Proceedings of the 1989 Winter Simulation Conference
E.A. MacNair, K.J. Musselman, P. Heidelberger (eds.)

NON-UNIFORM RANDOM NUMBER GENERATION: A SURVEY AND TUTORIAL

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ABSTRACT

The basic pseudo-random number generators on computers return deviates which are uniformly distributed in the interval between 0 and 1. For simulations and other applications other random variables are needed which follow given statistical distributions, for instance normal deviates. The survey will concentrate on the most important distributions arising in simulation applications.

The considered non-uniform distributions fall into two categories: continuous and discrete. In either class very efficient methods for sampling from general distributions are presented. Specific cases considered include the exponential, normal, gamma, beta and Cauchy distributions in the continuous, and Poisson, binomial and hypergeometric generators in the discrete category.

In selecting suitable specific algorithms for each distribution we rejected the 'easiest' methods which are not fast enough. On the other hand, some of the most efficient generators are rather difficult to implement. The selected algorithms are almost as fast as these, but not too complex. Their Fortran versions are portable except for the employed basic $(0,1)$-uniform generators for which, however, the user may substitute his or her own favorite. A number of the proposed methods are the author's recent developments. Some well-known alternatives will also be mentioned.

REFERENCES

Ahrens, J.H. and Dieter, U. (1989a), An alias method for sampling from the normal distribution. *Computing 42*, 159-170.

Ahrens, J.H. and Dieter, U. (1989b), A convenient sampling method with bounded computation times for Poisson distributions. *American Journal of Mathematical and Management Sciences* (to appear).

Devroye, L. (1986), *Non-uniform random variate generation*. Springer, New York.

Kinderman, A.J. and Monahan, J.F. (1977), Computer generation of random variables using the ratio of uniform deviates. *ACM Transactions on Mathematical Software 3*, 257-260.

Knuth, D.E. (1981), *The art of computer programming, Vol. II: seminumerical algorithms*, Addison-Wesley, Reading, Mass.

Marsaglia, G. (1977), The squeeze method for generating gamma variates. *Journal of Computational and Applied Mathematics 3*, 321-325.

Neumann, J. v. (1951), Various techniques used in connection with random digits. Monte Carlo methods. *National Bureau of Standards AMS, 12*, 36-38

Stadlober, E. (1989a), Binomial random variate generation: A method based on ratio of uniforms. *American Journal of Mathematical and Management Sciences* (to appear).

Stadlober, E. (1989b), Sampling from Poisson, binomial and hypergeometric distributions: Ratio of uniforms as a simple and fast alternative. *Mathematisch-Statistische Sektion 303*. Forschungsgesellschaft Joanneum, Graz.

Walker, A.J. (1977), An efficient method for generating discrete random variables with general distributions. *ACM Transactions on Mathematical Software 3*, 253-256.

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