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# Performance analysis of a generic naval $C^2$ battle group system by use of timed Petri nets

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## ABSTRACT

This paper addresses the modeling and performance analysis of a generic naval $C^2$ (command and control) battle group system. This work comprises three goals:

- Identification of human, equipment, and organizational factors that make up a $C^2$ system of a naval battle group that consists of at least one command ship together with several co-operation platforms such as ships, submarines and aircraft.
- Modeling of a generic naval $C^2$ battle group system as an asynchronous concurrent system.
- Performance analysis for this system according to dynamic response time, with system resources being limited by the capacity of the sytem and the maximum throughput rate of the generic naval $C^2$ battle group system and execution schedule, which determine the earliest instant at which the different tasks can occur in the $C^2$ system.

Using a model that simulates these measures, we compare systems and determine an optimal naval $C^2$ battle group system within limited constraints.

## 1. INTRODUCTION

The command and control ability of a naval $C^2$ system initially introduced by Shellard (1985), and defined by Choi and Kuo (1988), to carry out its tasks while defending itself against any threat that may arise is a critical factor in its effectiveness. In this paper, we are concerned with finding the computation rate of operations in generic naval $C^2$ battle group systems. Developed systems have been constructed from devices that have a finite operation speed. Since Petri nets developed by Peterson (1981) and Girault (1982) do not have time parameters as part of their definition, we can model the structure of systems by assuming that the corresponding activity in Petri nets has a finite, nonzero time duration.

Two types of constraints originally studied by Ramchandani (1974) and Hillion (1983) affect the time-related performance of a naval $C^2$ battle group system. The first type is related to the internal structure that determines how the various operations work in the system: some operations are processed one after another, as in sequential operations, and others are processed independently, as in concurrent operations. The sequential and concurrent operations are precisely performed by the naval $C^2$ battle group system. The second type of constraint involves time and resources. More precisely, a time constraint is derived from task execution times: it is the amount of time necessary to perform each operation. In addition, system resources are limited by the capacity of the system to handle certain amounts of input, such as threats from the enemy and offense targets, at the same time.

This set of constraints makes $C^2$ processes (Gally 1985) occur asynchronously and concurrently in real time. The Petri-net formalism (Tabak and Levis 1985) provides a convenient tool for performance analysis of naval $C^2$ systems that have asynchronous and concurrent properties (Ramchandani 1974). This paper, continuing previous work by Choi and Kuo (1988), uses timed Petri nets to model naval $C^2$ battle group systems both as asynchronous and concurrent systems.

In the analysis developed here, the dynamic process of the naval $C^2$ battle group system is investigated for successively arriving input. The objectives of this research task are (1) to identify human, equipment, and organizational factors that make up a naval $C^2$ system, (2) to design the naval $C^2$ system as an asynchronous and concurrent system that satisfies properties of the Petri net for a naval ship and a naval battle group, (3) to analyze the performance of the naval $C^2$ system with respect to the dynamic response time, the maximum throughput rate of the system, and the execution schedule and (4) to develop an optimal generic naval $C^2$ battle group system by using simulation.

### Nomenclature

CIC: Combat Information Center
ASUW: Anti-Surface Warfare
ASW: Anti-Submarine Warfare
AAW: Anti-Aircraft Warfare
EQ: Electronic Warfare

## 2. PETRI NET DESIGN OF NAVAL $C^2$ SYSTEMS

### 2.1. Command and Control System Design

In many engineering sciences, a design is based on mathematical equations, but in command and control systems introduced by Carrington (1973) and subsequently reviewed by Morgan (1985) there are a number of hypothetical elements, such as the human factor, that

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cannot be formulated mathematically. While it is possible to formulate equations related to human behavior by taking the average reaction time of a typical operator to a specific problem, this step would be of no avail to the system designer who is concerned both with human reactions in responding to problems and intuitive behavior. The designer, therefore, adopts a different approach, reducing all the hypothetical problems to a series of factors, by defining the data flow in the system, and showing all the possible paths or branches.

The system itself could be regarded as a black box (Figure 1) where only the input and output are considered in the initial design stages. Starting with the black box output, the designer first analyzes what is required of the system, that is, what information the system has to deliver and how the system responds. These are regarded as the definition of the situation to be displayed for command and control.

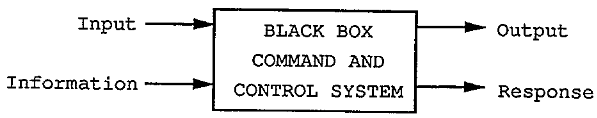


Fig. 1. Command and control system.

### 2.2. Human and Machine Factors of the $C^2$ System

In all command and control systems, the machine (equipment) appears to be the main implementing factor. This impression, however, is erroneous. The machine is not the dominating element but rather an instrument generally employed to relieve humans of the onerous load of repetitive and routine operations and to free them for analytical operations. In no sense is the machine intended to replace the human operator completely.

Command and control systems are human oriented; that is, persons are an essential part of the system, where their main task is decision making. Sensor handling, data collection, data editing, display generating, and the decision distribution can be transferred exclusively to the machine because these functions are routine and repetitive. It is the position of the humans in the decision-making process, studied by Boettcher (1983) and Levis (1984), that is uncertain. Numerous solutions have been proposed for utilizing the human element initially studied by Gundry (1985) in an operational system that is based on the philosophy of the system design. Many systems have failed because of the wrong placement of the human operators in the system. The possible different design philosophies in system are presented here.

1. Where only humans can make the decisions without intervention, work would cease and the operation would be on standby.

2. In a completely automatic system where the machine carries out all the operations, the humans play no active part.

3. An automated system, as in (2), operates with no human intervention. However, when there is a fault in the system, the operation is transferred to a manual backup system. In other words, when the automatic system is unable to carry out its tasks, it is transferred back to its original manual operation.

4. In an automated system, the routine and repetitive decisions are carried out and performed by the machine. Nevertheless, information for command and control must be handled by humans. This design philosophy will be used to design a generic naval $C^2$ system.

### 2.3. Design of the Naval $C^2$ Battle Group System

A generic naval $C^2$ model is developed for describing the structure of asynchronous, concurrent systems, and the finite-state human and machine organizational factors are used to describe the behavior of systems introduced by Tanish (1985). Therefore, each human, machine, and organizational factor of several platforms, such as ships, submarines and aircraft, is modeled as a component of the naval $C^2$ battle group system. After this model was developed, it was verified that the Petri net model of the naval $C^2$ battle group system satisfies Petri net theories and the properties of a well defined system, namely liveness and boundedness (studied by Chretienne and Carlier 1984 and Sifakis 1978): these properties guarantee that the system is deadlock free (liveness) and that information does not accumulate in the system (boundedness).

**2.3.1. Model of the Single Interacting System.** A naval ship carries a complement of sensors and weapons, together with supporting service, which are selected to fulfill the ship's intended operational role. Recently, the $C^2$ system design tended to concentrate on selection of the most up-to-date sensors and weapons on an individual basis, without paying enough attention to their integration or coordination.

Figure 2 shows the aggregate Petri-net model of the single interacting naval $C^2$ system. The process occurs in five parts: Sensor (S), Data base (DB), Man and Machine (MM) [which includes Command Interpretation (CI) and Decision (D)], and Command and Control (CC). Incoming input–either from the environment or from other systems–is processed by the sensor (first part) to detect and track. This information is then analyzed and predictions are made with a data base that is connected with other sensors. The resulting information is combined with commands received by the human or machine part so as to select a signal in the command and control part. Each of the five parts, which corresponds to a particular task performed by the system, is modeled by a transition. The firing of a transition represents the execution of a particular task performed by the system. According to the rules of operation of a Petri net, a transition can fire when it is enabled, that is, when there is a least one token in each of its input places. When a

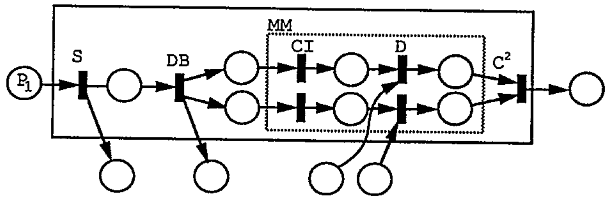


Fig. 2. Model of the interacting naval $C^2$ system.

transition fires, one token is moved from each input place and one token is added in each output place. If we assume that tokens represent information messages or signals, the flow of tokens models the flow of information or signals in the process (Bejjani 1983).

**2.3.2. Model of the Interacting Naval C2 System with Limited Resources.** The model of the interacting system does not take into account the limited capacity for information processing or command control's response that characterizes the system. As long as information messages are present (tokens are available in place $p_1$), the processing can start (i.e., the transition corresponding to the sensor part can fire). However, information processing of systems is subject to the bounded rationality constraint originally studied by Hillion (1983). In the information and theoretical approach, the amount of information processed is measured by the total activity, G, of the system, which also characterizes the system's workload. It is assumed that there exists an upper bound, $G_r$, above which the system becomes overloaded and performance degrades:

$$G \leq G_r$$

When the analysis is carried out for the steady-state process, the above constraint takes the form

$$G \leq F\,t = G_r \qquad (1)$$

where F is the processing rate constraint that characterizes the system, and 1/t (response t) is the average arrival rate of inputs (response average interarrival time). This constraint implies that the system must process inputs at a rate at least equal to their arrival. In this paper, time-related performance measures were dealt with and do not address the accuracy of the response, which is based on the comparison between the actual system's reponse and the ideal or desired response. In particular, the way a system reacts to information overload (which occurs when $G > F\ t$) and the extent to which it affects its performance are not matters of concern here. This remains allowable, however, insofar as the actual processing constraint can be modeled by writing inequality (1) in another form:

$$1/t \leq F/G \qquad (2)$$

When relation (2) is used, the bounded rationality limitation turns out to be a constraint on the allowable rate of incoming input, that is, on the maximum rate of input that can be handled by the system without overloading. We should not assume that input is processed at a rate at least equal to the rate at which it arrives in order to derive a bound on the total activity allowable, G (given F). Thus, no information overload occurs in the process and the corresponding constraint on the allowable rate of incoming input could be derived.

Let us see how it is possible to model this constraint using the Petri-net framework. The limited processing capabilities of the system result from limited resources available to perform the various processing tasks. In particular, the bounded rationality constraint is very much related to the limited capacity of the system. Indeed, this bound means that a system cannot properly handle too much input at the same time. This fact is important in evaluating a system that must precisely handle different inputs during the time necessary to complete the various processing tasks. This is the reason why the bound was identified, and the limited capacity of the single interacting naval C2 system is modeled by using the Petri net formalism as shown in Figure 3.

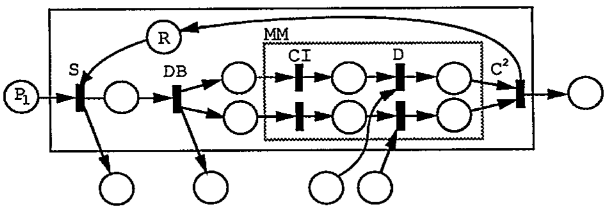


Fig. 3. Model of the interacting naval C2 system with limited resources.

### 2.4. A Generic Naval C2 System

Having analyzed the model of the simple interacting system, we present the aggregated Petri net model studied by Hillion (1983) for a single ship and a battle group. First, the interactions between the ship and the environment are described.

**2.4.1. Naval C2 System Configuration.** The naval C2 system is made up of several cooperating systems introduced by Hill (1985), each of which will have a specified operational function or functions. These cooperating systems may be sensor or weapons systems that are supported by such systems as the ship's reference information and the command control system (Figure 4). In this paper, an overall naval C2 system is classified according to the role of systems.

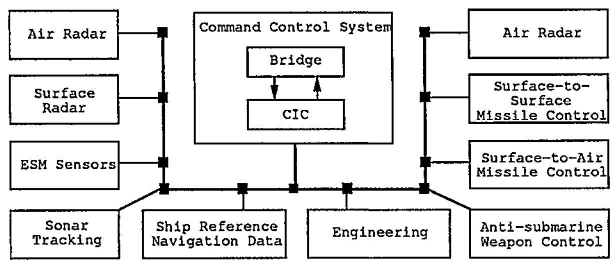


Fig. 4. The ship's command control system.

**2.4.2. Model of the Naval C2 System with the Environment.** The first processing part of the system partitions the external information or messages into a set of inputs that is assigned to different systems. Likewise, a set of outputs characterizes the last processing. Therefore, if the systems classified in the previous section interact directly with the environment, the corresponding Petri net representation is as shown on Figure 5. The place $P_s$ represents the source of information or messages, and the transition $t_1$ models the partitioning operation. Since it is the first processing part, $t_1$ will be called the input transition of the process. Each time that $t_1$ fires, one token is sent to the input place of the set of the interacting systems. The place $p_r$ represents the output transition, which is the overall system response.

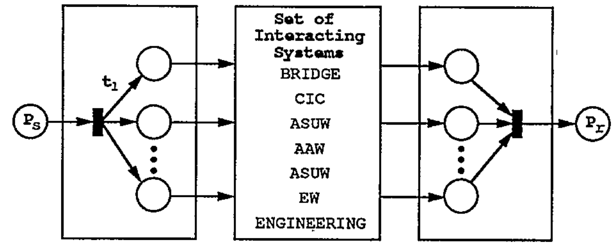


Fig. 5. Model of the naval C2 system with the environment.

**2.4.3. Model of the Naval C2 System with Limited Resources.** The analysis of the resources constraint–as carried out in the previous section for a single interacting system–can be applied to the overall system. The resources used by the overall system may have various forms, but at least a processing constraint always exists that comes from the limited capacity of the structural places. The resulting model is shown in Figure 6.

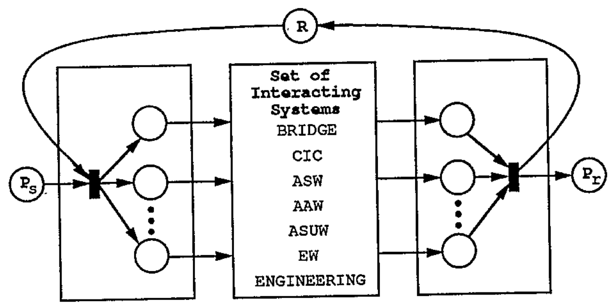


Fig. 6. Model of the naval C2 ystem with limited resources.

**2.4.4. A generic C2 System of a Naval Ship.** According to the previous section, a generic C2 system of a naval ship is modeled in Figure 7. Also, the task processing times are included in the model by assigning to each transition a corresponding firing time (execution time or processing time).

**2.4.5. A C2 System of a Naval Battle Group.** A naval battle group is defined as consisting of at least one command ship together with several co-operation platforms such as ships, submarines, and aircrafts. The command ship and their platforms contain a wide variety of sensors and weapon systems that allow the battle group to carry out defensive and offensive missions as prescribed by higher authority. The defense and offense of the battle group assets are clearly of paramount importance. In a defense case, the battle group threat consists of enemy submarines, surface ships, and canon projectiles. The defense and offense of the battle group involve anti-submarine warfare (ASW), anti-surface warfare (ASUW), anti-aircraft warfare (AAW), and electronic warfare (EW). According to the previous section, a C2 system of a naval battle group consists of two ships (command ship and escorting ship) as shown in Figure 8.

## 3. PERFORMANCE ANALYSIS OF THE NAVAL C2 SYSTEM

### 3.1. Maximum Throughput Rate of the System

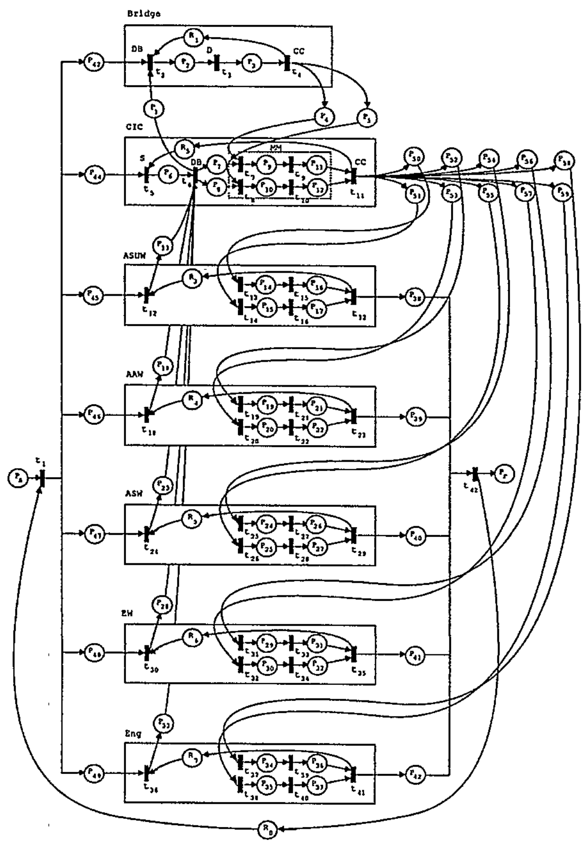


Fig. 7. A generic naval C2 system.

If external input is arriving continously at a rate that is low enough, the naval C2 system will be able to handle all input as soon as it arrives. Then, the rate at which input is being processed will precisely correspond to its arrival rate. However, beyond a certain input arrival rate, the naval C2 system will be overloaded: input will queue at the entry of the system, this queue will grow infinitely over time, and the naval C2 system will never catch up. This bound precisely determines the maximum throughput rate of the system, originally studied by Chretienne and Carlier (1984) and subsequently applied by Hillion (1983). This measure of performance, which characterizes the maximum rate of processing of the overall system, is important because it bounds the allowable rate of external input that can be handled by the naval C2 system. Given the internal structure of the system, the maximum throughput rate is characterized as a function of both time constraints (i.e., the various task processing times) and resource constraints. As a result, the system works better whenever the maximum throughput rate is larger.

### 3.2. The Execution Schedule

Assume that the processing starts at time = 0 and that it occurs repetitively at its maximum throughput rate. There is a need to determine the earliest instance of time at which the various tasks can occur in the repetitive process. In this paper, the naval C2 battle group system will be allowed to process simultaneously more than one input in

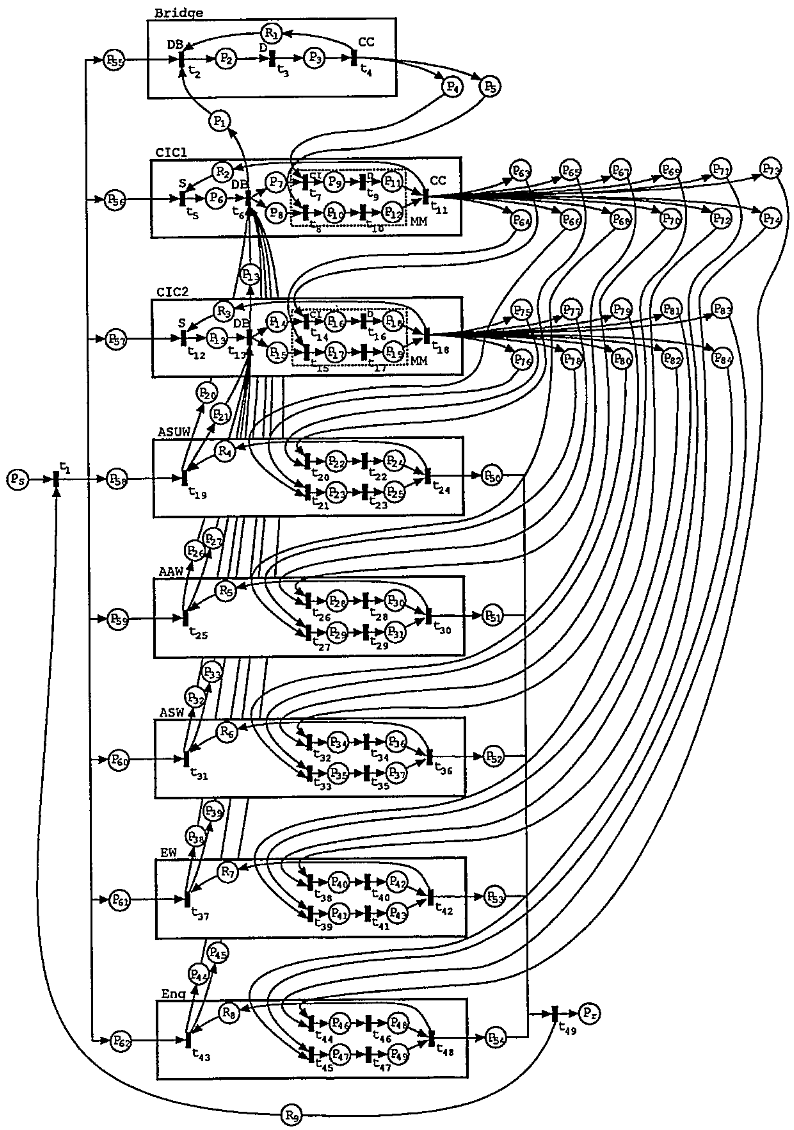


Fig. 8. A C2 system of a naval battle group consisting of the command ship and the escorting ship.

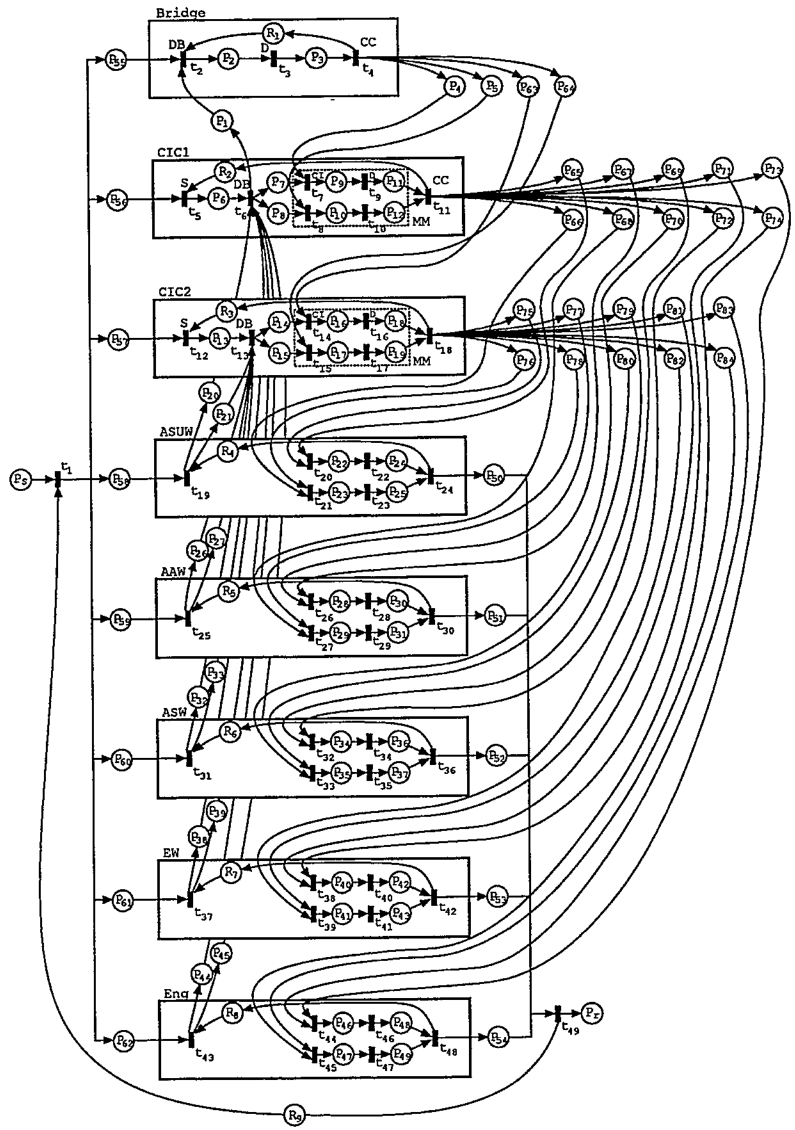


Fig. 9. Model of the C2 naval system.

order to obtain dynamic response. In particular, if various input arrive simultaneously, it will be possible to determine the response time, which corresponds to the time interval between the moment the input was received and the moment a response to each input was made. The execution schedule (i.e., the epoch of times at which the various operations occur in the process) is determined for allowable rates of incoming input. Note that the firing schedule computed here will characterize the dynamic behavior of the naval $C^2$ battle group system. Starting from an initial state, the process can occur repetitively, which assumes that input is always available at the entry of the system. Thus, the best performance will be obtained with respect to real-time processing. The measures of performance described above are important in the performance evaluation of the naval $C^2$ battle group system. Suppose that the goal of a naval $C^2$ system is to detect, track, analyze, predict, and allocate weapons, and respond to threats such as ships, submarines, and aircraft. If several threats arrive simultaneously, it will be possible from the execution schedule to evaluate the ability of the system to respond to all these threats in a certain time duration.

### 3.3. Performance Analysis Criteria

The generic naval $C^2$ battle group system, which was described in the previous section, is simulated by using a simulation model to analyze performance and to provide an optimal system within certain time durations according to the $C^2$ effectiveness studied by Georgiou and Lammers (1985). The structure of the system and the processing time of transitions are fixed, but the system resource, which is the capacity to handle a certain amount of inputs (threats) at the same time, is varied. Before the naval $C^2$ system is simulated, we need to determine concurrent tasks, which are unordered (parallel) tasks, sequential tasks which are strictly ordered, and input transitions.

The simulation model is described in Figure 9, and the results are obtained for the 5,754,801 unique cases (the generic $C^2$ of a naval ship) and for the 40,353,607 unique cases (the generic $C^2$ of a battle group) are considered. However, because of the large number, a single optimal generic $C^2$ system model of a naval ship and battle group case are presented in Table 1 and Table 2, respectively. They contain the following information:

- limited time duration corresponding to the amount of time that the overall system can have to respond against inputs (threats)
- the transition firing times, which correspond to the task processing times, which are assumed to be units of time, and the initial marking of the resource places corresponding to the resources available for processing is assumed to be one to seven units of capacity because the analysis of the human memory has shown that a maximum of only six or seven units of information can be held in this memory without any loss of information
- the firing schedule obtained for a total of N repetitions of the process that corresponds to the number of inputs (threats) processed (the choice of N is arbitrary and can be as large as desired).

  Accordingly, the firing schedule will correspond to the sequence ($S_n^i$), for i = 1,2,...,K (total number of transitions in the net) and n = 1,2,...,N.

From this information, the following measures of performance will be computed and compared:

- the maximum throughput rate
- the dynamic response time corresponding to the complete processsing of N ($S_n^i$ + processing time of the transition + K)

### 3.4. Performance Analysis

The performances obtained for cases where each resource limit is varied from one to seven are compared. The maximum throughput (the generic $C^2$ system of a naval ship) rate is largest in $R_0 = 7, R_1 = 2, R_2 = 5, R_3 = 5, R_4 = 5, R_6 = 5$, and $R_7 = 5$, and the dynamic response time is in the limited time duration. If there are more resources available for processing, the performance is not improved. The maximum throughput rate is exactly the same as the one obtained in the former case. This is a very important result because existing equipment and trained humans are needed to increase each system's resource limit. In addition, the $C^2$ system of the naval battle group that was developed in the previous section is simulated by using the same simulation model. As a result, a new $C^2$ system of the naval battle group (see Figure 10 and Table 3) is considered to be better than the existing $C^2$ system of the naval battle group (see Figure 8). This conclusion suggests that the new $C^2$ system of the naval battle group is better than the existing $C^2$ system in a real warfare situation, but we should consider certain operational constraints such as communication range, ability of the $C^2$ system, and the like.

## 4. CONCLUSION

In this paper, modeling of the generic naval $C^2$ battle group system as an asynchronous concurrent system was accomplished, and the maximum throughput rate was expressed directly as a function of both the task processing times and the resources available. In addition, the execution schedule of the system is determined when the processing of inputs occurs at its maximum throughput rate. These measures of performance characterize the best dynamic performance achievable by the naval $C^2$ battle group system, as determined by the maximum rate at which input can be processed and the earliest instance of time at which the various tasks can be executed. These measures provide a useful tool to analyze and compare systems that have different resource limits to develop an optimal generic naval $C^2$ battle group system. In particular, the precise characterization of the resource and time constraints makes it possible to investigate which of the constraints should be modified in the actual design so as to improve the time performance.

## ACKNOWLEDGMENTS

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Table 1. Computation of the Execution Schedule: Optimal Case.

Reource and Time Constraints

Marking of the Resource Places:

| $R_0$ | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ | $R_6$ | $R_7$ |
|---|---|---|---|---|---|---|---|
| 7 | 2 | 5 | 5 | 5 | 5 | 5 | 5 |

Limited Time Duration: 41 units of time

Transition Firing Times:

| $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_5$ | $t_6$ | $t_7$ | $t_8$ | $t_9$ | $t_{10}$ | $t_{11}$ | $t_{12}$ | $t_{13}$ | $t_{14}$ | $t_{15}$ | $t_{16}$ | $t_{17}$ | $t_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

| $t_{19}$ | $t_{20}$ | $t_{21}$ | $t_{22}$ | $t_{23}$ | $t_{24}$ | $t_{25}$ | $t_{26}$ | $t_{27}$ | $t_{28}$ | $t_{29}$ | $t_{30}$ | $t_{31}$ | $t_{32}$ | $t_{33}$ | $t_{34}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

| $t_{35}$ | $t_{36}$ | $t_{37}$ | $t_{38}$ | $t_{39}$ | $t_{40}$ | $t_{41}$ | $t_{42}$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $t_1$ | 0.0 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 13.0 | 14.0 | 16.0 | 17.0 | 19.0 | 24.0 | 25.0 | 27.0 |
| $t_5$ | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 9.0 | 10.0 | 14.0 | 15.0 | 17.0 | 20.0 | 21.0 | 25.0 | 26.0 | 28.0 |
| $t_{12}$ | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 12.0 | 13.0 | 15.0 | 16.0 | 18.0 | 23.0 | 24.0 | 26.0 | 27.0 | 29.0 |
| $t_{18}$ | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 12.0 | 13.0 | 15.0 | 16.0 | 18.0 | 23.0 | 24.0 | 26.0 | 27.0 | 29.0 |
| $t_{24}$ | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 12.0 | 13.0 | 15.0 | 16.0 | 18.0 | 23.0 | 24.0 | 26.0 | 27.0 | 29.0 |
| $t_{30}$ | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 12.0 | 13.0 | 15.0 | 16.0 | 18.0 | 23.0 | 24.0 | 26.0 | 27.0 | 29.0 |
| $t_{36}$ | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 12.0 | 13.0 | 15.0 | 16.0 | 18.0 | 23.0 | 24.0 | 26.0 | 27.0 | 29.0 |
| $t_6$ | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 13.0 | 14.0 | 16.0 | 17.0 | 19.0 | 24.0 | 25.0 | 27.0 | 28.0 | 30.0 |
| $t_2$ | 3.0 | 4.0 | 6.0 | 7.0 | 9.0 | 14.0 | 15.0 | 17.0 | 18.0 | 20.0 | 25.0 | 26.0 | 28.0 | 29.0 | 31.0 |
| $t_3$ | 4.0 | 5.0 | 7.0 | 8.0 | 10.0 | 15.0 | 16.0 | 18.0 | 19.0 | 21.0 | 26.0 | 27.0 | 29.0 | 30.0 | 32.0 |
| $t_4$ | 5.0 | 6.0 | 8.0 | 9.0 | 11.0 | 16.0 | 17.0 | 19.0 | 20.0 | 22.0 | 27.0 | 28.0 | 30.0 | 31.0 | 33.0 |
| $t_7$ | 6.0 | 7.0 | 9.0 | 10.0 | 12.0 | 17.0 | 18.0 | 20.0 | 21.0 | 23.0 | 28.0 | 29.0 | 31.0 | 32.0 | 34.0 |
| $t_8$ | 6.0 | 7.0 | 9.0 | 10.0 | 12.0 | 17.0 | 18.0 | 20.0 | 21.0 | 23.0 | 28.0 | 29.0 | 31.0 | 32.0 | 34.0 |
| $t_9$ | 7.0 | 8.0 | 10.0 | 11.0 | 13.0 | 18.0 | 19.0 | 21.0 | 22.0 | 24.0 | 29.0 | 30.0 | 32.0 | 33.0 | 35.0 |
| $t_{10}$ | 7.0 | 8.0 | 10.0 | 11.0 | 13.0 | 18.0 | 19.0 | 21.0 | 22.0 | 24.0 | 29.0 | 30.0 | 32.0 | 33.0 | 35.0 |
| $t_{11}$ | 8.0 | 9.0 | 11.0 | 12.0 | 14.0 | 19.0 | 20.0 | 22.0 | 23.0 | 25.0 | 30.0 | 31.0 | 33.0 | 34.0 | 36.0 |
| $t_{13}$ | 9.0 | 10.0 | 12.0 | 13.0 | 15.0 | 20.0 | 21.0 | 23.0 | 24.0 | 26.0 | 31.0 | 32.0 | 34.0 | 35.0 | 37.0 |
| $t_{14}$ | 9.0 | 10.0 | 12.0 | 13.0 | 15.0 | 20.0 | 21.0 | 23.0 | 24.0 | 26.0 | 31.0 | 32.0 | 34.0 | 35.0 | 37.0 |
| $t_{19}$ | 9.0 | 10.0 | 12.0 | 13.0 | 15.0 | 20.0 | 21.0 | 23.0 | 24.0 | 26.0 | 31.0 | 32.0 | 34.0 | 35.0 | 37.0 |
| $t_{20}$ | 9.0 | 10.0 | 12.0 | 13.0 | 15.0 | 20.0 | 21.0 | 23.0 | 24.0 | 26.0 | 31.0 | 32.0 | 34.0 | 35.0 | 37.0 |
| $t_{25}$ | 9.0 | 10.0 | 12.0 | 13.0 | 15.0 | 20.0 | 21.0 | 23.0 | 24.0 | 26.0 | 31.0 | 32.0 | 34.0 | 35.0 | 37.0 |
| $t_{26}$ | 9.0 | 10.0 | 12.0 | 13.0 | 15.0 | 20.0 | 21.0 | 23.0 | 24.0 | 26.0 | 31.0 | 32.0 | 34.0 | 35.0 | 37.0 |
| $t_{31}$ | 9.0 | 10.0 | 12.0 | 13.0 | 15.0 | 20.0 | 21.0 | 23.0 | 24.0 | 26.0 | 31.0 | 32.0 | 34.0 | 35.0 | 37.0 |
| $t_{32}$ | 9.0 | 10.0 | 12.0 | 13.0 | 15.0 | 20.0 | 21.0 | 23.0 | 24.0 | 26.0 | 31.0 | 32.0 | 34.0 | 35.0 | 37.0 |
| $t_{37}$ | 9.0 | 10.0 | 12.0 | 13.0 | 15.0 | 20.0 | 21.0 | 23.0 | 24.0 | 26.0 | 31.0 | 32.0 | 34.0 | 35.0 | 37.0 |
| $t_{38}$ | 9.0 | 10.0 | 12.0 | 13.0 | 15.0 | 20.0 | 21.0 | 23.0 | 24.0 | 26.0 | 31.0 | 32.0 | 34.0 | 35.0 | 37.0 |
| $t_{15}$ | 10.0 | 11.0 | 13.0 | 14.0 | 16.0 | 21.0 | 22.0 | 24.0 | 25.0 | 27.0 | 32.0 | 33.0 | 35.0 | 36.0 | 38.0 |
| $t_{16}$ | 10.0 | 11.0 | 13.0 | 14.0 | 16.0 | 21.0 | 22.0 | 24.0 | 25.0 | 27.0 | 32.0 | 33.0 | 35.0 | 36.0 | 38.0 |
| $t_{21}$ | 10.0 | 11.0 | 13.0 | 14.0 | 16.0 | 21.0 | 22.0 | 24.0 | 25.0 | 27.0 | 32.0 | 33.0 | 35.0 | 36.0 | 38.0 |
| $t_{22}$ | 10.0 | 11.0 | 13.0 | 14.0 | 16.0 | 21.0 | 22.0 | 24.0 | 25.0 | 27.0 | 32.0 | 33.0 | 35.0 | 36.0 | 38.0 |
| $t_{27}$ | 10.0 | 11.0 | 13.0 | 14.0 | 16.0 | 21.0 | 22.0 | 24.0 | 25.0 | 27.0 | 32.0 | 33.0 | 35.0 | 36.0 | 38.0 |
| $t_{28}$ | 10.0 | 11.0 | 13.0 | 14.0 | 16.0 | 21.0 | 22.0 | 24.0 | 25.0 | 27.0 | 32.0 | 33.0 | 35.0 | 36.0 | 38.0 |
| $t_{33}$ | 10.0 | 11.0 | 13.0 | 14.0 | 16.0 | 21.0 | 22.0 | 24.0 | 25.0 | 27.0 | 32.0 | 33.0 | 35.0 | 36.0 | 38.0 |
| $t_{34}$ | 10.0 | 11.0 | 13.0 | 14.0 | 16.0 | 21.0 | 22.0 | 24.0 | 25.0 | 27.0 | 32.0 | 33.0 | 35.0 | 36.0 | 38.0 |
| $t_{39}$ | 10.0 | 11.0 | 13.0 | 14.0 | 16.0 | 21.0 | 22.0 | 24.0 | 25.0 | 27.0 | 32.0 | 33.0 | 35.0 | 36.0 | 38.0 |
| $t_{40}$ | 10.0 | 11.0 | 13.0 | 14.0 | 16.0 | 21.0 | 22.0 | 24.0 | 25.0 | 27.0 | 32.0 | 33.0 | 35.0 | 36.0 | 38.0 |
| $t_{17}$ | 11.0 | 12.0 | 14.0 | 15.0 | 17.0 | 22.0 | 23.0 | 25.0 | 26.0 | 28.0 | 33.0 | 34.0 | 36.0 | 37.0 | 39.0 |
| $t_{23}$ | 11.0 | 12.0 | 14.0 | 15.0 | 17.0 | 22.0 | 23.0 | 25.0 | 26.0 | 28.0 | 33.0 | 34.0 | 36.0 | 37.0 | 39.0 |
| $t_{29}$ | 11.0 | 12.0 | 14.0 | 15.0 | 17.0 | 22.0 | 23.0 | 25.0 | 26.0 | 28.0 | 33.0 | 34.0 | 36.0 | 37.0 | 39.0 |
| $t_{35}$ | 11.0 | 12.0 | 14.0 | 15.0 | 17.0 | 22.0 | 23.0 | 25.0 | 26.0 | 28.0 | 33.0 | 34.0 | 36.0 | 37.0 | 39.0 |
| $t_{41}$ | 11.0 | 12.0 | 14.0 | 15.0 | 17.0 | 22.0 | 23.0 | 25.0 | 26.0 | 28.0 | 33.0 | 34.0 | 36.0 | 37.0 | 39.0 |
| $t_{42}$ | 12.0 | 13.0 | 15.0 | 16.0 | 18.0 | 23.0 | 24.0 | 26.0 | 27.0 | 29.0 | 34.0 | 35.0 | 37.0 | 38.0 | 40.0 |

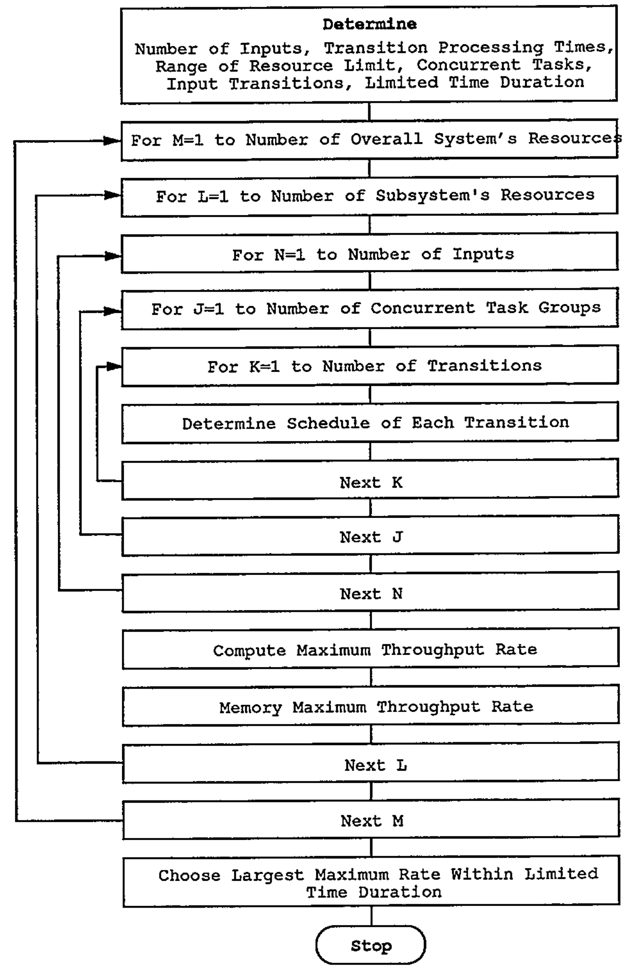


Fig. 10. Simulation model.

Table 2. Computation of the Execution Schedule: Optimal Case.

Reource and Time Constraints

| Marking of the Resource Places: | $R_0$ | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ | $R_6$ | $R_7$ |
|---|---|---|---|---|---|---|---|---|
| | 7 | 2 | 5 | 5 | 5 | 5 | 5 | 5 |

Limited Time Duration: 15 units of time

Transition Firing Times:

| $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_5$ | $t_6$ | $t_7$ | $t_8$ | $t_9$ | $t_{10}$ | $t_{11}$ | $t_{12}$ | $t_{13}$ | $t_{14}$ | $t_{15}$ | $t_{16}$ | $t_{17}$ | $t_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 1 | 1 | 3 | 2 | 4 | 1 | 2 | 3 | 4 | 2 | 4 | 2 | 3 | 2 | 3 | 1 |

| $t_{19}$ | $t_{20}$ | $t_{21}$ | $t_{22}$ | $t_{23}$ | $t_{24}$ | $t_{25}$ | $t_{26}$ | $t_{27}$ | $t_{28}$ | $t_{29}$ | $t_{30}$ | $t_{31}$ | $t_{32}$ | $t_{33}$ | $t_{34}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 1 | 3 | 4 | 3 | 1 | 3 | 2 | 4 | 1 | 1 | 3 | 1 | 2 | 4 | 3 |

| $t_{35}$ | $t_{36}$ | $t_{37}$ | $t_{38}$ | $t_{39}$ | $t_{40}$ | $t_{41}$ | $t_{42}$ | $t_{43}$ | $t_{44}$ | $t_{45}$ | $t_{46}$ | $t_{47}$ | $t_{48}$ | $t_{49}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 3 | 4 | 1 | 1 | 3 | 2 | 2 | 4 | 1 | 2 | 2 | 3 | 4 |

Execution Schedule

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $t_1$ | 0.0 | 2.0 | 4.0 | 6.0 | 8.0 | 10.0 | 12.0 | 40.0 | 44.0 | 48.0 | 52.0 | 56.0 | 72.0 | 76.0 | 80.0 | 84.0 | 88.0 | 104.0 | 108.0 | 112.0 |
| $t_5$ | 3.0 | 6.0 | 9.0 | 12.0 | 15.0 | 28.0 | 32.0 | 43.0 | 47.0 | 51.0 | 60.0 | 64.0 | 75.0 | 79.0 | 83.0 | 92.0 | 96.0 | 107.0 | 111.0 | 115.0 |
| $t_{12}$ | 2.0 | 4.0 | 6.0 | 8.0 | 10.0 | 12.0 | 14.0 | 42.0 | 46.0 | 50.0 | 54.0 | 58.0 | 74.0 | 78.0 | 82.0 | 86.0 | 90.0 | 106.0 | 110.0 | 114.0 |
| $t_{19}$ | 4.0 | 8.0 | 12.0 | 16.0 | 20.0 | 36.0 | 40.0 | 44.0 | 48.0 | 52.0 | 68.0 | 72.0 | 76.0 | 80.0 | 84.0 | 100.0 | 104.0 | 108.0 | 112.0 | 116.0 |
| $t_{25}$ | 3.0 | 6.0 | 9.0 | 12.0 | 15.0 | 36.0 | 40.0 | 44.0 | 48.0 | 52.0 | 68.0 | 72.0 | 76.0 | 80.0 | 84.0 | 100.0 | 104.0 | 108.0 | 112.0 | 116.0 |
| $t_{31}$ | 1.0 | 3.0 | 5.0 | 7.0 | 9.0 | 34.0 | 38.0 | 42.0 | 46.0 | 50.0 | 66.0 | 70.0 | 74.0 | 78.0 | 82.0 | 98.0 | 102.0 | 106.0 | 110.0 | 114.0 |
| $t_{37}$ | 3.0 | 6.0 | 9.0 | 12.0 | 15.0 | 35.0 | 39.0 | 43.0 | 47.0 | 51.0 | 67.0 | 71.0 | 75.0 | 79.0 | 83.0 | 99.0 | 103.0 | 107.0 | 111.0 | 115.0 |
| $t_{43}$ | 2.0 | 4.0 | 6.0 | 8.0 | 10.0 | 36.0 | 40.0 | 44.0 | 48.0 | 52.0 | 68.0 | 72.0 | 76.0 | 80.0 | 84.0 | 100.0 | 104.0 | 108.0 | 112.0 | 116.0 |
| $t_{13}$ | 8.0 | 12.0 | 16.0 | 20.0 | 24.0 | 40.0 | 44.0 | 48.0 | 52.0 | 56.0 | 72.0 | 76.0 | 80.0 | 84.0 | 88.0 | 104.0 | 108.0 | 112.0 | 116.0 | 120.0 |
| $t_6$ | 10.0 | 14.0 | 18.0 | 22.0 | 26.0 | 42.0 | 46.0 | 50.0 | 54.0 | 58.0 | 74.0 | 78.0 | 82.0 | 86.0 | 90.0 | 106.0 | 110.0 | 114.0 | 118.0 | 122.0 |
| $t_2$ | 13.0 | 17.0 | 21.0 | 25.0 | 29.0 | 45.0 | 49.0 | 53.0 | 57.0 | 61.0 | 77.0 | 81.0 | 85.0 | 89.0 | 93.0 | 109.0 | 113.0 | 117.0 | 121.0 | 125.0 |
| $t_3$ | 14.0 | 18.0 | 22.0 | 26.0 | 30.0 | 46.0 | 50.0 | 54.0 | 58.0 | 62.0 | 78.0 | 82.0 | 86.0 | 90.0 | 94.0 | 110.0 | 114.0 | 118.0 | 122.0 | 126.0 |
| $t_4$ | 15.0 | 19.0 | 23.0 | 27.0 | 31.0 | 47.0 | 51.0 | 55.0 | 59.0 | 63.0 | 79.0 | 83.0 | 87.0 | 91.0 | 95.0 | 111.0 | 115.0 | 119.0 | 123.0 | 127.0 |
| $t_7$ | 19.0 | 23.0 | 27.0 | 31.0 | 35.0 | 51.0 | 55.0 | 59.0 | 63.0 | 67.0 | 83.0 | 87.0 | 91.0 | 95.0 | 99.0 | 115.0 | 119.0 | 123.0 | 127.0 | 131.0 |
| $t_8$ | 16.0 | 20.0 | 24.0 | 28.0 | 32.0 | 48.0 | 52.0 | 56.0 | 60.0 | 64.0 | 80.0 | 84.0 | 88.0 | 92.0 | 96.0 | 112.0 | 116.0 | 120.0 | 124.0 | 128.0 |
| $t_9$ | 21.0 | 25.0 | 29.0 | 33.0 | 37.0 | 53.0 | 57.0 | 61.0 | 65.0 | 69.0 | 85.0 | 89.0 | 93.0 | 97.0 | 101.0 | 117.0 | 121.0 | 125.0 | 129.0 | 133.0 |
| $t_{10}$ | 19.0 | 23.0 | 27.0 | 31.0 | 35.0 | 51.0 | 55.0 | 59.0 | 63.0 | 67.0 | 83.0 | 87.0 | 91.0 | 95.0 | 99.0 | 115.0 | 119.0 | 123.0 | 127.0 | 131.0 |
| $t_{11}$ | 25.0 | 29.0 | 33.0 | 37.0 | 41.0 | 57.0 | 61.0 | 65.0 | 69.0 | 73.0 | 89.0 | 93.0 | 97.0 | 101.0 | 105.0 | 121.0 | 125.0 | 129.0 | 133.0 | 137.0 |
| $t_{18}$ | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 | 11.0 | 12.0 | 13.0 | 14.0 | 15.0 | 16.0 | 17.0 | 18.0 | 19.0 | 20.0 |
| $t_{14}$ | 27.0 | 31.0 | 35.0 | 39.0 | 43.0 | 59.0 | 63.0 | 67.0 | 71.0 | 75.0 | 91.0 | 95.0 | 99.0 | 103.0 | 107.0 | 123.0 | 127.0 | 131.0 | 135.0 | 139.0 |
| $t_{15}$ | 28.0 | 32.0 | 36.0 | 40.0 | 44.0 | 60.0 | 64.0 | 68.0 | 72.0 | 76.0 | 92.0 | 96.0 | 100.0 | 104.0 | 108.0 | 124.0 | 128.0 | 132.0 | 136.0 | 140.0 |
| $t_{20}$ | 26.0 | 30.0 | 34.0 | 38.0 | 42.0 | 58.0 | 62.0 | 66.0 | 70.0 | 74.0 | 90.0 | 94.0 | 98.0 | 102.0 | 106.0 | 122.0 | 126.0 | 130.0 | 134.0 | 138.0 |
| $t_{21}$ | 28.0 | 32.0 | 36.0 | 40.0 | 44.0 | 60.0 | 64.0 | 68.0 | 72.0 | 76.0 | 92.0 | 96.0 | 100.0 | 104.0 | 108.0 | 124.0 | 128.0 | 132.0 | 136.0 | 140.0 |
| $t_{26}$ | 27.0 | 31.0 | 35.0 | 39.0 | 43.0 | 59.0 | 63.0 | 67.0 | 71.0 | 75.0 | 91.0 | 95.0 | 99.0 | 103.0 | 107.0 | 123.0 | 127.0 | 131.0 | 135.0 | 139.0 |
| $t_{27}$ | 29.0 | 33.0 | 37.0 | 41.0 | 45.0 | 61.0 | 65.0 | 69.0 | 73.0 | 77.0 | 93.0 | 97.0 | 101.0 | 105.0 | 109.0 | 125.0 | 129.0 | 133.0 | 137.0 | 141.0 |
| $t_{32}$ | 27.0 | 31.0 | 35.0 | 39.0 | 43.0 | 59.0 | 63.0 | 67.0 | 71.0 | 75.0 | 91.0 | 95.0 | 99.0 | 103.0 | 107.0 | 123.0 | 127.0 | 131.0 | 135.0 | 139.0 |
| $t_{33}$ | 29.0 | 33.0 | 37.0 | 41.0 | 45.0 | 61.0 | 65.0 | 69.0 | 73.0 | 77.0 | 93.0 | 97.0 | 101.0 | 105.0 | 109.0 | 125.0 | 129.0 | 133.0 | 137.0 | 141.0 |
| $t_{38}$ | 29.0 | 33.0 | 37.0 | 41.0 | 45.0 | 61.0 | 65.0 | 69.0 | 73.0 | 77.0 | 93.0 | 97.0 | 101.0 | 105.0 | 109.0 | 125.0 | 129.0 | 133.0 | 137.0 | 141.0 |
| $t_{39}$ | 26.0 | 30.0 | 34.0 | 38.0 | 42.0 | 58.0 | 62.0 | 66.0 | 70.0 | 74.0 | 90.0 | 94.0 | 98.0 | 102.0 | 106.0 | 122.0 | 126.0 | 130.0 | 134.0 | 138.0 |
| $t_{44}$ | 29.0 | 33.0 | 37.0 | 41.0 | 45.0 | 61.0 | 65.0 | 69.0 | 73.0 | 77.0 | 93.0 | 97.0 | 101.0 | 105.0 | 109.0 | 125.0 | 129.0 | 133.0 | 137.0 | 141.0 |
| $t_{45}$ | 26.0 | 30.0 | 34.0 | 38.0 | 42.0 | 58.0 | 62.0 | 66.0 | 70.0 | 74.0 | 90.0 | 94.0 | 98.0 | 102.0 | 106.0 | 122.0 | 126.0 | 130.0 | 134.0 | 138.0 |
| $t_{16}$ | 29.0 | 33.0 | 37.0 | 41.0 | 45.0 | 61.0 | 65.0 | 69.0 | 73.0 | 77.0 | 93.0 | 97.0 | 101.0 | 105.0 | 109.0 | 125.0 | 129.0 | 133.0 | 137.0 | 141.0 |
| $t_{17}$ | 31.0 | 35.0 | 39.0 | 43.0 | 47.0 | 63.0 | 67.0 | 71.0 | 75.0 | 79.0 | 95.0 | 99.0 | 103.0 | 107.0 | 111.0 | 127.0 | 131.0 | 135.0 | 139.0 | 143.0 |
| $t_{22}$ | 30.0 | 34.0 | 38.0 | 42.0 | 46.0 | 62.0 | 66.0 | 70.0 | 74.0 | 78.0 | 94.0 | 98.0 | 102.0 | 106.0 | 110.0 | 126.0 | 130.0 | 134.0 | 138.0 | 142.0 |
| $t_{23}$ | 31.0 | 35.0 | 39.0 | 43.0 | 47.0 | 63.0 | 67.0 | 71.0 | 75.0 | 79.0 | 95.0 | 99.0 | 103.0 | 107.0 | 111.0 | 127.0 | 131.0 | 135.0 | 139.0 | 143.0 |
| $t_{28}$ | 28.0 | 32.0 | 36.0 | 40.0 | 44.0 | 60.0 | 64.0 | 68.0 | 72.0 | 76.0 | 92.0 | 96.0 | 100.0 | 104.0 | 108.0 | 124.0 | 128.0 | 132.0 | 136.0 | 140.0 |
| $t_{29}$ | 30.0 | 34.0 | 38.0 | 42.0 | 46.0 | 62.0 | 66.0 | 70.0 | 74.0 | 78.0 | 94.0 | 98.0 | 102.0 | 106.0 | 110.0 | 126.0 | 130.0 | 134.0 | 138.0 | 142.0 |
| $t_{34}$ | 30.0 | 34.0 | 38.0 | 42.0 | 46.0 | 62.0 | 66.0 | 70.0 | 74.0 | 78.0 | 94.0 | 98.0 | 102.0 | 106.0 | 110.0 | 126.0 | 130.0 | 134.0 | 138.0 | 142.0 |
| $t_{35}$ | 31.0 | 35.0 | 39.0 | 43.0 | 47.0 | 63.0 | 67.0 | 71.0 | 75.0 | 79.0 | 95.0 | 99.0 | 103.0 | 107.0 | 111.0 | 127.0 | 131.0 | 135.0 | 139.0 | 143.0 |
| $t_{40}$ | 30.0 | 34.0 | 38.0 | 42.0 | 46.0 | 62.0 | 66.0 | 70.0 | 74.0 | 78.0 | 94.0 | 98.0 | 102.0 | 106.0 | 110.0 | 126.0 | 130.0 | 134.0 | 138.0 | 142.0 |
| $t_{41}$ | 29.0 | 33.0 | 37.0 | 41.0 | 45.0 | 61.0 | 65.0 | 69.0 | 73.0 | 77.0 | 93.0 | 97.0 | 101.0 | 105.0 | 109.0 | 125.0 | 129.0 | 133.0 | 137.0 | 141.0 |
| $t_{46}$ | 31.0 | 35.0 | 39.0 | 43.0 | 47.0 | 63.0 | 67.0 | 71.0 | 75.0 | 79.0 | 95.0 | 99.0 | 103.0 | 107.0 | 111.0 | 127.0 | 131.0 | 135.0 | 139.0 | 143.0 |
| $t_{47}$ | 28.0 | 32.0 | 36.0 | 40.0 | 44.0 | 60.0 | 64.0 | 68.0 | 72.0 | 76.0 | 92.0 | 96.0 | 100.0 | 104.0 | 108.0 | 124.0 | 128.0 | 132.0 | 136.0 | 140.0 |
| $t_{24}$ | 32.0 | 36.0 | 40.0 | 44.0 | 48.0 | 64.0 | 68.0 | 72.0 | 76.0 | 80.0 | 96.0 | 100.0 | 104.0 | 108.0 | 112.0 | 128.0 | 132.0 | 136.0 | 140.0 | 144.0 |
| $t_{30}$ | 33.0 | 37.0 | 41.0 | 45.0 | 49.0 | 65.0 | 69.0 | 73.0 | 77.0 | 81.0 | 97.0 | 101.0 | 105.0 | 109.0 | 113.0 | 129.0 | 133.0 | 137.0 | 141.0 | 145.0 |
| $t_{36}$ | 33.0 | 37.0 | 41.0 | 45.0 | 49.0 | 65.0 | 69.0 | 73.0 | 77.0 | 81.0 | 97.0 | 101.0 | 105.0 | 109.0 | 113.0 | 129.0 | 133.0 | 137.0 | 141.0 | 145.0 |
| $t_{42}$ | 32.0 | 36.0 | 40.0 | 44.0 | 48.0 | 64.0 | 68.0 | 72.0 | 76.0 | 80.0 | 96.0 | 100.0 | 104.0 | 108.0 | 112.0 | 128.0 | 132.0 | 136.0 | 140.0 | 144.0 |
| $t_{48}$ | 34.0 | 38.0 | 42.0 | 46.0 | 50.0 | 66.0 | 70.0 | 74.0 | 78.0 | 82.0 | 98.0 | 102.0 | 106.0 | 110.0 | 114.0 | 130.0 | 134.0 | 138.0 | 142.0 | 146.0 |
| $t_{49}$ | 38.0 | 42.0 | 46.0 | 50.0 | 54.0 | 70.0 | 74.0 | 78.0 | 82.0 | 86.0 | 102.0 | 106.0 | 110.0 | 114.0 | 118.0 | 134.0 | 138.0 | 142.0 | 146.0 | 150.0 |

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Table 3. Computation of the Execution Schedule: Optimal Case.

Resource and Time Constraints

| Marking of the Resource Places: | $R_0$ | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ | $R_6$ | $R_7$ |
|---|---|---|---|---|---|---|---|---|
| | 7 | 2 | 5 | 5 | 5 | 5 | 5 | 5 |

Limited Time Duration: 135 units of time

Transition Firing Times:

| $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_5$ | $t_6$ | $t_7$ | $t_8$ | $t_9$ | $t_{10}$ | $t_{11}$ | $t_{12}$ | $t_{13}$ | $t_{14}$ | $t_{15}$ | $t_{16}$ | $t_{17}$ | $t_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 1 | 1 | 3 | 2 | 4 | 1 | 2 | 3 | 4 | 2 | 4 | 2 | 3 | 2 | 3 | 1 |

| $t_{19}$ | $t_{20}$ | $t_{21}$ | $t_{22}$ | $t_{23}$ | $t_{24}$ | $t_{25}$ | $t_{26}$ | $t_{27}$ | $t_{28}$ | $t_{29}$ | $t_{30}$ | $t_{31}$ | $t_{32}$ | $t_{33}$ | $t_{34}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 1 | 3 | 4 | 3 | 1 | 3 | 2 | 4 | 1 | 1 | 3 | 1 | 2 | 4 | 3 |

| $t_{35}$ | $t_{36}$ | $t_{37}$ | $t_{38}$ | $t_{39}$ | $t_{40}$ | $t_{41}$ | $t_{42}$ | $t_{43}$ | $t_{44}$ | $t_{45}$ | $t_{46}$ | $t_{47}$ | $t_{48}$ | $t_{49}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 3 | 4 | 1 | 1 | 3 | 2 | 2 | 4 | 1 | 2 | 2 | 3 | 4 |

Execution Schedule

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $t_1$ | 0.0 | 2.0 | 4.0 | 6.0 | 8.0 | 10.0 | 12.0 | 36.0 | 40.0 | 44.0 | 48.0 | 52.0 | 64.0 | 68.0 | 72.0 | 76.0 | 80.0 | 92.0 | 96.0 | 100.0 |
| $t_5$ | 3.0 | 6.0 | 9.0 | 12.0 | 15.0 | 24.0 | 28.0 | 39.0 | 43.0 | 47.0 | 52.0 | 56.0 | 67.0 | 71.0 | 75.0 | 80.0 | 84.0 | 95.0 | 99.0 | 103.0 |
| $t_{12}$ | 2.0 | 4.0 | 6.0 | 8.0 | 10.0 | 20.0 | 24.0 | 38.0 | 42.0 | 46.0 | 50.0 | 54.0 | 66.0 | 70.0 | 74.0 | 78.0 | 82.0 | 94.0 | 98.0 | 102.0 |
| $t_{19}$ | 4.0 | 8.0 | 12.0 | 16.0 | 20.0 | 32.0 | 36.0 | 40.0 | 44.0 | 48.0 | 60.0 | 64.0 | 68.0 | 72.0 | 76.0 | 88.0 | 92.0 | 96.0 | 100.0 | 104.0 |
| $t_{25}$ | 3.0 | 6.0 | 9.0 | 12.0 | 15.0 | 32.0 | 36.0 | 40.0 | 44.0 | 48.0 | 60.0 | 64.0 | 68.0 | 72.0 | 76.0 | 88.0 | 92.0 | 96.0 | 100.0 | 104.0 |
| $t_{31}$ | 1.0 | 3.0 | 5.0 | 7.0 | 9.0 | 30.0 | 34.0 | 38.0 | 42.0 | 46.0 | 58.0 | 62.0 | 66.0 | 70.0 | 74.0 | 86.0 | 90.0 | 94.0 | 98.0 | 102.0 |
| $t_{37}$ | 3.0 | 6.0 | 9.0 | 12.0 | 15.0 | 31.0 | 35.0 | 39.0 | 43.0 | 47.0 | 59.0 | 63.0 | 67.0 | 71.0 | 75.0 | 87.0 | 91.0 | 95.0 | 99.0 | 103.0 |
| $t_{43}$ | 2.0 | 4.0 | 6.0 | 8.0 | 10.0 | 32.0 | 36.0 | 40.0 | 44.0 | 48.0 | 60.0 | 64.0 | 68.0 | 72.0 | 76.0 | 88.0 | 92.0 | 96.0 | 100.0 | 104.0 |
| $t_6$ | 6.0 | 10.0 | 14.0 | 18.0 | 22.0 | 34.0 | 38.0 | 42.0 | 46.0 | 50.0 | 62.0 | 66.0 | 70.0 | 74.0 | 78.0 | 90.0 | 94.0 | 98.0 | 102.0 | 106.0 |
| $t_{13}$ | 8.0 | 12.0 | 16.0 | 20.0 | 24.0 | 36.0 | 40.0 | 44.0 | 48.0 | 52.0 | 64.0 | 68.0 | 72.0 | 76.0 | 80.0 | 92.0 | 96.0 | 100.0 | 104.0 | 108.0 |
| $t_2$ | 9.0 | 13.0 | 17.0 | 21.0 | 25.0 | 37.0 | 41.0 | 45.0 | 49.0 | 53.0 | 65.0 | 69.0 | 73.0 | 77.0 | 81.0 | 93.0 | 97.0 | 101.0 | 105.0 | 109.0 |
| $t_3$ | 10.0 | 14.0 | 18.0 | 22.0 | 26.0 | 38.0 | 42.0 | 46.0 | 50.0 | 54.0 | 66.0 | 70.0 | 74.0 | 78.0 | 82.0 | 94.0 | 98.0 | 102.0 | 106.0 | 110.0 |
| $t_4$ | 11.0 | 15.0 | 19.0 | 23.0 | 27.0 | 39.0 | 43.0 | 47.0 | 51.0 | 55.0 | 67.0 | 71.0 | 75.0 | 79.0 | 83.0 | 95.0 | 99.0 | 103.0 | 107.0 | 111.0 |
| $t_7$ | 15.0 | 19.0 | 23.0 | 27.0 | 31.0 | 43.0 | 47.0 | 51.0 | 55.0 | 59.0 | 71.0 | 75.0 | 79.0 | 83.0 | 87.0 | 99.0 | 103.0 | 107.0 | 111.0 | 115.0 |
| $t_8$ | 12.0 | 16.0 | 20.0 | 24.0 | 28.0 | 40.0 | 44.0 | 48.0 | 52.0 | 56.0 | 68.0 | 72.0 | 76.0 | 80.0 | 84.0 | 96.0 | 100.0 | 104.0 | 108.0 | 112.0 |
| $t_{14}$ | 13.0 | 17.0 | 21.0 | 25.0 | 29.0 | 41.0 | 45.0 | 49.0 | 53.0 | 57.0 | 69.0 | 73.0 | 77.0 | 81.0 | 85.0 | 97.0 | 101.0 | 105.0 | 109.0 | 113.0 |
| $t_{15}$ | 14.0 | 18.0 | 22.0 | 26.0 | 30.0 | 42.0 | 46.0 | 50.0 | 54.0 | 58.0 | 70.0 | 74.0 | 78.0 | 82.0 | 86.0 | 98.0 | 102.0 | 106.0 | 110.0 | 114.0 |
| $t_9$ | 17.0 | 21.0 | 25.0 | 29.0 | 33.0 | 45.0 | 49.0 | 53.0 | 57.0 | 61.0 | 73.0 | 77.0 | 81.0 | 85.0 | 89.0 | 101.0 | 105.0 | 109.0 | 113.0 | 117.0 |
| $t_{10}$ | 15.0 | 19.0 | 23.0 | 27.0 | 31.0 | 43.0 | 47.0 | 51.0 | 55.0 | 59.0 | 71.0 | 75.0 | 79.0 | 83.0 | 87.0 | 99.0 | 103.0 | 107.0 | 111.0 | 115.0 |
| $t_{16}$ | 15.0 | 19.0 | 23.0 | 27.0 | 31.0 | 43.0 | 47.0 | 51.0 | 55.0 | 59.0 | 71.0 | 75.0 | 79.0 | 83.0 | 87.0 | 99.0 | 103.0 | 107.0 | 111.0 | 115.0 |
| $t_{17}$ | 17.0 | 21.0 | 25.0 | 29.0 | 33.0 | 45.0 | 49.0 | 53.0 | 57.0 | 61.0 | 73.0 | 77.0 | 81.0 | 85.0 | 89.0 | 101.0 | 105.0 | 109.0 | 113.0 | 117.0 |
| $t_{11}$ | 21.0 | 25.0 | 29.0 | 33.0 | 37.0 | 49.0 | 53.0 | 57.0 | 61.0 | 65.0 | 77.0 | 81.0 | 85.0 | 89.0 | 93.0 | 105.0 | 109.0 | 113.0 | 117.0 | 121.0 |
| $t_{18}$ | 18.0 | 22.0 | 26.0 | 30.0 | 34.0 | 46.0 | 50.0 | 54.0 | 58.0 | 62.0 | 74.0 | 78.0 | 82.0 | 86.0 | 90.0 | 102.0 | 106.0 | 110.0 | 114.0 | 118.0 |
| $t_{20}$ | 22.0 | 26.0 | 30.0 | 34.0 | 38.0 | 50.0 | 54.0 | 58.0 | 62.0 | 66.0 | 78.0 | 82.0 | 86.0 | 90.0 | 94.0 | 106.0 | 110.0 | 114.0 | 118.0 | 122.0 |
| $t_{21}$ | 24.0 | 28.0 | 32.0 | 36.0 | 40.0 | 52.0 | 56.0 | 60.0 | 64.0 | 68.0 | 80.0 | 84.0 | 88.0 | 92.0 | 96.0 | 108.0 | 112.0 | 116.0 | 120.0 | 124.0 |
| $t_{26}$ | 23.0 | 27.0 | 31.0 | 35.0 | 39.0 | 51.0 | 55.0 | 59.0 | 63.0 | 67.0 | 79.0 | 83.0 | 87.0 | 91.0 | 95.0 | 107.0 | 111.0 | 115.0 | 119.0 | 123.0 |
| $t_{27}$ | 25.0 | 29.0 | 33.0 | 37.0 | 41.0 | 53.0 | 57.0 | 61.0 | 65.0 | 69.0 | 81.0 | 85.0 | 89.0 | 93.0 | 97.0 | 109.0 | 113.0 | 117.0 | 121.0 | 125.0 |
| $t_{32}$ | 23.0 | 27.0 | 31.0 | 35.0 | 39.0 | 51.0 | 55.0 | 59.0 | 63.0 | 67.0 | 79.0 | 83.0 | 87.0 | 91.0 | 95.0 | 107.0 | 111.0 | 115.0 | 119.0 | 123.0 |
| $t_{33}$ | 25.0 | 29.0 | 33.0 | 37.0 | 41.0 | 53.0 | 57.0 | 61.0 | 65.0 | 69.0 | 81.0 | 85.0 | 89.0 | 93.0 | 97.0 | 109.0 | 113.0 | 117.0 | 121.0 | 125.0 |
| $t_{38}$ | 25.0 | 29.0 | 33.0 | 37.0 | 41.0 | 53.0 | 57.0 | 61.0 | 65.0 | 69.0 | 81.0 | 85.0 | 89.0 | 93.0 | 97.0 | 109.0 | 113.0 | 117.0 | 121.0 | 125.0 |
| $t_{39}$ | 22.0 | 26.0 | 30.0 | 34.0 | 38.0 | 50.0 | 54.0 | 58.0 | 62.0 | 66.0 | 78.0 | 82.0 | 86.0 | 90.0 | 94.0 | 106.0 | 110.0 | 114.0 | 118.0 | 122.0 |
| $t_{14}$ | 25.0 | 29.0 | 33.0 | 37.0 | 41.0 | 53.0 | 57.0 | 61.0 | 65.0 | 69.0 | 81.0 | 85.0 | 89.0 | 93.0 | 97.0 | 109.0 | 113.0 | 117.0 | 121.0 | 125.0 |
| $t_{45}$ | 22.0 | 26.0 | 30.0 | 34.0 | 38.0 | 50.0 | 54.0 | 58.0 | 62.0 | 66.0 | 78.0 | 82.0 | 86.0 | 90.0 | 94.0 | 106.0 | 110.0 | 114.0 | 118.0 | 122.0 |
| $t_{22}$ | 26.0 | 30.0 | 34.0 | 38.0 | 42.0 | 54.0 | 58.0 | 62.0 | 66.0 | 70.0 | 82.0 | 86.0 | 90.0 | 94.0 | 98.0 | 110.0 | 114.0 | 118.0 | 122.0 | 126.0 |
| $t_{23}$ | 27.0 | 31.0 | 35.0 | 39.0 | 43.0 | 55.0 | 59.0 | 63.0 | 67.0 | 71.0 | 83.0 | 87.0 | 91.0 | 95.0 | 99.0 | 111.0 | 115.0 | 119.0 | 123.0 | 127.0 |
| $t_{28}$ | 24.0 | 28.0 | 32.0 | 36.0 | 40.0 | 52.0 | 56.0 | 60.0 | 64.0 | 68.0 | 80.0 | 84.0 | 88.0 | 92.0 | 96.0 | 108.0 | 112.0 | 116.0 | 120.0 | 124.0 |
| $t_{29}$ | 26.0 | 30.0 | 34.0 | 38.0 | 42.0 | 54.0 | 58.0 | 62.0 | 66.0 | 70.0 | 82.0 | 86.0 | 90.0 | 94.0 | 98.0 | 110.0 | 114.0 | 118.0 | 122.0 | 126.0 |
| $t_{34}$ | 26.0 | 30.0 | 34.0 | 38.0 | 42.0 | 54.0 | 58.0 | 62.0 | 66.0 | 70.0 | 82.0 | 86.0 | 90.0 | 94.0 | 98.0 | 110.0 | 114.0 | 118.0 | 122.0 | 126.0 |
| $t_{35}$ | 27.0 | 31.0 | 35.0 | 39.0 | 43.0 | 55.0 | 59.0 | 63.0 | 67.0 | 71.0 | 83.0 | 87.0 | 91.0 | 95.0 | 99.0 | 111.0 | 115.0 | 119.0 | 123.0 | 127.0 |
| $t_{40}$ | 26.0 | 30.0 | 34.0 | 38.0 | 42.0 | 54.0 | 58.0 | 62.0 | 66.0 | 70.0 | 82.0 | 86.0 | 90.0 | 94.0 | 98.0 | 110.0 | 114.0 | 118.0 | 122.0 | 126.0 |
| $t_{41}$ | 25.0 | 29.0 | 33.0 | 37.0 | 41.0 | 53.0 | 57.0 | 61.0 | 65.0 | 69.0 | 81.0 | 85.0 | 89.0 | 93.0 | 97.0 | 109.0 | 113.0 | 117.0 | 121.0 | 125.0 |
| $t_{46}$ | 27.0 | 31.0 | 35.0 | 39.0 | 43.0 | 55.0 | 59.0 | 63.0 | 67.0 | 71.0 | 83.0 | 87.0 | 91.0 | 95.0 | 99.0 | 111.0 | 115.0 | 119.0 | 123.0 | 127.0 |
| $t_{47}$ | 24.0 | 28.0 | 32.0 | 36.0 | 40.0 | 52.0 | 56.0 | 60.0 | 64.0 | 68.0 | 80.0 | 84.0 | 88.0 | 92.0 | 96.0 | 108.0 | 112.0 | 116.0 | 120.0 | 124.0 |
| $t_{24}$ | 28.0 | 32.0 | 36.0 | 40.0 | 44.0 | 56.0 | 60.0 | 64.0 | 68.0 | 72.0 | 84.0 | 88.0 | 92.0 | 96.0 | 100.0 | 112.0 | 116.0 | 120.0 | 124.0 | 128.0 |
| $t_{30}$ | 29.0 | 33.0 | 37.0 | 41.0 | 45.0 | 57.0 | 61.0 | 65.0 | 69.0 | 73.0 | 85.0 | 89.0 | 93.0 | 97.0 | 101.0 | 113.0 | 117.0 | 121.0 | 125.0 | 129.0 |
| $t_{36}$ | 29.0 | 33.0 | 37.0 | 41.0 | 45.0 | 57.0 | 61.0 | 65.0 | 69.0 | 73.0 | 85.0 | 89.0 | 93.0 | 97.0 | 101.0 | 113.0 | 117.0 | 121.0 | 125.0 | 129.0 |
| $t_{42}$ | 28.0 | 32.0 | 36.0 | 40.0 | 44.0 | 56.0 | 60.0 | 64.0 | 68.0 | 72.0 | 84.0 | 88.0 | 92.0 | 96.0 | 100.0 | 112.0 | 116.0 | 120.0 | 124.0 | 128.0 |
| $t_{48}$ | 30.0 | 34.0 | 38.0 | 42.0 | 46.0 | 58.0 | 62.0 | 66.0 | 70.0 | 74.0 | 86.0 | 90.0 | 94.0 | 98.0 | 102.0 | 114.0 | 118.0 | 122.0 | 126.0 | 130.0 |
| $t_{49}$ | 34.0 | 38.0 | 42.0 | 46.0 | 50.0 | 62.0 | 66.0 | 70.0 | 74.0 | 78.0 | 90.0 | 94.0 | 98.0 | 102.0 | 106.0 | 118.0 | 122.0 | 126.0 | 130.0 | 134.0 |


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