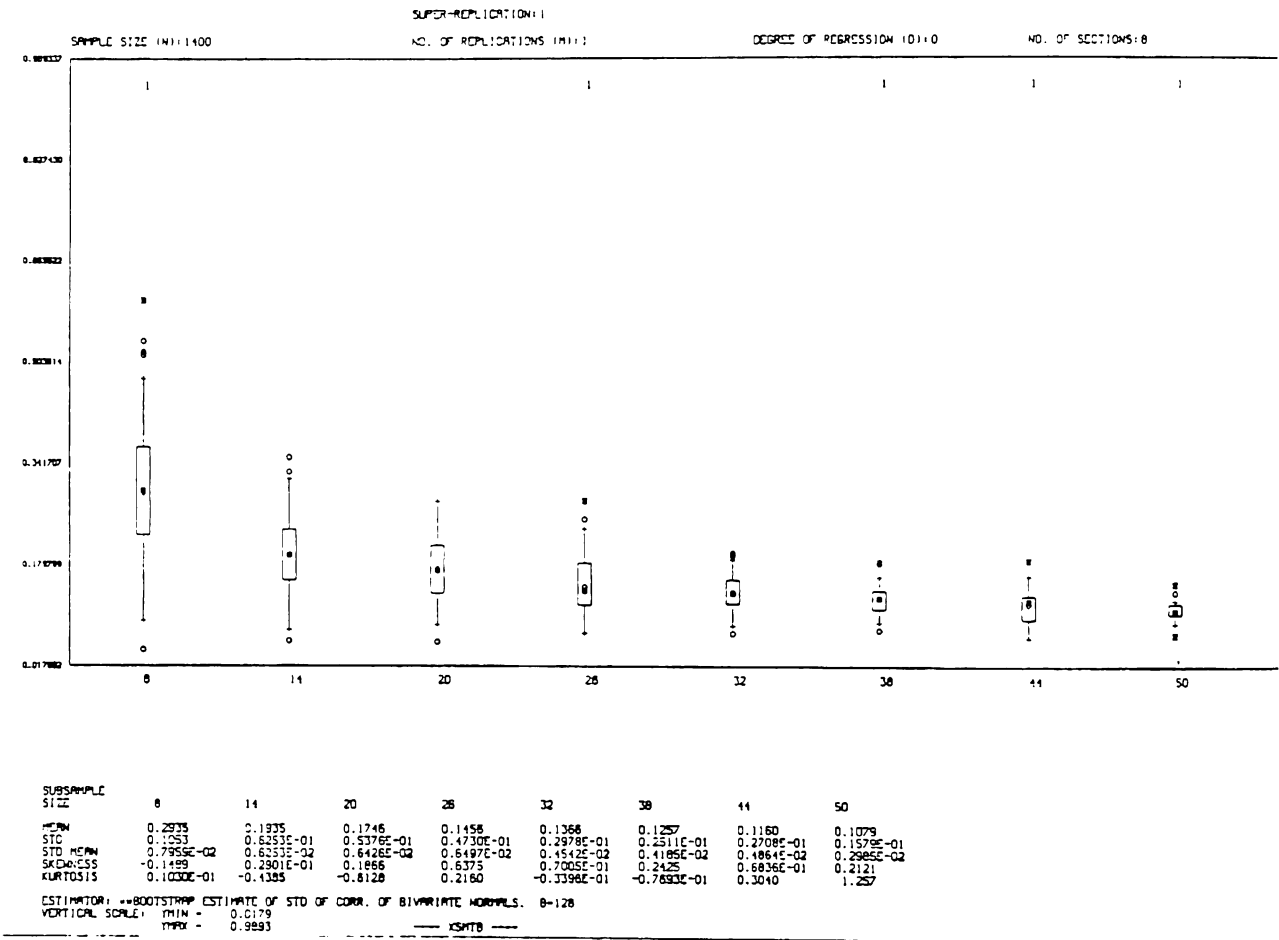
SUPER-REPLICATION: 1

SUBSAMPLE SIZE	8	14	20	26	32	38	44	50
MEAN	0.2975	0.1935	0.1746	0.1456	0.1368	0.1257	0.1160	0.1079
STD	0.1053	0.6253E-01	0.5376E-01	0.4730E-01	0.2978E-01	0.2511E-01	0.2708E-01	0.1579E-01
STD MEAN	0.795SE-02	0.6253E-02	0.6426E-02	0.6497E-02	0.4542E-02	0.4185E-02	0.4864E-02	0.2985E-02
SKEWNESS	-0.1499	0.2301E-01	0.1866	0.6375	0.7005E-01	0.2425	0.6836E-01	0.2121
KURTOSIS	0.1030E-01	-0.4385	-0.8128	0.2160	-0.3398E-01	-0.7893E-01	0.3040	1.257

ESTIMATOR: **BOOTSTRAP ESTIMATE OF STD OF CORR. OF BIVARIATE NORMALS. B=128
VERTICAL SCALE: MIN = 0.0179
                  MAX = 0.9893   —— XSMTB ——

Figure 3: SIMTBED run to investigate the properties of the standard bootstrap estimate of the standard deviation of the correlation $\hat{\rho}$ in a bivariate Normal sample ($\rho = 0.5$) for $B = 128$ (number of replications in the bootstrap simulation)

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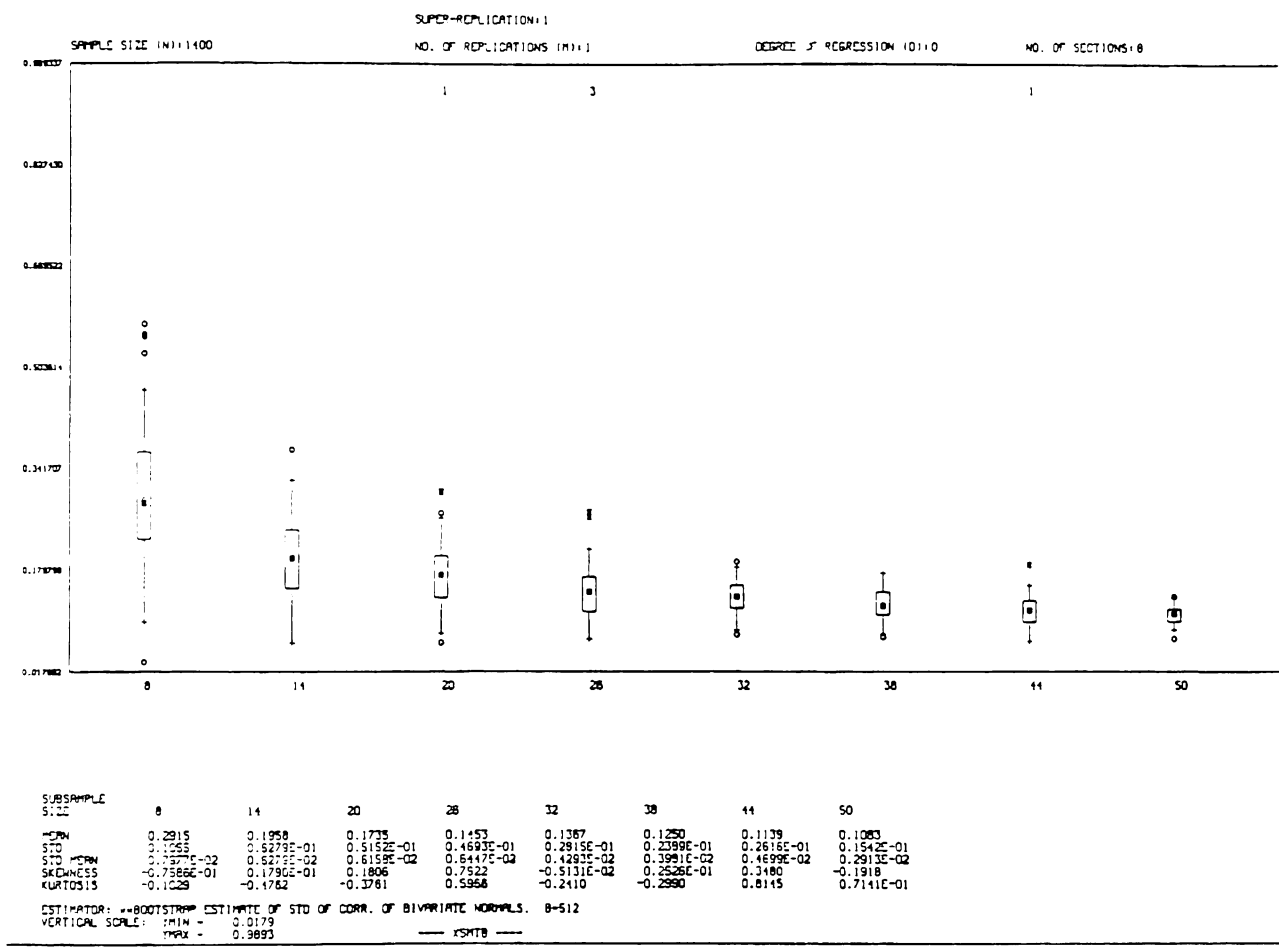
Figure 4: SIMTBED run to investigate the properties of the standard bootstrap estimate of the standard deviation of the correlation $\hat{\rho}$ in a bivariate Normal sample ($\rho = 0.5$) for $B = 512$ (number of replications in the bootstrap simulation)
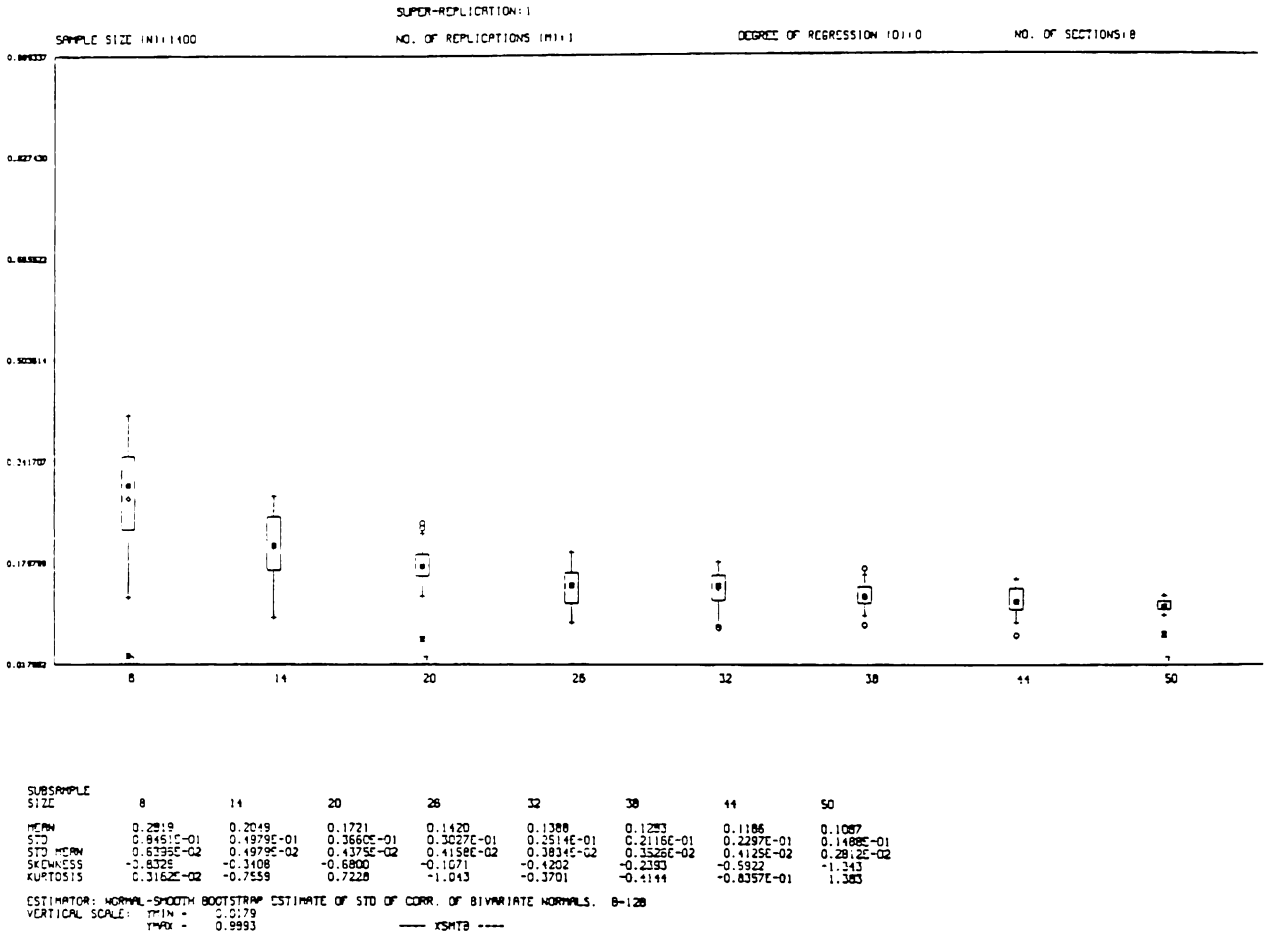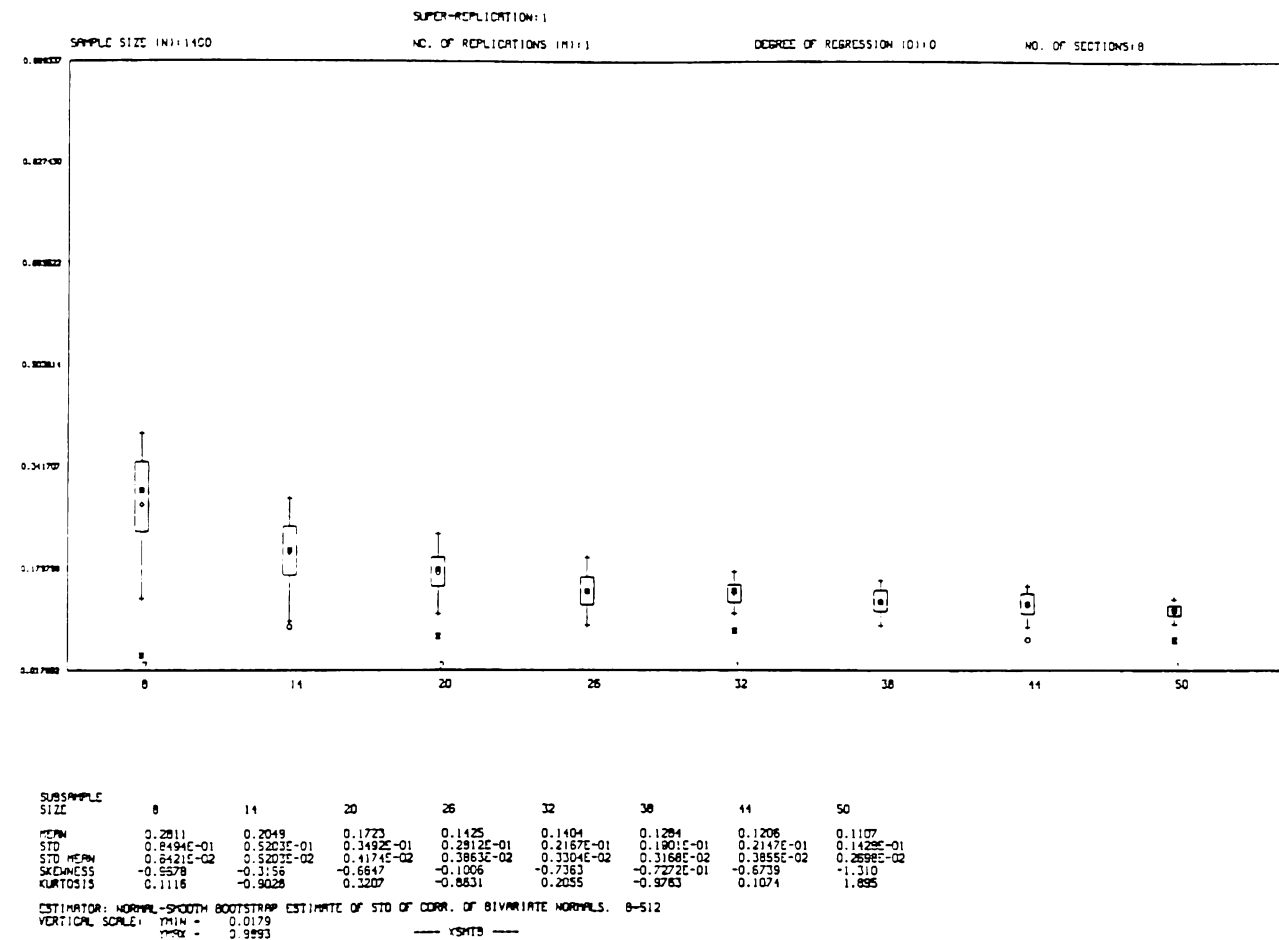
Figure 5: SIMTBED run to investigate the properties of the Normal-smoothed bootstrap estimate of the standard deviation of the correlation $\hat{\rho}$ in a bivariate Normal sample ($\rho = 0.5$). (Here the number of replications in the bootstrap simulation is $b = 128$.)
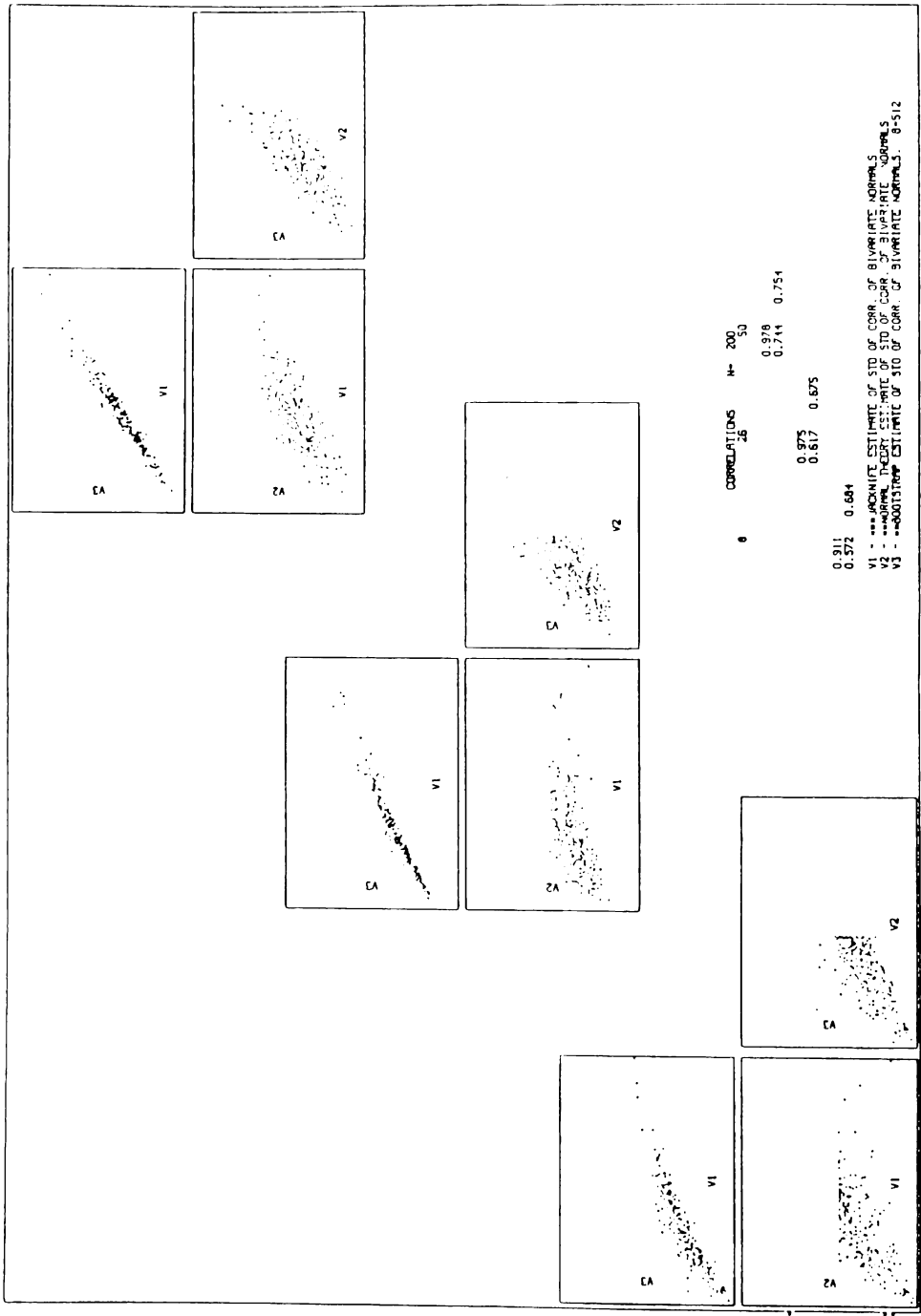
Figure 6: SIMTBED run to investigate the properties of the Normal-smoothed bootstrap estimate of the standard deviation of the correlation $\hat{\rho}$ in a bivariate Normal sample ($\rho = 0.5$). (Here the number of replications in the bootstrap simulation is $b = 512$.)

Figure 7: Experimental graphics to show the correlation and dependence between various statistics, and to show the change in the dependence as a function of sample size.

ACKNOWLEDGMENTS

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AUTHOR'S BIOGRAPHY

PETER A. W. LEWIS is Distinguished Professor of Statistics and Operations Research in the Department of Operations Research at the Naval Postgraduate School in Monterey, California. He holds an A.B. degree (1954) from Columbia University, B.S. and M.S. degrees (1955 and 1957) from Columbia School of Engineering, and a Ph. D. in Statistics from the University of London in 1964. He is co-author, with D.R. Cox, of the book 'The Statistical Analysis of Series of Events' (1966), editor of the Volume entitled 'Stochastic Point Processes: Statistical Analysis, Theory and Applications' (1972) and co-author, with E.J.Orav, of the book 'Simulation Methodology for Statisticians, Operations Analysts and Engineers' (1989). He has also published a software package 'Enhanced Simulation and Statistics Package' (1989) with E.J.Orav and L.Uribe. He is the author of over eighty refereed papers in technical journals, and is a Fellow of the American Statistical Association and the Institute of Mathematical Statistics.

Peter A.W. Lewis
Code 55LW
Naval Postgraduate School
Monterey, CA 93943
(408) 646-2283