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Optimization over a finite number of system designs with one-stage sampling and multiple comparisons with the best

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ABSTRACT

Multiple comparisons with the best, which is applicable to single-stage experiments, is introduced as a method for choosing the best of a finite number of system designs. Examples are given.

1. INTRODUCTION

When designing systems, it is natural to attempt to design the best possible system relative to some performance criteria, but subject to structural or resource constraints. When system behavior is uncertain, stochastic models may be employed to aid the design process. In that case, the criterion often becomes mean or expected system performance. This paper considers optimization of stochastic models via simulation with respect to minimum or maximum expected performance.

In his survey of stochastic optimization, Glynn (1986) classified stochastic optimization problems based on their decision space. Within his classification, this paper addresses problems with finite-dimensional, discrete decision spaces, in which the number of possible decisions (system designs) is finite. This is a common situation in practice when system designs arise from choosing among competing machines, schedules, or facilities, subject to constraints on available budget or technology.

The standard methods used to search for the best design in such problems come from the statistical literature on ranking and selection. However, indifference zone selection was thought to require two-stage sampling. In simulation experiments, this means simulating each system design, calculating a second-stage sample size based on the initial results, and then restarting each simulation to obtain additional data. The technique introduced below, called *multiple comparisons with the best (MCB)*, can be applied in a single-stage experiment, and implies ranking and selection inference. Also included are two examples that demonstrate how MCB can be used as an alternative to ranking and selection.

2. MULTIPLE COMPARISONS WITH THE BEST

We assume that k competing systems are to be compared in terms of their expected performance. Denote the k systems by $\pi_1, \pi_2, \ldots, \pi_k$, and their expected performance by $\theta_1, \theta_2, \ldots, \theta_k$, respectively.

Suppose that a larger expected performance implies a better system. For each system π_i, consider the quantity $\theta_i - \max_{j \neq i} \theta_j$, which can be termed "system i performance minus the best of the other systems' performance." We claim that, to assess the systems, very often the parameters $\theta_i - \max_{j \neq i} \theta_j$ for $i = 1, \ldots, k$ are the quantities of primary interest. This can be seen as follows: If $0 < \theta_i - \max_{j \neq i} \theta_j$, then system π_i is the best, for it is better than the best of the other systems. If $\theta_i - \max_{j \neq i} \theta_j < 0$, then system π_j is not the best, since there is another better system. Even if $\theta_i - \max_{j \neq i} \theta_j < 0$, if $-\delta < \theta_i - \max_{j \neq i} \theta_j$ where δ is a small postive number, then system π_i is within δ of the best. Thus, for multiple comparisons with the best, the relevent parameters are $\theta_i - \max_{j \neq i} \theta_j$ for $i = 1, \ldots, k$.

Because the systems are stochastic and estimates are based on (necessarily finite) samples, the quantities $\theta_i - \max_{j \neq i} \theta_j$ are not known precisely. MCB gives two-sided $(1 - \alpha)100\%$ simultaneous confidence intervals for $\theta_i - \max_{j \neq i} \theta_j$ for all i. The subset selection aspect of ranking and selection decides which systems are not the best (i.e., $\theta_i - \max_{j \neq i} \theta_j \leq 0$). The indifference zone selection aspect of ranking and selection decides whether system $\pi_{[k]}$, that appears to be the best according to the data, can indeed be inferred to be the best system (i.e., $\theta_{[k]} - \max_{j \neq [k]} \theta_j \geq 0$), or at least good enough (i.e., $\theta_{[k]} - \max_{j \neq [k]} \theta_j \geq -\delta$). It would be natural to presume that, at the same confidence level $1 - \alpha$, (a) one-sided subset selection inference would be sharper than MCB upper bound inference; (b) one-sided indifference zone inference would be sharper than MCB lower bound inference; and (c) the joint confidence level of subset selection and indifference zone selection would be less than $1 - \alpha$. Surprisingly, (a), (b), and (c) are false, as we show below.

2.1 Derivation of MCB Simultaneous Confidence Intervals

Suppose for each system π_i a random sample $Y_{i1}, Y_{i2}, \ldots, Y_{in}$ is generated, where each system is simulated independently. Then under the usual normality and equality of variance assumptions we have the oneway model

$$Y_{i\ell} = \theta_i + \epsilon_{i\ell}, i = 1, \ldots, k, \ \ell = 1, \ldots, n,$$

where $\epsilon_{11}, \ldots, \epsilon_{kn}$ are independent and identically distributed (i.i.d.) normal with mean 0 and variance σ^2 unknown. We use the following notation

$$\begin{aligned} \bar{Y}_i &= n^{-1}\sum_{\ell=1}^{n} Y_{i\ell} \\ s^2 &= \text{MSE} = (k(n-1))^{-1}\sum_{i=1}^{k}\sum_{\ell=1}^{n}(Y_{i\ell}-\bar{Y}_i)^2 \end{aligned}$$

for the sample means and the pooled sample variance, respectively. Also, let $(1), \ldots, (k)$ denote the unknown indices such that $\theta_{(1)} \leq \theta_{(2)} \leq \cdots \leq \theta_{(k)}$, and let $[1], \ldots, [k]$ denote the random indices such that $\bar{Y}_{[1]} < \bar{Y}_{[2]} < \cdots < \bar{Y}_{[k]}$.

Define the event E as follows:

$$E = \{\bar{Y}_{(k)} - \theta_{(k)} \geq \bar{Y}_i - \theta_i - ds/\sqrt{n} \ \ \forall i, i \neq (k)\}.$$

The probability content of the event E is independent of $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_k)'$, and σ^2. Thus, one can find the critical value d, which depends only on k, α and $\nu = k(n-1)$, the degrees of freedom for s^2, such that

$$\Pr\{E\} = \Pr\{\bar{Y}_{(k)} - \theta_{(k)} \geq \bar{Y}_i - \theta_i - ds/\sqrt{n} \ \ \forall i, i \neq (k)\} = 1 - \alpha. \tag{1}$$

Suppose a larger expected performance is better. Let $-x^- = x$ if x is negative, 0 otherwise; and $x^+ = x$ if x is positive, 0 otherwise.

Theorem 1 (Hsu 1984b) *Under the oneway model above, the closed intervals*

$$[-(\bar{Y}_i - \max_{j\neq i}\bar{Y}_j - ds/\sqrt{n})^-, (\bar{Y}_i - \max_{j\neq i}\bar{Y}_j + ds/\sqrt{n})^+], i = 1, \ldots, k$$

form a set of $(1-\alpha)100\%$ *simultaneous confidence intervals for* $\theta_i - \max_{j\neq i}\theta_j$.

Proof: The lower MCB confidence bounds are derived by noting that

$$\begin{aligned} E &= \{\bar{Y}_{(k)} - \theta_{(k)} \geq \bar{Y}_i - \theta_i - ds/\sqrt{n} \ \ \forall i, i \neq (k)\} \\ &= \{\theta_i - \theta_{(k)} \geq \bar{Y}_i - \bar{Y}_{(k)} - ds/\sqrt{n} \ \ \forall i, i \neq (k)\} \\ &= \{\theta_i - \max_{j\neq i}\theta_j \geq \bar{Y}_i - \bar{Y}_{(k)} - ds/\sqrt{n} \ \ \forall i, i \neq (k)\} \\ &\subseteq \{\theta_i - \max_{j\neq i}\theta_j \geq \bar{Y}_i - \max_{j\neq i}\bar{Y}_j - ds/\sqrt{n} \ \ \forall i, i \neq (k)\} \\ &= \{\theta_i - \max_{j\neq i}\theta_j \geq \bar{Y}_i - \max_{j\neq i}\bar{Y}_j - ds/\sqrt{n} \ \ \forall i, i \neq (k) \\ &\qquad \text{and } \theta_i - \max_{j\neq i}\theta_j \geq 0 \text{ for } i = (k)\} \\ &\subseteq \{\theta_i - \max_{j\neq i}\theta_j \geq -(\bar{Y}_i - \max_{j\neq i}\bar{Y}_j - ds/\sqrt{n})^- \ \ \forall i\} \\ &= E_1. \end{aligned}$$

Similiarly, the upper MCB confidence bounds are derived by noting that

$$\begin{aligned} E &= \{\bar{Y}_{(k)} - \theta_{(k)} \geq \bar{Y}_i - \theta_i - ds/\sqrt{n} \ \ \forall i, i \neq (k)\} \\ &= \{\theta_{(k)} - \theta_i \leq \bar{Y}_{(k)} - \bar{Y}_i + ds/\sqrt{n} \ \ \forall i, i \neq (k)\} \\ &\subseteq \{\theta_{(k)} - \max_{j\neq(k)}\theta_j \leq \bar{Y}_{(k)} - \bar{Y}_i + ds/\sqrt{n} \ \ \forall i, i \neq (k)\} \\ &= \{\theta_{(k)} - \max_{j\neq(k)}\theta_j \leq \bar{Y}_{(k)} - \max_{j\neq(k)}\bar{Y}_j + ds/\sqrt{n}\} \\ &= \{\theta_{(k)} - \max_{j\neq(k)}\theta_j \leq \bar{Y}_{(k)} - \max_{j\neq(k)}\bar{Y}_j + ds/\sqrt{n} \\ &\qquad \text{and } \theta_i - \max_{j\neq i}\theta_j \leq 0 \ \ \forall i, i \neq (k)\} \\ &\subseteq \{\theta_i - \max_{j\neq i}\theta_j \leq +(\bar{Y}_i - \max_{j\neq i}\bar{Y}_j + ds/\sqrt{n})^+ \ \ \forall i\} \\ &= E_2. \end{aligned}$$

We have thus shown that $1 - \alpha = \Pr\{E\} \leq \Pr_{\boldsymbol{\theta}}\{E_1 \cap E_2\} =$

$$\Pr_{\boldsymbol{\theta}}\{-(\bar{Y}_i - \max_{j\neq i}\bar{Y}_j - ds/\sqrt{n})^- \leq \theta_i - \max_{j\neq i}\theta_j \leq (\bar{Y}_i - \max_{j\neq i}\bar{Y}_j + ds/\sqrt{n})^+ \ \ \forall i\} \tag{2}$$

which completes the proof. □

Hsu (1984a) noted that equality is attained in (2) when $\theta_1 = \cdots = \theta_k$, or when $\theta_{(k)} - \theta_{(k-1)} > ds/\sqrt{n}$, but not generally otherwise.

2.2 MCB Upper Bounds Imply Subset Selection

Gupta's (1956, 1965) subset selection selects system π_i if and only if

$$\bar{Y}_i - \max_{j\neq i}\bar{Y}_j + ds/\sqrt{n} \geq 0. \tag{3}$$

The crticial value d in subset selection is the same d as in (1) of MCB. Comparing (3) with (2), one sees that a system is rejected (not selected) by subset selection if and only if its MCB upper bound is 0. Subset selection guarantees, with a proability at least $1-\alpha$, that the true best system is contained in the selected subset; i.e.

$$\Pr_{\boldsymbol{\theta}}[(k) \in \{i : \bar{Y}_i - \max_{j\neq i}\bar{Y}_j \geq -ds/\sqrt{n}\}] \geq 1 - \alpha. \tag{4}$$

We now show that (4) is implied by (2) if it is assumed that $\theta_{(k)} > \theta_{(k-1)}$:

$$\begin{aligned} E_1 \cap E_2 &= \{\theta_i - \max_{j\neq i}\theta_j \in \pm(\bar{Y}_i - \max_{j\neq i}\bar{Y}_j \pm ds/\sqrt{n})^\pm \ \ \forall i\} \\ &\subseteq \{\theta_i - \max_{j\neq i}\theta_j \leq +(\bar{Y}_i - \max_{j\neq i}\bar{Y}_j + ds/\sqrt{n})^+ \ \ \forall i\} \\ &\subseteq \{\theta_i - \max_{j\neq i}\theta_j \leq 0 \ \ \forall i \text{ such that} \\ &\qquad \bar{Y}_i - \max_{j\neq i}\bar{Y}_j < -ds/\sqrt{n}\} \\ &\subseteq \{(k) \notin \{i : \bar{Y}_i - \max_{j\neq i}\bar{Y}_j < -ds/\sqrt{n}\}\} \end{aligned}$$

$$= \{(k) \in \{i : \bar{Y}_i - \max_{j \neq i} \bar{Y}_j \geq -ds/\sqrt{n}\}\}$$
$$= E_3.$$

Thus $\Pr_{\boldsymbol{\theta}}\{E_3\} \geq \Pr_{\boldsymbol{\theta}}\{E_1 \cap E_2\} \geq 1 - \alpha$.

2.3 MCB Lower Bounds Imply Indifference Zone Selection

When σ^2 is known, Bechhofer's (1954) indifference zone selection selects system $\pi_{[k]}$ as the best system and guarantees

$$\Pr\{0 = \theta_{[k]} - \theta_{(k)}\} \geq 1 - \alpha \text{ if } \theta_{(k)} - \theta_{(k-1)} \geq d\sigma/\sqrt{n}. \quad (5)$$

We now show (5) is implied by (2):

$$\begin{aligned} E_1 \cap E_2 &= \{\theta_i - \max_{j \neq i} \theta_j \in \pm(\bar{Y}_i - \max_{j \neq i} \bar{Y}_j \pm ds/\sqrt{n})^{\pm} \ \forall i\} \\ &\subseteq \{-(\bar{Y}_i - \max_{j \neq i} \bar{Y}_j - ds/\sqrt{n})^{-} \leq \theta_i - \max_{j \neq i} \theta_j \ \forall i\} \\ &\subseteq \{-(\bar{Y}_{[k]} - \max_{j \neq [k]} \bar{Y}_j - ds/\sqrt{n})^{-} \leq \theta_{[k]} - \max_{j \neq [k]} \theta_j\} \\ &\subseteq \{-ds/\sqrt{n} \leq \theta_{[k]} - \max_{j \neq [k]} \theta_j\} \\ &= E_4. \end{aligned}$$

Thus $\Pr_{\boldsymbol{\theta}}\{E_4\} \geq \Pr_{\boldsymbol{\theta}}\{E_1 \cap E_2\} \geq 1 - \alpha$. This holds for any degrees of freedom ν. In the case $\nu = \infty$, we have $s^2 = \sigma^2$ and

$$\Pr_{\boldsymbol{\theta}}\{-d\sigma/\sqrt{n} \leq \theta_{[k]} - \max_{j \neq [k]} \theta_j\} \geq 1 - \alpha$$

$$\Longrightarrow \Pr_{\boldsymbol{\theta}}\{0 = \theta_{[k]} - \theta_{(k)}\} \geq 1 - \alpha \text{ if } \theta_{(k)} - \theta_{(k-1)} > d\sigma/\sqrt{n}$$

which completes the proof.

Notice that when σ^2 is unknown, if one selects $\pi_{[k]}$ as the best system only if $\bar{Y}_{[k]} - \max_{j \neq [k]} \bar{Y}_j - ds/\sqrt{n} > 0$, then $\Pr\{0 = \theta_{[k]} - \theta_{(k)}$ when $\bar{Y}_{[k]} - \max_{j \neq [k]} \bar{Y}_j - ds/\sqrt{n} > 0\} \geq \Pr\{E_4\} \geq 1 - \alpha$. This shows indifference zone selection is possible in single-stage experiments with σ^2 unknown without assuming any indifference zone if one allows the possibility of no selection when the data is too noisy, in much the same way one allows a test of a null hypothesis to not reject when the data is inconclusive. Comparing with (2), one sees that a system is selected as the best if its MCB lower bound is 0.

2.4 Joint Subset Selection and Indifference Zone Selection Inference

Subset selection inference is based on the event E_3; indifference zone selection inference is based on the event E_4; while MCB inference is based on $E_1 \cap E_2$. We have shown $E_1 \cap E_2 \subseteq E_3 \cap E_4$. Therefore, since the MCB confidence intervals are guaranteed to cover the parameters $\theta_i - \max_{j \neq i} \theta_j$ simultaneously with a probability of at least $1 - \alpha$, subset selection inference and indifference zone selection inference can be given simultaneously with the guarantee that both aspects are correct with a probability of at least $1 - \alpha$. In fact, as noted in Hsu (1981), since the two aspects of ranking and selection correspond to upper and lower MCB bounds, MCB inference and (both aspects of) ranking and selection inference can be given simultaneously with the guarantee that *all* the inferences are correct with a probability of at least $1 - \alpha$. This recent realization made it possible to write a single computer package for ranking, selection, and mutliple comparisons with the best (Aubuchon, Gupta, and Hsu 1986).

2.5 R and S Values

For each system, in addition to reporting whether that system is rejected at the chosen confidence level $1 - \alpha$, it is convenient to report the smallest α for which that system can be rejected. This is called the *R-value* for that system. Of course, it would be useless to report the R-value of the system that appears to be the best. For that system, in addition to reporting whether it is selected as the best at the chosen confidence level $1 - \alpha$, we also report the smallest α for which that system can be selected as the best. This is called the *S-value* of that system. R and S-values are particularly suited for computer implementation (see Hsu 1984a).

2.6 When Smaller Expected Performance is Better

Now consider the case where a smaller expected performance implies a better system. By symmetry with the earlier discussion, the parameter of primary interest for each system π_i is $\theta_i - \min_{j \neq i} \theta_j$, which is "system i performance minus the best of the other systems' performance." Now, if $0 < \theta_i - \min_{j \neq i} \theta_j$, then system π_i is not the best system. If $\theta_i - \min_{j \neq i} \theta_j < 0$, then system π_i is the best system. Even if $0 < \theta_i - \min_{j \neq i} \theta_j$, suppose $\theta_i - \min_{j \neq i} \theta_j < \delta$, where δ is a small positive number. Then system π_i is close to the best.

MCB inference obtains, for any specified confidence level $1 - \alpha$, the simultaneous confidence intervals

$$[-(\bar{Y}_i - \min_{j \neq i} \bar{Y}_j - ds/\sqrt{n})^{-}, (\bar{Y}_i - \min_{j \neq i} \bar{Y}_j + ds/\sqrt{n})^{+}]$$

for $\theta_i - \min_{j \neq i} \theta_j$, $i = 1, \ldots, k$.

For ranking and selection inference, subset selection rejects system π_i if and only if $0 < \bar{Y}_i - \min_{j \neq i} \bar{Y}_j - ds/\sqrt{n}$; i.e., when the MCB lower bound for system π_i is 0. Indifference zone selection inference selects system π_i as the best system if and only if $\bar{Y}_i - \min_{j \neq i} \bar{Y}_j + ds/\sqrt{n} < 0$; i.e., if the MCB upper bound for system π_i is 0. Again, for each system except the one that appears to be the best, the R-value is the smallest α for which that system can be rejected as best. The S-value for the system that appears to be the best represents the smallest α for which it can be selected as best.

3. EXAMPLES

In this section we give two examples of optimization using MCB. These examples were originally used to illustrate two-stage ranking and selection procedures.

3.1 Machine-Repair System

The first example is the classical machine-repair problem. Iglehart (1977) used this example to illustrate a two-stage indifference zone selection procedure for determining the system with the smallest steady-state mean response.

Consider a system composed of $n+m$ identical machines and n machine operators. When in use, machines are subject to failure, and the time until failure is modeled as an exponentially distributed random variable with mean $1/\lambda$ time units. Since there are only n machine operators, there are at most n machines in use at any time. When a machine fails, it is replaced by one of the m spares, if there is one available. When there are fewer than n machines available, some operators are idle.

A failed machine is repaired by one of s identical repairmen; the repairmen are identical in the sense that they each work at rate μ machines repaired/unit time, with the time to complete a repair being exponentially distributed. Machines are repaired on a first-come-first-served basis, and repaired machines return to active use if there is an idle operator; otherwise they join the pool of spares.

Let $\{X_t; t \geq 0\}$ be a stochastic process representing the number of machines being repaired or waiting to be repaired at time t; thus, $X_t \in \{0, 1, \ldots, n+m\}$. Under the assumptions above, X_t is a birth-death process, which is a special case of a Markov process. Let λ_j and μ_j be the birth rate and death rate in state j, respectively. Then $\lambda_j = n\lambda$, if $j = 0, 1, \ldots, m$, or $\lambda_j = (m+n-j)\lambda$, if $j = m+1, \ldots, n+m$; also $\mu_j = j\mu$, if $j = 1, 2, \ldots, s$, or $\mu_j = s\mu$, if $j = s+1, \ldots, n+m$. For any initial state X_0, the process X_t converges weakly to a random variable X, denoted $X_t \Longrightarrow X$. The parameter of interest is $\theta = \mathrm{E}[X]$.

We fixed $n = 10$, $m = 4$, and $\lambda = 1$, and considered the (s, μ) combinations shown in Table 1. In all cases $s\mu = 12$, so that the total repair capacity of all three systems is the same.

Let θ_i be the parameter associated with (s, μ) combination i. Then we are interested in simultaneous confidence intervals for $\theta_i - \min_{j \neq i} \theta_j$, $i = 1, 2, 3$. This constrasts with Iglehart who attempted to find i^* such that $\theta_{i^*} = \min_i \theta_i$. The values of θ_i given in the table were determined using standard results for birth-death processes.

Table 1: System Parameters and Expected System State

i	s	μ	θ_i
1	2	6	3.0782
2	3	4	3.4708
3	4	3	3.8903

The simulation experiment consisted of generating 10 i.i.d. estimates of θ_i for each of the three (s, μ) combinations in the following way: Let $Y_{i\ell}$ be the ℓth estimate of θ_i, for $\ell = 1, 2, \ldots, 10$, and let $X_t^{i\ell}$ be a stochastic process corresponding to system i with $X_0^{i\ell} = 0$; the index ℓ denotes the ℓth replication of the simulation. Then

$$Y_{i\ell} = \int_{2000}^{5000} X_t^{i\ell} dt / 3000.$$

That is, $Y_{i\ell}$ is the time average number of machines under repair or waiting to be repaired for the last 3000 time units of a simulation run of length 5000 time units. The design of the experiment assured that $Y_{i\ell}$, $\ell = 1, 2, \ldots, 10$ were i.i.d., and that $Y_{i\ell}$, $i = 1, 2, 3$ were independent for all ℓ. Subroutine rnexp from the IMSL Library was used to generate values from the exponential distribution.

The data is plotted in Figure 1. Little from the plot suggests that the variances are heterogeneous. The sample means are given in Table 2. To assess the normality assumption, a quantile-quantile plot was made of the combined residuals $Y_{i\ell} - \bar{Y}_i$, $\ell = 1, 2, \ldots, 10$, $i = 1, 2, 3$, against the normal distribution, with a robust regression line fitted through the points. There was no evidence against the normality assumption. The pooled root mean squared error (RMS) for this data is 0.2444, with $3(10-1) = 27$ degrees of freedom. Applying the MCB function in S with $\alpha = 0.05$, we obtain the results in Table 2.

The R-values for systems 2 and 3 are less than $\alpha = 0.05$. Therefore, we can infer these systems are not the best. The same conclusion can be arrived at by noting that the lower confidence bounds for these systems are 0. Gupta's subset selection procedure would thus select system 1 as the best system.

System 1 appears to be the best from the data. Its S-value of 0.0007 is much less than $\alpha = 0.05$, indicating evidence that system 1 is the best system. The conclusion can also be reached by noting that the 95% upper confidence bound for $\theta_1 - \min_{j \neq 1} \theta_j$ is 0. Bechoffer's indifference zone selection procedure, modified for single-stage variance-unknown experiments, would also select system 1. The MCB intervals are plotted in Figure 2.

Table 2: MCB Results for Machine-Repair Problem

i	$\bar{Y}_i$	$\bar{Y}_i - \min_{j \neq i} \bar{Y}_j$	R-value	S-value	interval
1	3.1346	-0.4195	-	0.0007	(-0.6378, 0.0)
2	3.5541	0.4195	0.0007	-	(0.0, 0.6378)
3	3.8543	0.7196	0.0001	-	(0.0, 0.9380)

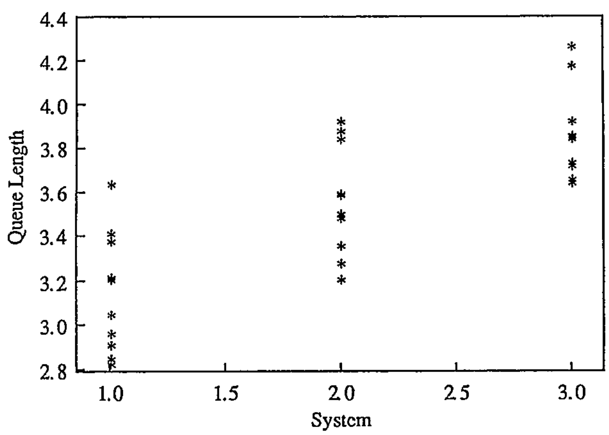

Figure 1: Data for Machine-Repair Problem

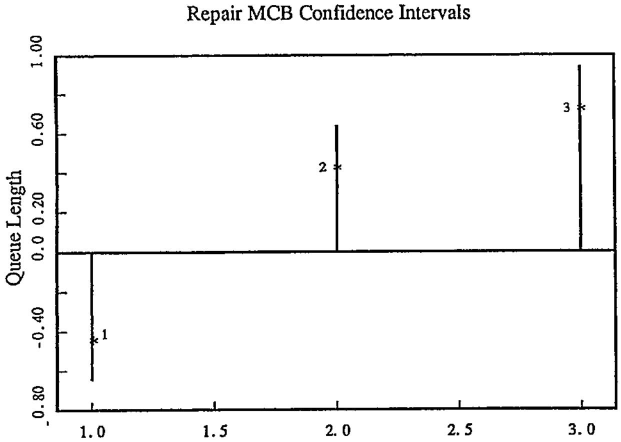

Figure 2: MCB Intervals for Machine-Repair Problem

3.2 Inventory System

The second example is an (s, S) inventory model. Koenig and Law (1985) used this example to illustrate a two-stage subset selection procedure for determining a subset of possible inventory policies that contains the least-expected-cost policy.

An (s, S) inventory system is one in which the level of inventory of some discrete item is reviewed periodically. If the inventory level is found to be below s units, then enough additional inventory is ordered to bring the inventory level up to S units. When the inventory position at a review period is found to be above s units, no additional items are ordered.

Let $\{I_t; t = 1, 2, \ldots\}$ be the inventory position just after a review at period t. Orders are filled immediately, so $I_t \in \{s, s+1, s+2, \ldots, S\}$. Let $\{D_t; t = 1, 2, \ldots\}$ be a stochastic process representing the demand for units of inventory in period t. The inventory position I_t changes in the following way: $I_{t+1} = S$ if $I_t - D_t < s$, or $I_{t+1} = I_t - D_t$ if $I_t - D_t \geq s$. We assume that $I_1 = S$ and $\{D_t; t = 1, 2, \ldots\}$ is a sequence of i.i.d. Poisson random variables with mean 25. Under these assumptions $\{I_t; t = 1, 2, \ldots\}$ is a Markov chain.

In each period there are costs associated with the inventory position. If $I_t - D_t < s$, then in period $t+1$ a cost of $32 + 3(S - (I_t - D_t))$ is incurred, which is a fixed cost plus a per unit cost of bringing the inventory position up to S. In period $t+1$, if $I_{t+1} \geq D_{t+1}$, then a holding cost of $I_{t+1} - D_{t+1}$ dollars is incurred; otherwise a shortage cost of $5(D_{t+1} - I_{t+1})$ dollars is incurred.

Let C_t^i be the cost incurred in period t under policy i. The quantity of interest is the expected total cost of the inventory system for 30 periods under several (s, S) policies, with a smaller expected total cost being preferred. The five policies considered by Koenig and Law are given in Table 3. Let

$$\theta_i = \mathrm{E}\left[\sum_{t=1}^{30} C_t^i\right]$$

be the expected cost for policy i. We are interested in simultaneous confidence intervals for $\theta_i - \min_{j \neq i} \theta_j$, $i = 1, 2, 3, 4, 5$. This constrasts with Koenig and Law who attempted to find a subset of policies that contains $\theta_{i^*} = \min_i \theta_i$. The values of θ_i given in the table, which were taken from Koenig and Law (1985), can be obtained using standard Markov chain analysis.

The simulation experiment consisted of generating 30 i.i.d. estimates of θ_i for each of the five (s, S) combinations in the following way: Let $Y_{i\ell}$ be the ℓth estimate of θ_i, for $\ell = 1, 2, \ldots, 30$, and let $C_t^{i\ell}$ be the cost in period t correspond-

Table 3: Parameters and Expected Cost

i	s	S	θ_i
1	20	40	114.18
2	20	80	112.74
3	40	60	130.55
4	40	100	130.70
5	60	100	147.38

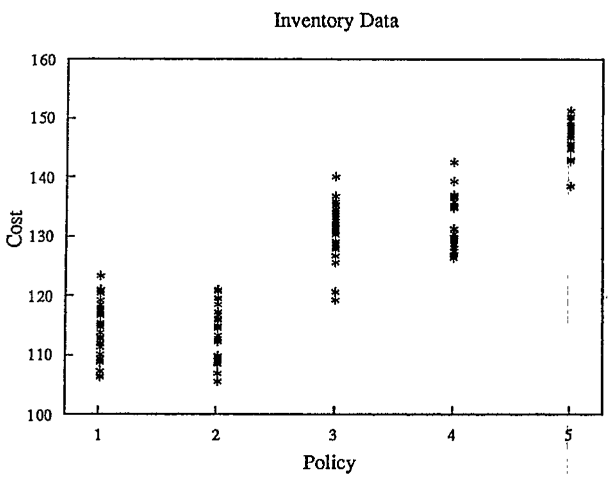

Figure 3: Data for Inventory Problem

ing to system i; the index ℓ denotes the ℓth replication of the simulation. Then

$$Y_{i\ell} = \sum_{t=1}^{30} C_t^{i\ell}.$$

That is, $Y_{i\ell}$ is the total cost of 30 periods of operation under policy i. The design of the experiment assured that all $Y_{i\ell}$, $\ell = 1, 2, \ldots, 30$ were i.i.d., and that $Y_{i\ell}$, $i = 1, 2, 3, 4, 5$, were independent for all ℓ. Subroutine rnpoi from the IMSL Library was used to generate values from the Poisson distribution.

The data is plotted in Figure 3. Little from the plot suggests that the variances are heterogeneous. The sample means are given in Table 4. To assess the normality assumption, a quantile-quantile plot was made of the combined residuals $Y_{i\ell} - \bar{Y}_i$, $\ell = 1, 2, \ldots, 30$, $i = 1, 2, 3, 4, 5$, against the normal distribution, with a robust regression line fitted through the points. There was no evidence against the normality assumption. The RMS for this data is 4.11014, with $5(30-1) = 145$ degrees of freedom. Applying the MCB function in S with $\alpha = 0.05$, we obtain the results in Table 4.

The R-values for policies 3, 4 and 5 are less than $\alpha = 0.05$. Therefore, we can infer these policies are not the best. The same conclusion can be arrived at by noting that the lower

Table 4: MCB Results for Inventory Problem

i	$\bar{Y}_i$	$\bar{Y}_i - \min_{j \neq i} \bar{Y}_j$	R-value	S-value	interval
1	114.043	1.046	0.3808	-	(-1.267, 3.359)
2	112.998	-1.046	-	0.3808	(-3.359, 1.267)
3	131.055	18.057	0.0000	-	(0.0, 20.370)
4	131.749	18.751	0.0000	-	(0.0, 21.064)
5	146.715	33.717	0.0000	-	(0.0, 36.030)

confidence bounds for these systems are 0. Gupta's subset selection procedure would thus select policies 1 and 2 to be in the subset.

Policy 2 appears to be the best from the data. However, since its S-value 0.3808 is greater than $\alpha = 0.05$, the evidence is insufficient to conclude policy 2 is the best policy, which is not unreasonable considering the closeness of the sample means for policies 1 and 2. The 95% upper confidence bound for $\theta_2 - \min_{j \neq 2} \theta_j$ indicates that policy 2 may be worse than the true best policy by as much as 1.267. Bechoffer's indifference zone selection procedure, modified for single-stage variance-unknown experiments, would thus choose the option not to select any policy as the best policy. The MCB intervals are plotted in Figure 4.

4. CONCLUSION

We have presented an alternative to standard ranking and selection methods for stochastic optimization problems when the number of system designs is finite. Extensions of the MCB methodology that would be useful in simulation experiments include methods for applying MCB in steady-state simulation using single-run experiment designs, and theory for sharpening MCB inference by using the common random numbers variance reduction technique. Both problems are being considered by the authors.

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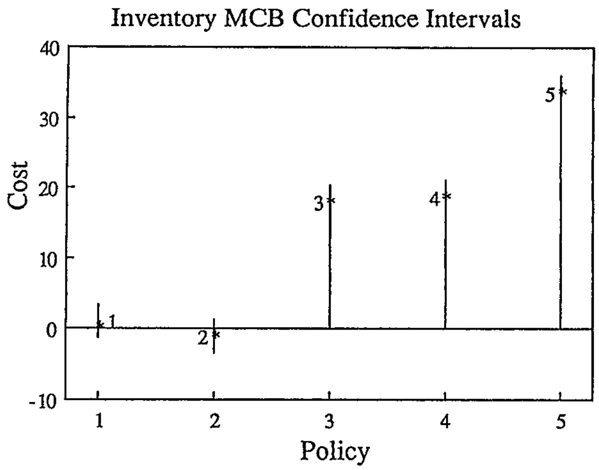

Figure 4: MCB Intervals for Inventory Problem

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