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PERTURBATION ANALYSIS
- Theoretical, Experimental, and Implementational Aspects -

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EXTENDED ABSTRACT

Perturbation Analysis (PA) is a technique for the computation of the gradient of performance measure (PM) of a discrete event dynamic system with respect to its parameters (θ) using only one sample path or Monte Carlo experiment of the system. It is probably the most developed of the several techniques of single run gradient estimation [1,2,3,4]. There are now close to forty papers on the subject matter.

When first presented, one's immediate reaction to PA has often been incredulity stemming from the belief that "one cannot get something for nothing". Later this disbelief may be developed into a more sophisticated and technical objection involving the legitimacy of interchanging differentiation and expectation operators or the probabilistic convergence of the PA estimate to its true value . In less technical terms, these translate to "How can you squeeze out information about a trajectory / sample-path operating under one value of the system parameter from that of another operating under a different value? Don't the two trajectories behave entirely dissimilarly?" This is primarily a **theoretical** and conceptual question for which there are now fairly concrete as well as intuitively pleasing answers. References [4,5] can be consulted for details.

Experimentally, PA is ahead of theory in the sense that there are experimental results which we cannot satisfactorily explain. These algorithms are often arrived at via intuitive and heuristic reasoning. We can safely say that the results they produce are not statistical accidents. Yet no rigorous proofs are available. A typical example is the case of PA and aggregation in queueing networks. PA can often be successfully applied to the aggregated version of network for which PA is known to fail if applied to the original complex network. Another experimental observation is the excellent variance reduction associated with PA calculation of a gradient estimate. These experimental findings, in fact, provide challenges and clues to future theoretical developments. This is in contrast with an axiomatic and purely mathematical development of the subject matter.

Lastly, the **implementation** of PA in simulation and real time setting involve another type of effort. There does not exist a general purpose PA algorithm or routine which is totally data driven and transparent to the user. It is not a trivial exercise to develop such software. Yet this author is convinced that this must be carried out if PA is to be really effective in everyday applications.

Although the theoretical aspects of the matter have understandibly received the most attention. The other two aspects of experimentation and implementation may be more important in the long run both in terms of utility and long term health of this research topic.

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