Proceedings of the 1987 Winter Simulation Conference
A. Thesen, H. Grant, W. David Kelton (eds.)

MARGINALLY SPECIFIC ALTERNATIVES TO NORMAL ARMA PROCESSES

Lee S. Dewald, Sr.
Department of Mathematics
United States Military Academy
West Point, N. Y.

Peter A. W. Lewis
Department of Operations Research
Naval Postgraduate School
Monterey, CA 93940.

Ed McKenzie
Department of Mathematics
University of Strathclyde
Glasgow, Scotland

ABSTRACT

In many practical cases in time series analysis, marginal distributions in stationary situations are not Gaussian. It is therefore necessary to be able to generate and analyze non-Gaussian time series. Several non-Gaussian time series models are discussed in this paper. The marginal distributions are Laplace or l-Laplace distributions, and the correlation structure of the processes mimics that of the standard additive, linear, constant coefficient ARMA(p,q) models.

1. INTRODUCTION.

Random coefficient time series models and ordinary linear time series models with non-Gaussian marginal distributions have been developed for a variety of interesting situations in time series analysis to offer viable alternatives to the standard Gaussian assumptions. A problem with the ordinary linear models with non-Gaussian error structure is that the marginal distributions change with the correlation structure, and in particular tend to become Gaussian as the correlation in the processes gets large (Mallows, 1967).

There are, however, important exceptions to this result, and these exceptions lead to important alternatives. Gastwirth and Wolff (1965) developed a linear first order autogressive process with a Laplace marginal distribution (called LAR(1)). Although it is a linear model, the marginal distributions are Laplace for all values of the correlation. This is because the distribution of the errors in the model is not absolutely continuous, which is a condition for the Mallows' results to hold. Independently Gaver and Lewis (1980) developed a linear first order autoregressive process with a Gamma marginal distribution, called GAR(1). Again the distribution of the errors in the model are not absolutely continuous. Subsequently both of these processes have been shown to be special cases of more general linear additive *discrete-random-coefficient* autoregressive models (Dewald and Lewis, 1985 and Lawrance and Lewis, 1981 and 1985). Other time series models using *continuous-random-coefficients* and having a specified marginal distribution are, for Gamma distributions, due to Lewis (1981), Hugus (1982) and Lewis, McKenzie and Hugus (1987), and, with Beta distributions, due to McKenzie (1985).

The Gamma models in particular provide a broad class of time series models for use in fields such as Operations Analysis, Hydrology and Meteorology. Nicholls and Quinn (1982) discuss general random coefficient autoregressive processes without reference to a particular marginal distribution; their results have applications in assessing statistical and asymptotic properties of all the models discussed here.

2. THE LAPLACE LAR(1) MODEL.

The Laplace LAR(1) model, and its generalization to higher order autoregressive and moving average correlation structures, was put forward as a model where two-sided symmetrical random variables had larger kurtosis and longer tails than could be expected from Gaussian time series. A particular example is that of position errors in a large navigation system which were found by Hsu (1979) to have Laplace distributions. Again the N-S or E-W components of wind velocity data are often symmetric and long-tailed, especially in the tropics. For a summary of these applications and a summary of methods of generating the LAR(1) and its generalizations, see Dewald and Lewis (1985). In particular we note that generalizations to NLAR(1) and NLAR(2) models which are the analogs of the NEAR(1) and NEAR(2) processes of Lawrence and Lewis (1981 and 1985) are available and are discussed in that paper. All of these models are *discrete-random-coefficient* linear models.

The random coefficient linear process approach is not the only way to generate Laplace variables with a specified correlation structure. The literature contains numerous articles on the generation of random sequences. One approach put forward in several papers (Gujar and Kavanagh, 1968; Haddad and Valisalo, 1970; Li and Hammond, 1975, and Sondhi, 1983) involves passing white noise through a linear filter followed by a zero-memory nonlinear transformation. This is a general procedure that produces exactly the required marginal distribution and a good approximation to the autocorrelation structure. However, the scheme lacks the simplicity of the methods proposed for Laplace processes, which are just a random linear combination of Laplacian random variables. Moreover, the filtering approach produces, for example, in the first order autoregressive case, only *one* process. It is important to note in non-Gaussian time series there are infinitely many processes with the given marginal and autocorrelation structure. This dictates perhaps the use of higher order moments and an approach to this using a higher order residual analysis has been proposed by Lawrance and Lewis (1987).

3. THE l-LAPLACE MODELS.

Another application of the Laplace models arises in those cases where positive-valued time series are differenced to remove trends. The resulting marginal random variables are two-sided and, in particular, differenced Gamma variables result in the l-Laplace family of distributions which we now consider.

The l-Laplace family of distributions is a natural generalization of the Laplace distribution, this being the l-Laplace distribution when l=1. The l-Laplace distribution is infinitely divisible, additive, symmetric and has extremely thick tails for small values of the parameter l. This case would be appropriate for modelling components of wind velacity data. These data often exhibit very sporadic, long-tailed behavior. On the other hand, as l approaches infinity, the l-Laplace distribution approaches a Gaussian distribution. Thus if the data is Gaussian, no loss of generality arises from using the l-Laplace distribution.

A square-root-Beta-Laplace transform is introduced which allows one to transform one member of the l-Laplace family into another in a simple way. As a result one can obtain an l-Laplace process with first-order autoregressive structure, the continuous-random-parameter just being the square root of an appropriate Beta variable. These l-Laplace processes are Markovian processes with the geometrically decaying autocorrelation function which is typical of the Gaussian, first order autoregressive (AR(1)) process. The basic structure is a random coefficient autoregression, and we generalise the structure of this model to encompass moving average (MA) and mixed autoregressive moving average (ARMA) processes. Again, both the structure of the process and their autocorrelation functions mirror those of the Gaussian ARMA processes.

ACKNOWLEDGEMENTS

This research was supported by the Office of Naval Research under Grant RR014-05-01 at the Naval Postgraduate School.

REFERENCES

Dewald, L.S. and Lewis, P.A.W. (1985). A New Laplace Second-Order Autoregressive Time Series Model – NLAR(2). *IEEE Transactions on Information Theory,* **Vol. IT-31**, 645-652.

Gastwirth, J.L. and Wolff, S.S. (1965). A Characterization of the Laplace Distribution. *John Hopkins University Department of Statistics Report 28.*

Gujar, V. G. and Kavanagh, R. J.. Generation of random signals with specified probability density functions and power density spectra. *IEEE Transactions on Automatic Control.* **Vol. AC-13**, 716-719.

Haddad, A.H. and Valisalo, P.E. (1970). Generation of random time series through hybrid computation. In: *Proceedings Sixth International Hybrid Computation Meetings*, 193-200.

Hsu, D.A. (1979). Long-tailed distributions for position errors in navigation. *Journal of the Royal Statistical Society*, **C, Vol. 28**, 62-72.

Hugus, D.K. (1982). Extensions of some models for positive-valued time series. Ph.D. Thesis, Naval Postgraduate School, Monterey, CA.

Lawrance, A.J. and Lewis, P.A.W. (1981). A new autoregressive time series model in exponential variables (NEAR(1)), *Advances in Applied Probability*, **Vol. 13**, 826-845.

Lawrance, A.J. and Lewis, P.A.W. (1985). Modelling and residual analysis of nonlinear autoregressive time series in exponential variables. *Journal of the Royal Statistical Society*, **Vol. B47**, 165-202.

Lawrance, A.J. and Lewis, P.A.W. (1987). Higher-Order residual analysis for nonlinear time series with autoregressive correlation structures. *International Statistical Review*, **Vol. 55**, 1, 21-35.

Lewis, P.A.W. (1981). Simple multivariate time series for simulation of complex systems. In: *Proceedings Winter Simulation Conference*, Ed. T. T. Oren, C. M. Delfosse and C. M. Shub. New York: *IEEE Press*, 389-390.

Lewis, P.A.W., McKenzie, E., and Hugus, D.K. (1986). Gamma processes. Naval Postgraduate School Technical Report NPS-55-86-002. Submitted for publication.

Li, S.T. and Hammond, J.L. (1975). Generation of pseudo-random numbers with specified univariate distributions and correlation coefficients. *IEEE Transactions on Systems, Man, Cybernetics*, **Vol. SMC-5**, 557-561.

Liu, B. and Munson, D.C. (1982). Generation of random sequence having a jointly specified marginal distribution and autocovariance. *IEEE Transactions on Acoustics and Speech, Signal Processing.* **Vol. ASSP-30**, 973-983.

Mallows, C. (1967). Linear processes are nearly Gaussian. *Journal of Applied Probability*, **Vol. 19**, 181-189.

McKenzie, E. (1985). An autoregressive process for beta random variables. *Management Science*, **Vol. 31, 8**, 988-997.

Nicholls, D.F. and Quinn, B.G. (1982). *Random coefficient autoregressive models: an introduction.* Springer verlag, Berlin.

Sondhi, M.M. (1983). Random processes with specified spectral density and first-order probability density. *Bell System Technical Journal*, **Vol. 62, no. 3**, 679-701.

AUTHORS' BIOGRAPHIES

LEE S. DEWALD, Sr., is a member of the Mathematics Faculty at the United States Military Academy, West Point, N.Y.. He received a Ph.D. degree in Operations research from the Naval Postgraduate school in 1985.

PETER A. W. LEWIS is Professor of Statistics and Operations research at the Naval Postgraduate School in Monterey, California. He has the A.B., B.S. and M.S. degrees in Electronic Engineering from the School of Engineering at Columbia University and a Ph.D. in Statistics from the University of London. He is the co-author, with D.R. Cox, of the book entitled 'The Statistical Analysis of Series of Events' and the author, with E.J. Orav, of the forthcoming book 'Simulation Methodology for Statisticians, Operations Analysts and Engineers'.

ED McKENZIE is a Lecturer in the Department of Mathematics University of Strathclyde in Glasgow, Scotland. He has a Ph.D. degree in Statistics from the University of Strathclyde. He is the author of numerous articles in the fields of time series analysis, forecasting and applied probability.