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SDLC PROTOCOL THROUGHPUT ANALYSIS

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ABSTRACT

A mathematical model is presented to determine large message throughput utilizing Synchronous Data Link Control (SDLC) protocol in local or long haul networks. The model includes provisions for wait time which includes processor service times, plus satellite and/or land based propagation delays. Other inputs include the probability of ACK/NAK timeout, block size, and assumed bit error rate.

Bit errors may be precipitated by electromechanical equipment malfunctions, lightning, power surges, crosstalk, thermal noise, and numerous other causes. Although bit errors are, therefore, bursty in nature, this analysis assumes that the bit error rate is random rather than bursty which will provide conservative results.

Basic feedback theory is applied to both clarify and simplify the analysis. A detailed example illustrates use of the model in a Communication Satellite application to illustrate large message throughput degradation caused by the Satellite propagation delay for various bit error rates.

An empirically derived simple expression to determine optimum block size for maximum throughput as a function of bit error rate is presented for the stated assumptions.

INTRODUCTION

SDLC is one of the more common network protocols available. One of its uses is with local or longhaul terrestrial communication networks. Transmission of data via a Communication Satellite using SDLS protocol is another application. Although modulo 8 SDLC is not the most efficient protocol for a Comsat link, it is more efficient than Bisynchronous Communication (BSC) protocol. More efficient protocols from a throughput standpoint are those which facilitate a greater number of frames per block such as modulo 128, and require retransmission of only those frames in error. The improved throughput efficiency usually is obtained at the expense of a more complex protocol with additional overhead. Modulo 128 has been available for the 3725 communications controller since June 29, 1984. It is considered appropriate to develop algorithms which can be utilized with Simulation models of SDLC or BSC protocols. The techniques used in this analysis may prove beneficial in developing mathematical models for other protocols of interest.

ASSUMPTIONS

- Bit errors are random, rather than "bursty" as they actually occur. This assumption will produce conservative results.
- Maximum frames per block may be set from 1 to 7 inclusive for modulo 8 analysis. The algorithms may be appropriately modified to facilitate other modulos, such as modulo 128.
- "Nominal" frame size for this analysis is 1024 bytes. The first frame of the SDLC block of this analysis will contain 977 bytes of message content and 11 bytes of overhead. Subsequent frames of each block will contain 1024 bytes of message content and 7 bytes of overhead.
- Messages are sent via a 3705 Communication Controller over a 56 kilobaud Communication Satellite channel to a 3271 dumb terminal.
- Size of acknowledgment frame is 11 bytes.
- On receipt of a negative acknowledgment (NAKJK), the retransmission will consist of the frame in error plus all remaining frames of the block. For example, when using a 7-frame block size, a NAK75 would result in frames 5, 6, and 7 being retransmitted. In the event of more than one bad frame per block, retransmission will be requested starting with the lowest numbered bad frame.
- Round trip propagation delay of 540 milliseconds plus a mean service time of 260 milliseconds constitutes the wait time contribution to wasted bandwidth. This value of service time may be too large for most applications, and will, therefore, produce conservative results in this analysis.
- Unlimited retransmissions on request (NAKJK's). In reality, a maximum number of attempts may be specified.
- Undetected errors as a function of the CRC polynomial utilized is not considered in this analysis.
- No network or equipment contention. The only factors affecting transmission time are transmission frequency, propagation delay, mean service time, and retransmissions due to bit errors or timeout as defined herein.
- Impact of SDLC zero bit insertion algorithm is negligible. This implies that the occurrence of a 01111110 bit pattern imbedded in the message content is minimal or non-existent.
- Partial frames not considered in this analysis.

- Analysis based on large, multimegabyte messages.

ANALYSIS

The message block and frame structure described in the Assumptions will be used for this analysis. The key variables which influence message transmission efficiency are:

- Frame size of first and subsequent frames of block.
- Frame overhead of first and subsequent frames of block.
- Maximum frames per block considered (up to 7)
- ACK/NAK frame size
- Channel capacity
- Bit error rate
- Wait time (equals propagation delays plus service times)
- Probability of timeout

The channel capacity is assumed to be 56 kilobaud for the examples considered herein since this is the highest channel capacity normally available for commercial terrestrial communications use, and will facilitate direct comparison at the same frequency to illustrate the degrading effect of propagation delay for satellite transmissions.

Message transmission efficiency, for this analysis, equals the time required for error free continuous transmission of the referenced message without overhead, service times, or propagation delays divided by the calculated time to transmit the message with the specified overhead, service time, propagation delay, bit error rate, and protocol assumptions.

Figure 1 illustrates the effect of a large message composed of many single frame message blocks, such as used with BSC protocol, being transmitted originally (input stage), and then defectively transmitted blocks being retransmitted indefinitely with as many additional stages as required to obtain a completely correct output message which must equal the input message. In reality, each stage is the same portion of a communication network, but repeated stages as illustrated help visualize the process and the equation derivation.

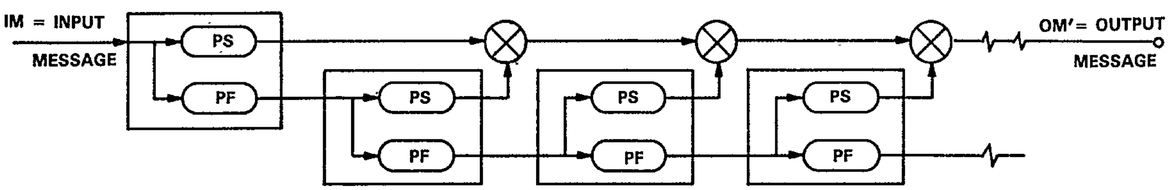
Equation (1) states that the output message equals the input message times a function of the probability of a successful message transmission and the probability of an unsuccessful message transmission. Equation (1) reduces to equation (2), which contains a portion (in parentheses) of one of the Maclaurin Power Series. Equation (3) represents the input/output transfer function of the network, and includes the closed equation form of the referenced Power Series. The value of equation (3) must always equal 1, providing the probability of success plus the probability of failure equals 1. This is true for the single frame block format of BSC protocol, as well as multiframe block protocols such as SDLC.

It is observed that equation (3) resembles the closed loop output/input transfer function of a feedback amplifier; namely, $A' = Eo'/Es = A/(1-A\beta)$, where A' is the closed circuit gain, Eo' is the closed circuit output, Es is the input, A is the open circuit gain, and β is the portion of the output which is fed back into the input in the closed loop configuration.

Equation (4) represents the total expected number of equivalent complete message transmissions including retransmissions. Equation (5) represents the expected number of equivalent complete retransmissions required to obtain exactly one good output message identical to the input message.

The equivalent equation for the model of Figure 2 is $OM'/IM = PS/(1-\beta PS)$, where $\beta = PF/PS$, and by substitution $OM'/IM = PS/(1-PF)$ which is identical to equation (3).

Equations (6) and (7) are defined by inspection of Figure 2. Equation (8) is derived by substitution.



ASSUMPTION: LARGE MESSAGE COMPOSED OF MANY SINGLE FRAME BLOCKS. PROBABILITIES RELATE TO INDIVIDUAL BLOCKS.

$$OM' = IM \times (PS + PF\,PS + (PF)^2 PS + (PF)^3 PS + \ldots) \quad (1)$$

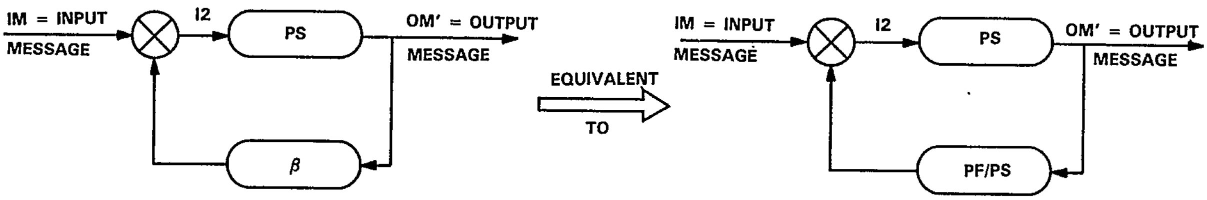
$$OM' = IM \times PS \times (1 + PF + (PF)^2 + (PF)^3 + \ldots) \quad (2)$$

$$OM'/IM = PS/(1-PF) = 1 \text{ (PROVIDING } PS + PF = 1) \quad (3)$$

$1/(1-PF)$ = EXPECTED NUMBER OF EQUIVALENT COMPLETE MESSAGE TRANSMISSIONS AND RETRANSMISSIONS (4)

$PF/(1-PF)$ = EXPECTED NUMBER OF EQUIVALENT COMPLETE RETRANSMISSIONS REQUIRED TO OBTAIN EXACTLY ONE GOOD OUTPUT MESSAGE. (5)

Figure 1: Repeated Message Transmission Representation



$$OM' = I2 \times PS \tag{6}$$

$$I2 = IM + \beta \times OM' \tag{7}$$

$$OM'/IM = PS/(1-\beta PS) \tag{8}$$

$$\beta = PF/PS \tag{9}$$

$$OM'/IM = PS/(1-PF) = PS \times 1/(1-PF) = 1 \tag{10}$$

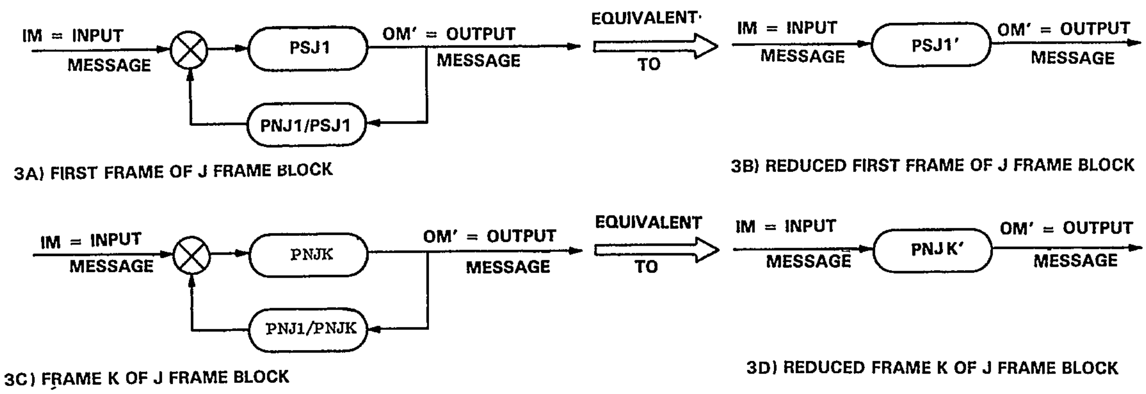
Figure 2: Equivalent Repeated Message Transmission Model

Equation (9) is derived by realizing that equation (8) must equal 1 for successful receipt of one good output message. Substitution yields equation (10), and the equivalent network is illustrated on the right side of Figure 2. Observe that equation (10) is identical to equation (3).

Figure 2 may be used with protocols such as Bisynchronous which transmit single frame blocks originally and retransmit the entire block upon receipt of a negative acknowledgment. Protocols such as SDLC, which retransmit only the portion of a block starting with the frame in error through and including the last frame of the block, require additional definition.

Figure 3 illustrates reduction of the repeated message transmission model as applied to SDLC protocol for ease of more concisely and comprehensively portraying the SDLC multiframe block model. Equation (11) is the SDLC first frame equivalent of equation (10). Equation (12) is the SDLC equivalent of equation (10) for frames other than frame 1. Observe that the numerator is the probability of a NAKJK acknowledgment which requests a retransmission of all remaining block frames including frame K of the original J frame block.

Equation (13) states that the open loop probability of a successful transmission (ACK) of a J frame block plus the summation of open loop probabilities from NAKJ1 through and including NAKJJ equals 1 in accordance with basic probability theory (PS + PF = 1). Note that the integer J denotes the number of frames which constitute an SDLC block of data. The number of frames per block for SDLC modulo 8 protocol



$$OM'/IM = PSJ1/(1-PNJ1) = PSJ1' \text{ (REFER TO FIGURE 3B)} \tag{11}$$

$$OM'/IM = PNJK/(1-PNJ1) = PNJK' \text{ (REFER TO FIGURE 3D)} \tag{12}$$

$$\left[PSJ1 + \sum_{K=1}^{J} PNJK = 1 \right] \tag{13}$$

J FRAME SDLC BLOCK OPEN LOOP PROBABILITY VALUES:
PSJI = OPEN LOOP PROBABILITY OF SUCCESSFUL 1ST FRAME XMISSION
PNJK = OPEN LOOP PROBABILITY OF A NAKJK FOR FRAME K OF J FRAME BLOCK RETRANSMIT FRAMES K THROUGH J

$$\left[PSJ1' + \sum_{K=2}^{J} PNJK' = 1 \right] \tag{14}$$

J FRAME SDLC BLOCK CLOSED LOOP PROBABILITY VALUES:
PSJ1' = CLOSED LOOP PROBABILITY OF SUCCESSFUL 1ST FRAME XMISSION
PNJK' = CLOSED LOOP PROBABILITIES OF A NAKJK FOR FRAME K OF J FRAME BLOCK

NOTE: J REPRESENTS THE NUMBER OF FRAMES WHICH COMPRISE THE BLOCK IN QUESTION

Figure 3: Repeated Message Transmission Model Reduction (SDLC Protocol)

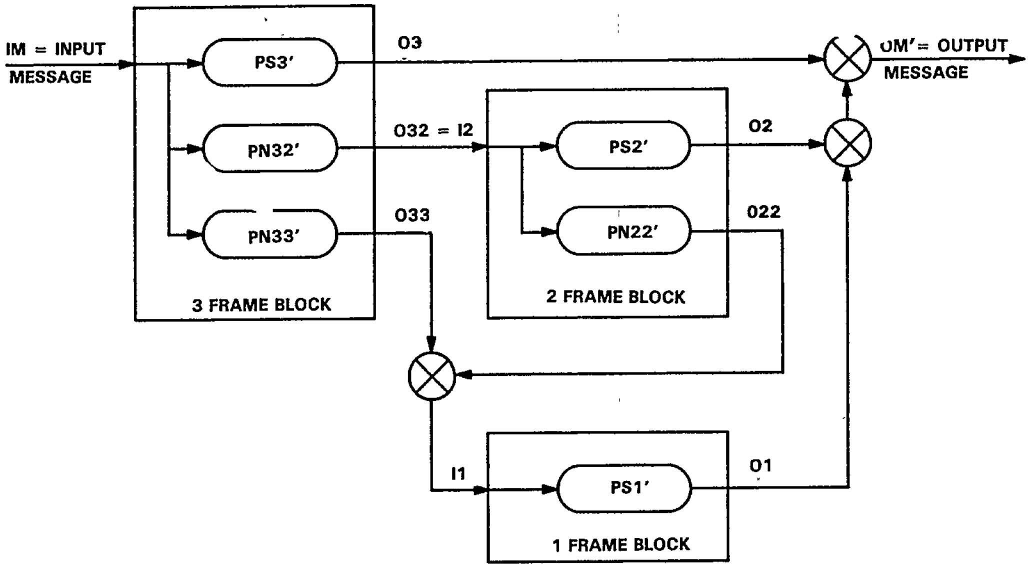
may be set to any integer value from 1 to 7, inclusive, by the user.

When the loop is closed and feedback applied, equation (14) states that for the closed loop condition, the probability of a successful transmission of a J frame block plus the summation of probabilities from NAKJ2 through and including NAKJJ equals 1 in accordance with basic probability theory. Observe that in this equation NAKJ1 is incorporated in the closed loop equation by means of PSJ1' which is defined in equation (11).

Assumed values of bit error rate, frame size and probability of no timeout are as indicated in the tables. The probability of successful transmission of a given frame is:

$$SJK = (1-e)^{**}b \qquad (15)$$

where e is the bit error rate, K is the frame number, b the number of bits in the frame, J is the number of frames in the block, and the double asterisk denotes exponentiation.



PS3' + PN32' + PN33' = 1; PER EQUATION (14) (25)

O3 + O32 + O33 = IM; MULTIPLYING EQUATION (25) BY IM (26)

O3 + O2 + O1 = IM; FOR OM = IM (27)

O32 + O33 = O2 − O1 (28)

O2 = PN33' (36)

O1 = PN22' = PN32' (37)

Figure 4: SDLC Three-Frame Block Model (Stage 1)

THREE FRAME SDLC BLOCK MODEL DEVELOPMENT (Stage 1)

The algorithm development for the 3-frame SDLC block model will consist of two stages. The first stage will determine a set of variables which have a recognizable pattern and may be easily derived for blocks of larger frame sizes such as the 7-frame SDLC block of this example, or the 127-frame SDLC modulo 128 block referenced earlier. These variables can then be readily applied to the final algorithms developed in the second stage. It is believed that this two-stage approach makes the analysis as well as the software coding more manageable for the larger frame sizes encountered with both similar and different type protocols. It is emphasized that the values obtained in the stage 1 development are not all actual protocol values, but rather represent a convenient method to facilitate stage 2 algorithm development.

Figure 4 illustrates the closed loop message flow for a 3-frame SDLC block. The equations for the open loop output/input transfer functions of the one, two and three frame block are derived from Table 1. Observe that in each case, equation (13) is satisfied.

$$S31 = (1-0.00001)^{**}7904 = \quad 0.924002598$$

$$S32 = S33 = (1-0.00001)^{**}8248 = \quad 0.920829475$$

There are 7904 bits in the first frame, and 8248 bits in frames 2 and 3 of this example.

The probability of a successful acknowledgment being received prior to timeout is expressed as:

$$SA = PNT((1-e)^{**}afb) \qquad (16)$$

where PNT is the probability of no timeout occurring, and afb represents the number of bits contained in the ACK/NAK frame, 88 in this example.

$$SA = (0.99)((1-0.00001)^{**}88) = \quad 0.989129179$$

The probability of an unsuccessful frame transmission is expressed as:

$$FJK = 1-SJK = 1-(1-e)^{**}b \qquad (17)$$

$$FJ1 = 1-SJ1 = \quad 0.075997402$$

$$FJ2 = FJ3 = 1-SJ3 = \quad 0.079170526$$

The probability of an unsuccessful ACK/NAK frame or a timeout is expressed as:

$$FA = 1-SA = 1-(PNT((1-e)^{**}afb)) \qquad (18)$$

$$FA = 1-SA = \qquad 0.010870821$$

For any given J frame SDLC block model, only the first frame of the J frame block configuration will contain a different number of bytes than all of the other frames. For example, when considering the entire 3-frame block SDLC model of Figure 4, only the first frame of the 3-frame block contains a different number of bytes (988 compared to 1031 in this example) than either of the other 2 blocks illustrated. S31 and F31 of Tables 1 and 2, for this reason, have different values than any other SJK's or FJK's of these tables.

Table 1 illustrates the probability breakdown for all possible transmission event occurrences for a 3, 2 and 1 frame block, respectively. The total number of event occurrences for a block is 2** (J+1), where J represents the number of frames per block. Each of the individual event probabilities represents the combined product of each SJK, FJK, SA or FA associated with that event. The probability of a NAKJK in Table 1 for a given J frame block is the sum of the individual probabilities associated with the NAKJK consequences of the block size in question. The probability of a successful J frame block transmission plus the probability of an unsuccessful transmission (summation of NAKJK's) equals 1 according to equation (13), and basic probability theory.

Table 1: One, Two and Three-Frame Block Probabilities

J	FRAME 1	FRAME 2	FRAME 3	ACK FRAME	CONSE-QUENCE	PROBABILITY
3	S31	S32	S33	SA	ACK	0.774969535
3	S31	S32	F33	SA	NAK33	0.066629867
3	S31	F32	S33	SA	NAK32	0.066629867
3	S31	F32	F33	SA	NAK32	0.005728663
3	S31	S32	S33	FA	NAK31	0.008517144
3	S31	S32	F33	FA	NAK31	0.000732282
3	S31	F32	S33	FA	NAK31	0.000732282
3	S31	F32	F33	FA	NAK31	0.000062960
3	F31	S32	S33	SA	NAK31	0.063739724
3	F31	S32	S33	FA	NAK31	0.000700518
3	F31	S32	F33	SA	NAK31	0.005480176
3	F31	S32	F33	FA	NAK31	0.000060229
3	F31	F32	S33	SA	NAK31	0.005480176
3	F31	F32	S33	FA	NAK31	0.000060229
3	F31	F32	F33	SA	NAK31	0.000471171
3	F31	F32	F33	FA	NAK31	0.000005178
2	S21	S22		SA	ACK	0.838709268
2	S21	F22		SA	NAK22	0.072110042
2	S21	S22		FA	NAK21	0.009217662
2	S21	F22		FA	NAK21	0.000792511
2	F21	S22		SA	NAK21	0.072110042
2	F21	S22		FA	NAK21	0.000792511
2	F21	F22		SA	NAK21	0.006199834
2	F21	F22		FA	NAK21	0.000068138
1	S11			SA	ACK	0.910819303
1	S11			FA	NAK11	0.010010172
1	F11			SA	NAK11	0.078309876
1	F11			FA	NAK11	0.000860649

PROBABILITY OF ACK (SUCCESSFUL 3 FRAME BLOCK XMISSION) = 0.774969535
PROBABILITY OF NAK31 (RESEND 3 FRAME BLOCK) = 0.086042069
PROBABILITY OF NAK32 (RESEND FRAMES 2 AND 3) = 0.072358530
PROBABILITY OF NAK33 (RESEND FRAME 3) = 0.066629867
PROBABILITY OF SUCCESSFUL 3 FRAME BLOCK XMISSION OR NAK31 OR NAK32 OR NAK33 = 1.000000000

PROBABILITY OF ACK (SUCCESSFUL 2 FRAME BLOCK XMISSION) = 0.838709260
PROBABILITY OF NAK21 (RSEND 2 FRAME BLOCK) = 0.089180698
PROBABILITY OF NAK22 (RESEND FRAME 2) = 0.072110042
PROBABILITY OF SUCCESSFUL 2 FRAME BLOCK XMISSION OR NAK21 OR NAK22 = 1.000000000

PROBABILITY OF ACK (SUCCESSFUL 1 FRAME BLOCK XMISSION) = 0.910819303
PROBABILITY OF NAK11 (RESEND 1 FRAME BLOCK) = 0.089180697
PROBABILITY OF SUCCESSUFL 1 FRAME BLOCK XMISSION OR A NAK11 = 1.000000000

NOTES:

J = NUMBER OF FRAMES PER PLOCK
K = FRAME NUMBER
SJK = PROB OF SUCCESSFUL FRAME K XMISSION
FJK = PROB OF UNSUCCESSFUL FRAME K XMISSION
SA = PROB OF SUCCESSFUL ACK FRAME XMISSION AND NO TIMEOUT
FA = PROB OF UNSUCCESSFUL ACK FRAME XMISSION AND/OR TIMEOUT
NAKJK = RESEND REMAINDER OF BLOCK STARTING WITH FRAME K. FOR EXAMPLE, NAK32 REQUESTS THE SENDER TO RETRANSMIT FRAMES 2 AND 3 OF THE 3 FRAME BLOCK
PNT = PROBABILITY OF NO TIMEOUT = 0.99
S31 = (0.99999)**7904 = 0.924002598
F31 = 1-S31 = 0.075997402
S32 = S33 = S22 = S21 = S11 = (0.99999)**8248 = 0.920829475
F32 = F33 = F22 = F21 = F11 = 1-S32 = 0.079170525
SA = 0.99(0.99999)**88 = 0.989129179
FA = 1-SA = 0.010870821

The closed loop output/input transfer functions are readily derived by dividing the open loop output/input transfer functions by the "feedback divisor", (1-PNJ1) for each J frame block per equations (11) and (12). Accordingly, for this example:

$$PS3' = PS3/(1-PN31) = \qquad 0.847926921 \qquad (19)$$

$$PN32' = PN32/(1-PN31) = \qquad 0.079170526 \qquad (20)$$

$$PN33' = PN33/(1-PN31) = \qquad 0.072902553 \qquad (21)$$

$$PS2' = PS2/(1-PN21) = \qquad 0.920829475 \qquad (22)$$

$$PS22' = PS22/(1-PN21) = \qquad 0.079170526 \qquad (23)$$

$$PS1' = PS1/(1-PN11) = \qquad 1.000000000 \qquad (24)$$

Observe that PS1' = PS2'+PN22' = PS3'+PN32'+PN33' = 1 per equation (14).

Equation (25) of Figure 4 results directly from equation (14). Equation (25) multiplied by IM yields equation (26). By inspection of Figure 4, O3+O2+O1 = OM' = IM for the output message to exactly equal the input message. This results in equation (27). Subtracting equation (27) from (26) results in equation (28).

By setting the input message equal to 1:

$$O3 = PS3' = \qquad 0.847926921 \qquad (29)$$

$$O32 = PN32' = \qquad 0.079170526 \qquad (30)$$

$$O33 = PN33' = \qquad 0.072902553 \qquad (31)$$

$$O2 = O32 \times PS2' = \qquad 0.072902553 \qquad (32)$$

$$O22 = O32 \times PN22' = \qquad 0.006267972 \qquad (33)$$

$$I1 = O33 + O22 = \qquad 0.079170526 \qquad (34)$$

$$O1 = I1 \times PS1' = \qquad 0.079170526 \qquad (35)$$

Observe that equations (25) through (28) of Figure 4 are satisfied. Also, observe that O1 = PN22' = PN32', and that O2 = PN33'. This pattern appears to indicate the possibility of an inductive solution for larger frame blocks.

By substituting the appropriate probability equations of Table 2 and then replacing occurrences of FJK with (1-SJK), it can be demonstrated that:

$$PN33' = O2 = PN32' \times PS2' \qquad (36)$$

$$PN32' = PN22' = O1 \qquad (37)$$

THREE FRAME SDLC BLOCK MODEL DEVELOPMENT (Stage 2)

The OJK (NAKJK) outputs of Figure 4 are each illustrated going to a single point. In reality, each output is split into two portions. One part goes to the output (OJ) of the block, and the other part goes to the same point as indicated in Figure 4. Figure 5 illustrates the statistical message flow for a 3-frame SDLC block with a "nominal" frame size of 1024 bytes. Observe, for example, that 032 results in a good first frame being transmitted to the output in the form of the lower input to the 03 summing point, and the remaining two frames (NAK32) being retransmitted in the form of the input equation of the 2-frame block. The one-frame block, in this analysis, continues to retransmit NAK11 messages until the output of this block equals the input to the block.

As illustrated in Figure 5, the output message consists of the 03 output plus approximately 1/3 of the 032 output, plus approximately 2/3 of the 033 output, plus approximately 2/3 of the Figure 4 02 output, plus approximately 1/3 of the Figure 4 022 output, plus approximately 1/3 of the Figure 4 01 output, and is represented by equation (41). If all frames were the same size, rather than the differently sized frame of the SDLC block caused by greater first frame overhead but less first frame message content, then all occurrences of the word "approximately" in the last sentence would be replaced with "exactly".

Table 2 illustrates the frame-by-frame breakdown at a byte and partial byte level of message transmission and retransmission due to NAKs. It is realized that frames will not be altered as indicated, nor will partial bytes be transmitted, but these values represent the statistical expected average values for a large message transmission of 1024 byte "nominal" frames in three frame blocks, and may, therefore, be utilized to determine large message throughput.

The same probability values are used as those of Table 1. Column 1 contains the frames per block which may vary from 3 to 1, inclusive. Column 2 contains the input message and overhead bytes for each of the frames in column 1. Note that the input to the 2-frame block consists of the 3-frame block NAK 2 output, and the input to the 1-frame blocks consists of the 3-frame block NAK 3 output plus the 2-frame block NAK 2 output.

Column 3 contains the probability equation plus the actual probability value used to determine the message throughput. Column 4 contains the consequence of the probability equation of column 3 which may be a good block or partial block transmission attempt resulting from a NAKJ2 or a NAKJ3. In the case of a NAKJK, the remaining frame or frames of the original block will be retransmitted as smaller blocks. NAKJ1's are not illustrated directly in Table 2 since they are accounted for in the feedback factors R3, R2, and R1. The variable R was selected to represent Retransmission. Observe that the value of RJ is obtained by utilizing equation (4) of Figure 1, where PF (probability of failure) is PNJ1, the probability of a NAKJ1 resulting from a failure to correctly transmit the first frame of a J-frame block, or the ACK frame.

Columns 5, 6 and 7 contain the frame 1, 2 and 3 message and overhead bytes received at the output for each of the applicable block transmission configurations. Finally, the individual good frame sums of column 5, column 6 and column 7 equal the originally transmitted message and overhead bytes for each of the respective frames.

SEVEN-FRAME SDLC BLOCK MODEL DEVELOPMENT

The SDLC 7-frame "stage 1" block model is illustrated in Figure 6. The same values will be used in this example as in the previous example, namely:

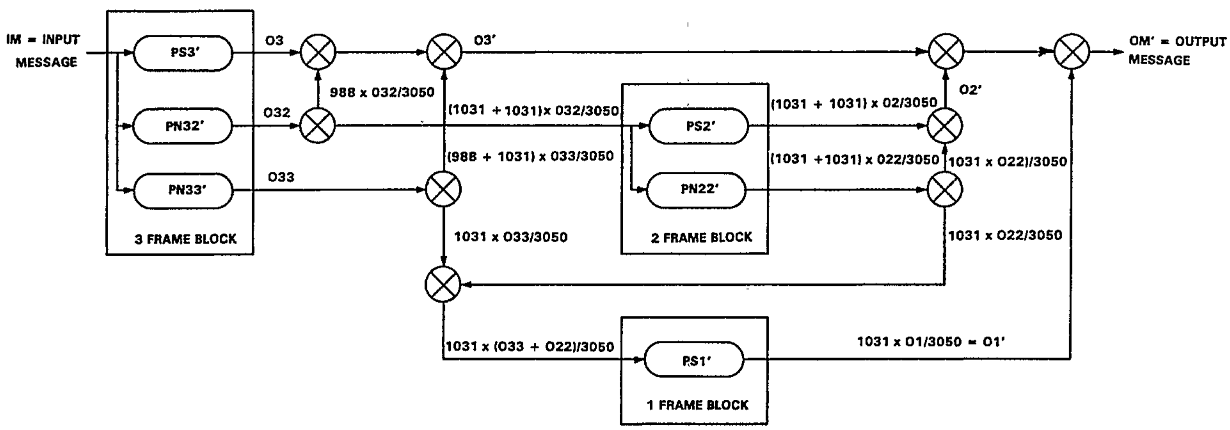
bit error rate = 0.00001

frame 1 = 988 bytes

frames 2 to 7, inclusive, = 1031 bytes

wait time value = 0.80 seconds

PNT = 0.99



$$O3'/IM = O3 \times (988 + 1031 + 1031)/3050 + O3 \times (988 + 1031)/3050 + O33 \times 988/3050 \quad (38)$$

$$O2'/IM = O2 \times (1031 + 1031)/3050 + O22 \times 1031/3050 \quad (39)$$

$$O1'/IM = O1 \times 1031/3050 \quad (40)$$

$$OM'/IM = (O3' + O2' + O1')/IM \quad (41)$$

Figure 5: SDLC Three-Frame Block Model
(Stage 2)

Table 2: Three-Frame SDLC Block Example

FRAMES PER BLOCK	INPUT BYTES FRAME 1/ FRAME 2/ FRAME 3	PROBABILITY EQUATION/ PROBABILITY	CONSEQUENCE	FRAME 1 MESSAGE/ OVERHEAD OUTPUT (BYTES)	FRAME 2 MESSAGE/ OVERHEAD OUTPUT (BYTES)	FRAME 3 MESSAGE/ OVERHEAD OUTPUT (BYTES)
3	977 + 11 1024 + 7 1024 + 7	S31S32S33SAR3 0.847926921	GOOD BLOCK	828.4246019 9.327196132	868.2771672 5.935488448	868.2771672 5.935488448
		S31S32F33SAR3 0.072902553	NAK33 - GOOD FRAMES 1 AND 2	71.22579466 0.801928087	74.65221467 0.510317874	
			RETRANSMIT FRAME 3 AS A ONE FRAME BLOCK			74.65221467 0.510317874
		S31F32SAR3 0.079170525	NAK32 - GOOD FRAME 1	77.34960341 0.870875781		
			RETRANSMIT FRAMES 2 & 3 AS A TWO FRAME BLOCK		81.07061811 0.554193679	81.07061811 0.554193697
2	0 + 0 81.07061811 + 0.554193579 81.07061811 + 0.554193679	S12S22SAR2 0.920829474	GOOD BLOCK		74.65221467 0.510317874	74.65221467 0.510317874
		S12F22SAR2 0.079170525	NAK22- GOOD FIRST FRAME		6.418403439 0.043875805	
			RETRANSMIT SECOND FRAME AS A ONE FRAME BLOCK			6.418403439 0.043875805
1	0 + 0 0 + 0 81.07861811 + 0.554193679	S11SAR1 1.000000000	GOOD BLOCK			81.07061811 0.554193679
ORIGINALLY TRANSMITTED BYTES AND FINAL TOTAL OUTPUT BYTES				977 11	1024 7	1024 7

S31 = (0.99999)**7904	= 0.924002598
F31 = 1-S31	= 0.075997402
S32 = S33 = S22 = S21 = S11 = (0.99999)**8248	= 0.920829475
F32 = F33 = F22 = F21 = F11 = 1-S32	= 0.079170525
SA = 0.99(0.99999)**88	= 0.989129179
FA = 1-SA	= 0.010870821
R3 = 1/(1 + (F31FA) - (F31 + FA))	= 1.094142264
R2 = R1 = 1/(1 + F21FA) - (F21 + FA))	= 1.897812613

These values result in the same preliminary probabilities as the previous example, namely:

S71 =	0.924002598
S72=S73=S74=S75=S76=S77 =	0.920829475
SA =	0.989129179
F71 =	0.075997402
F72=F73=F74=F75=F76=F77 =	0.079170526
FA =	0.010870821

A program was coded on the HP-41C to determine the output/input transfer functions and the 7 differently sized block output messages which produce the final correct total output message. Inspection of the results confirms the inductive solution pattern previously hypothesized, namely:

01=PN22'=PN32'=PN42'=PN52'=PN62'=PN72' =	0.079170526	(42)
02=PN33'=PN43'=PN53'=PN63'=PN73' =	0.072902553	(43)
03=PN44'=PN54'=PN64'=PN74' =	0.067130820	(44)
04 = PN55' = PN65' = PN75' =	0.061816038	(45)
05 = PN66' = PN76' =	0.056922029	(46)
06 = PN77' =	0.052415482	(47)
07 = 1-(01+02+03+04+05+06) =	0.609642552	(48)

Observe that except for the maximum size block (7 frames in this example), the ratio of adjacent block transfer functions in descending order is constant, and is equal to the open loop probability of a successful frame transmission for each but the first frame of a J-frame block. The transfer function of the maximum size block equals one minus the sum of all other size block transfer functions.

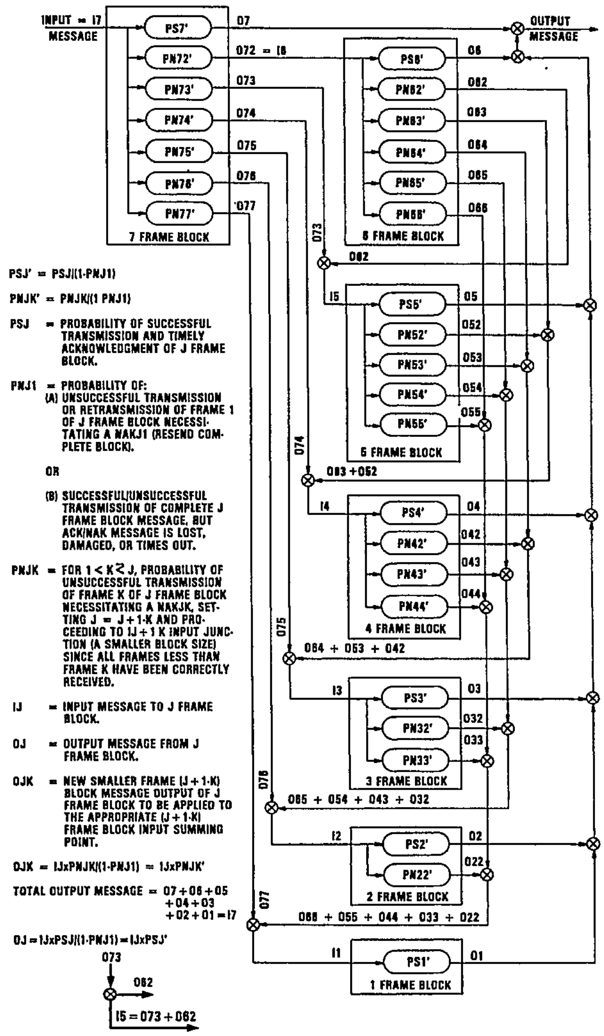


Figure 6: SDLC Seven-Frame Block Model

As before, the following equations represent the various block and overall output/input transfer functions assuming a 1024 byte "nominal" frame size.

$$07'/IM=07 + (988+(5\times1031))\times077/7174 + (988+(4\times1031))\times076/7174 + (988+(3\times1031))\times075/7174 + (988+(2\times1031))\times074/7174 + (988+1031)\times073/7174 + 988\times072/7174 \quad (49)$$

$$06'/IM = (6\times1031)\times06/7174 + (5\times1031)\times066/7174 + (4\times1031)\times065/7174 + (3\times1031)\times064/7174 + (2\times1031)\times063/7174 + 1031\times062/7174 \quad (50)$$

05'/IM = (5x1031)x05/7174 + (4x1031)x055/7174 (51)
+ (3x1031)x054/7174 + (2x1031)x053/7174
+ 1031x052/7174

04'/IM = (4x1031)x04/7174 + (3x1031)x044/7174 (52)
+ (2x1031)x043/7174 + 1031x042/7174

03'/IM = (3x1031)x03/7174 + (2x1031)x033/7174 (53)
+ 1031x032/7174

02'/IM = (2x1031)x02/7174 + 1031x022/7174 (54)

01'/IM = 1031x01/7174 (55)

OM'/IM=(07'+06'+05'+04'+03'+02'+01')/IM (56)

After completing the original analysis, making runs, and plotting results, it was observed by examination of Figure 7 that a simple equation could be expressed to closely represent the optimum block size in bytes as a function of bit error rate for the example variables assumed, namely:

optimum block size = 351407/((e x 10**8) (57)
**[log(351407/10290)]/3)

where e is the bit error rate; and, as before, the double asterisks denote exponentiation. This empirically derived equation is accurate to within less than 10% error for bit error rates between 10**(-8) and 10**(-5), inclusive. The error could be made essentially negligible at the expense of a more complex equation. Figure 11 illustrates optimum block size as a function of bit error rate utilizing equation (57).

RESULTS

Tables 3 and 4 illustrate various analytical results for both 3- and 7-frame SDLC modulo 8 transmission of a 10 megabyte message over a 56 KB Comsat channel. The results of the last 5 columns may be modified by the appropriate ratio to determine values for other message sizes.

Column 1 contains the frame numbers in the block transmitted or retransmitted. Column 2 contains the block size in bytes. Column 3 contains the total number of bytes per block required to be transmitted and retransmitted in order for the output message to exactly equal the input message. These values may be obtained from the "GOOD BLOCK" data of Table 2. The 2-frame block value of 179.23 in Table 3, for

Table 3: Three-Frame SDLC Block Detailed Example

FRAME #'S IN BLOCK	BLOCK SIZE (BYTES)	BYTES XMITTED PER BLOCK	OUTPUT MESSAGE CONTENT (BYTES) PER BLOCK	BYTES XMITTED PER 10 MEG MESSAGE	BLOCKS XMITTED PER 10 MEG MESSAGE	WAIT (1) TIME (SEC)	MSG (2) XMISSION TIME (SEC)	TOTAL TIME (SEC)
1-3	3050	3337.13	2700.21	11566731	3792.70	3034.16	1658.49	4692.65
2-3	2062	179.23	155.72	621290	301.30	241.04	89.23	330.27
3	1031	89.62	81.07	310645	301.30	241.04	44.85	285.90
TOTAL			3025.00	12499666	4395.31	3516.25	1792.57	5308.82

EFFICIENCY = 28.22%
CHANNEL RATE = 56 KB
BIT ERROR RATE = 10**(-5)
PROBABILITY OF TIMEOUT = 0.01

(1) WAIT TIME EQUALS PROPAGATION DELAY + SERVICE TIME = 800 MILLISECONDS
(2) INCLUDES 11 BYTE ACK/NAK FRAME MESSAGE TRANSMISSION TIME

Table 4: Seven-Frame SDLC Block Detailed Example

FRAME #'S IN BLOCK	BLOCK SIZE (BYTES)	BYTES XMITTED PER BLOCK	OUTPUT MESSAGE CONTENT (BYTES) PER BLOCK	BYTES XMITTED PER 10 MEG MESSAGE	BLOCKS XMITTED PER 10 MEG MESSAGE	WAIT (1) TIME (SEC)	MSG (2) XMISSION TIME (SEC)	TOTAL TIME (SEC)
1-7	7174	7849.38	5626.20	11558304	1611.14	1288.91	1653.73	2942.61
2-7	6186	537.70	399.73	791772	127.99	102.40	113.31	215.71
3-7	5155	448.08	346.05	659810	127.99	102.40	94.46	196.86
4-7	4124	358.47	287.76	527848	127.99	102.40	75.61	178.00
5-7	3093	268.85	224.46	396886	127.99	102.40	56.76	159.15
6-7	2062	179.32	155.72	263924	127.99	102.40	37.90	140.30
7	1031	89.62	81.07	131962	127.99	102.40	19.05	121.45
TOTAL			7121.00	14329506	2379.10	1903.28	2050.81	3954.09

EFFICIENCY = 37.88%
CHANNEL RATE = 56 KB
BIT ERROR RATE = 10**(-5)
PROBABILITY OF TIMEOUT = 0.01

(1) WAIT TIME EQUALS PROPAGATION DELAY + SERVICE TIME = 800 MILLISECONDS
(2) INCLUDES 11 BYTE ACK/NAK FRAME MESSAGE TRANSMISSION TIME

example, equals the sum of the frame 2 and 3 message plus overhead bytes of Table 2, 2 frames per block, "GOOD BLOCK" data divided by S12S22SA. Observe that the Retransmission factor R2 is not used in this determination. Alternatively, the 2-frame block value, for example, may be determined by multiplying the total input to the 2 frames per block model by the two block retransmission factor, R2.

Column 4 contains the number of received message bytes per block. These values may be derived by using equations (38) to (41), and (49) to (56), after first replacing all occurrences of 988 and 1031 with 977 and 1024, respectively, to obtain message content without overhead. Observe that the column 4 total figure in both tables equals the original block message content. The difference between these values and the complete block size of column 2, row 1 represents overhead bytes of 11 bytes for the first frame and 7 bytes for additional frames in the block.

Column 5 contains the total bytes per block required to be transmitted in order for the complete 10 megabyte message received to exactly equal the message sent. These values are obtained by dividing the message size of 10 megabytes by the message content block size of 3025 bytes, and multiplying the results by the bytes transmitted per block values of column 3.

Column 6 contains the blocks transmitted per 10 megabyte message, and is obtained by dividing column 5 by column 2.

Column 7 contains the wait time, and is obtained by multiplying the column 6 values of blocks per message by the wait time per block. The wait time per block in these examples is 800 milliseconds, consisting of round-trip propagation delay plus mean service time.

Column 8 contains message (including ACKs and NAKs) transmission time determined by adding the total ACK/NAK frame bytes of 11 times column 6 values to the transmitted message and overhead bytes of column 5, multiplying these values by 8 to convert bytes to bits, and dividing the results by the channel rate of 56000 bits per second.

Column 9 values are obtained by adding column 8 and column 7 values. The result is total time including propagation delay, service time, plus message transmission and retransmission for each of the 1 to J-frame block configurations of these examples.

Columns 4 through 9 are summed individually to produce total results for these columns. Efficiency is determined by dividing the time required to transmit the 10 megabyte message as one error free block without overhead by the estimated time determined, then multiplying by 100 to convert to percent.

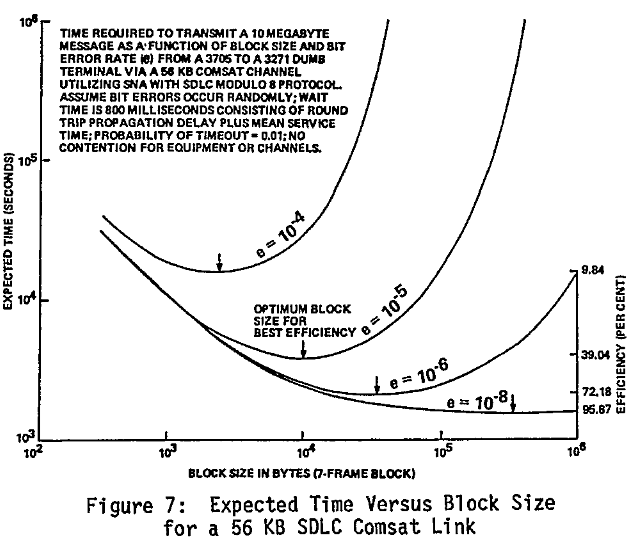


Figure 7: Expected Time Versus Block Size for a 56 KB SDLC Comsat Link

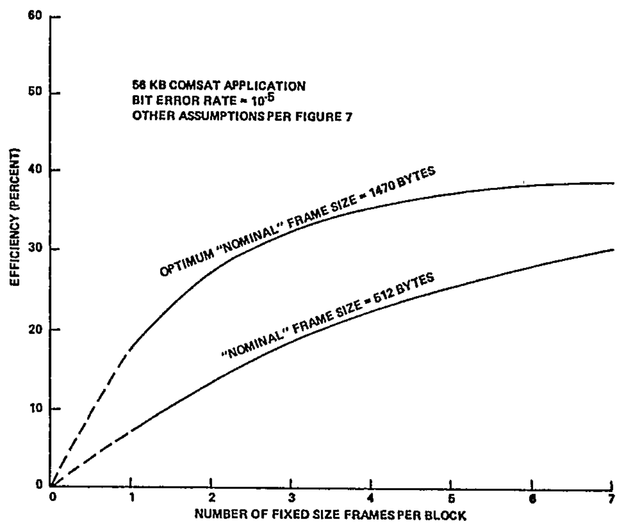


Figure 8: Efficiency Versus Frames Per Block for $e = 10^{-5}$

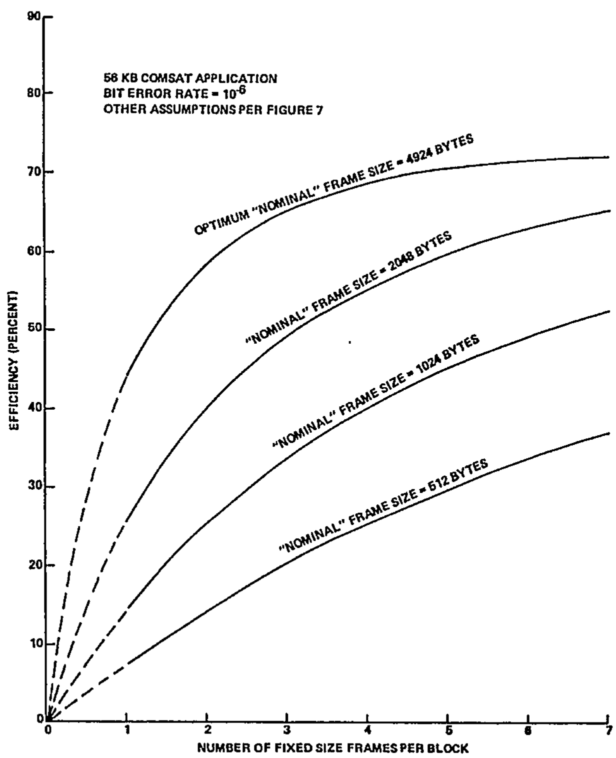


Figure 9: Efficiency Versus Frames Per Block for $e = 10^{-6}$