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A BONFERRONI SELECTION PROCEDURE WHEN USING COMMON RANDOM NUMBERS WITH UNKNOWN VARIANCES

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ABSTRACT

This paper presents a Bonferroni procedure for selecting the alternative with the largest mean when the variances are unknown and unequal and correlation is induced among the observations for each alternative by common random numbers. Simulation results show that the Bonferroni procedure is more efficient than Dudewicz and Dalal's procedure when the percentage of variance reduction is high.

Key Words: Selection, Variance Reduction, Bonferroni Inequality, Common Random Numbers

INTRODUCTION

Given k competing alternatives, a frequently occurring application of Monte-Carlo simulation is to identify the best alternative in the sense that it has the largest expected value for a specified output performance measure. We refer in this paper to a value of an output performance measure as an observation. There are two possible types of correlation among the observations. First, the observations produced by different alternatives may be correlated, and we call this cross alternative correlation. When comparing alternatives with simulation, one may be able to achieve a significant improvement in efficiency by introducing positive correlation across the observations from different alternatives using the easily implemented variance reduction technique of common random numbers (Kleijnen [1974]). When not employing common random numbers, we assume that the observations across alternatives are independent. The second type of correlation is within alternative correlation where the observations produced by a given alternative may be correlated particularly when the observations are a time series such as waiting times for customers in a queue. In this paper we assume there is no within alternative correlation and that the observations for an individual alternative are independent. In addition to correlation, another complicating factor is that the variances of the observations produced by different alternatives are usually unequal and unknown.

Dudewicz and Dalal [1975] consider the case of unequal and unknown variances but complete independence both for observations within individual alternatives and across alternatives. They present a procedure for specifying the number of independent observations for each alternative that guarantees a minimum probability of selecting the best alternative when the difference in expected response between the best and the next most competitive alternative is at least as large as a specified constant. Their procedure assumes that the observations for each alternative are normally distributed. With common random numbers, Dudewicz and Dalal's procedure will likely specify larger simulation sample sizes than is actually required to identify the best alternative.

In this paper, we develop a procedure using the Bonferroni inequality for selecting the alternative with the largest mean value when the variances are unknown and unequal and correlation is induced across the observations for each alternative by common random numbers. We assume unknown and unequal variances and correlations across alternatives. In addition, we assume that observations are normally distributed;

however, departures from this assumption are significant only when the t statistic has a distribution that cannot be approximated by the student t distribution. We compare the performance of the Bonferroni procedure with that obtained from Dudewicz and Dalal's procedure using a number of simulation experiments.

BONFERRONI PROCEDURE

We first define the statistical model used to represent the problem of selecting the best alternative from k alternatives, and then we specify the Bonferroni procedure. Denote the k alternatives as $\pi_1, \pi_2, \ldots, \pi_k$, where observations from π_i are independent and normally distributed with mean μ_i and variance σ^2_i for $i=1,2,\ldots,k$, with μ_i and σ^2_i unknown. Suppose we want to select the alternative that has the largest mean $\mu_{[k]}$, where $\mu_{[k]} > \mu_{[k-1]} > \cdots > \mu_{[1]}$ denote the ranked alternative means. We call achieving this goal making a correct selection, and the probability of this event is Pr(CS). Before conducting the simulation experiments we have to specify a pair of constants,

(δ^*, P^*) where $0 < \delta^*$ and $1/k < P^* < 1$ with the intention of achieving the following probability requirement:

$$Pr(CS) > P^* \text{ whenever } \mu_{[k]} - \mu_{[k-1]} > \delta^*$$

The Bonferroni procedure attempts to satisfy the above probability requirement using data from alternatives having correlated normal observations with unknown and unequal variances. Let

$X_{i,j}$ = the value of observation j from alternative i

We assume that $X_{i,j}$ and $X_{s,j}$ are correlated for $s \neq i$, but the pair $(X_{u,j}, X_{v,j})$ is independent of all other pairs $(X_{s,w}, X_{t,w})$ so long as $w \neq j$. The procedure is a means procedure, that is, it selects the alternative with the largest sample mean as the best alternative. Since Dudewicz [1971] proved that for the case of unknown variances any single-sample procedure can not satisfy the probability requirement, we apply Stein's two-stage sampling scheme described by Lehman [1959]. That is, we take an initial sample of size n for each alternative and then calculate a sample size for each alternative that guarantees the probability requirement.

The Bonferroni procedure is listed below.

1. From the initial sample of size n for each alternative, calculate

$$\overline{X}_i = \sum_{j=1}^{n} X_{i,j}/n \text{ for } i=1,2,\ldots,k$$

$$S^2_{i,j} = \sum_{u=1}^{n}((X_{i,u} - X_{j,u}) - (\overline{X}_i - \overline{X}_j))^2/(n-1)$$

for $i=1,2,\ldots,k$; $j=1,2,\ldots,k$; and $i \neq j$

$$S^2_i = \text{Max}(S^2_{i,j}: j=1,2,\ldots,k;\ j \neq i)$$

$$m_i = \text{Max}(n, [S^2_i/(\delta^*/h)^2 + .5])$$

for $i=1,2,\ldots,k$
where [x] is the greatest integer less than or equal to x
h satisfies $Pr(t_{n-1} < -h) = (1-P^*)/(k-1)$
t_{n-1} is Student's t statistic with n-1 degrees of freedom

2. Take an additional $m_i - n$ observations for alternatives $i=1,2,\ldots,k$.

Calculate $\hat{X}_i = \sum_{j=1}^{m_i} X_{i,j}/m_i$ for $i=1,2,\ldots,k$.

3. Select the alternative that yields the largest $\hat{X}_i$ as the best alternative.

RESULTS

The Bonferroni procedure is a conservative procedure in that it will specify more observations than actually required to meet the probability requirement. Also, Dudewicz and Dalal's procedure tends to be a conservative procedure when introducing positive correlation across the alternatives by using common random numbers. Simulation experiments indicate that the Bonferroni procedure is more efficient, i.e., less conservative, is more efficient, i.e., less conservative, than Dudewicz and Dalal's procedure when the percentage of variance reduction is high, e.g., 80 to 90%. Moreover, the Bonferroni procedure is more efficient in cases where the percentage of variance reduction is as low as 35% so long as k is small, e.g. three, and P^* is high, e.g., 0.95.

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