Proceedings of the 1985 Winter Simulation Conference
D. Gantz, G. Blais, S. Solomon (eds.)

ON THE USE OF MANOVA IN THE ANALYSIS OF MULTIPLE-RESPONSE SIMULATION EXPERIMENTS

Linda Weiser Friedman
Assistant Professor
Department of Statistics and Computer Information Systems
Baruch College of the City University of New York
New York, New York 10010

ABSTRACT

The purpose of this paper is to demonstrate the applicability of multivariate analysis of variance (MANOVA) to simulation analysis. The paper discusses an illustrative simulation experiment of an M/M/s queueing system, in which three system configurations are crossed with two queue disciplines to produce a 3 x 2 factorial design. Since there are several output variables of interest, namely average number of waiting demands, average time in system, and the proportion of demands forced to wait more than a specified amount of time, a multivariate statistical technique must be used. The necessity and advantage of using this multivariate tool, rather than treating the simulation experiment as several univariate-response experiments, is discussed.

INTRODUCTION

Over the past twenty years or so, a growing body of literature has focused on the statistical analysis of simulation output data (see, e.g., 15, 11, 3, 4, 5, 12, 13). Virtually all of the published reasearch has approached the problem from a univariate point of view, that is, assuming only a single response variable output from the simulation. However, the simulation researcher often must analyze data on more than one dimension as often more than a single measure of effectiveness is of interest. Recently, there has been some interest in the multiple response problem in simulation analysis (17, 18, 8).

Analysis of variance (ANOVA) models can be used anytime a simulationist wishes to determine the effect of various treatments (factors) or treatment combinations on the response variable of interest. It also enables one to determine whether two or more treatments interact with one another.

If there is more than one response variable of interest, a multivariate statistical test should be used. Treating a simulation with **p** response variables as if it were **p** experiments with one response variable each is inferior to multivariate analysis which not only improves the significance level but also enables the researcher to get a better idea of how the responses behave together (see 6,7).

Multivariate analysis of variance (MANOVA) is the multivariate extension of univariate ANOVA and is potentially a very useful tool for analyzing the results of multivariate simulation experiments.

ILLUSTRATION

For the purpose of illustration, a simple queueing system simulation model was developed using SIMSCRIPT II.5 (2). Demands were assumed to arrive according to a Poisson process with a constant average arrival rate, and service times were assumed to follow an exponential distribution with a constant average service time. The arrival rate, the service rate, and the number of servers were treated as variables input to the program. Thus, the system was modeled as an M/M/s queue with a single service facility and a single (initially, FIFO) waiting line.

The simulation model was validated by comparing the values of time-averages generated by the simulation program at the end of a collection of 15-week runs with the steady-state averages expected using queueing theory. This comparison was done via the one-sample Hotelling's T^2 test. Results indicated that the simulation estimates were what one would expect as output from an M/M/s queueing system.

In order to illustrate the use of MANOVA by simulationists, consider the following simple example: A firm is in the process of planning a new system, e.g., a machinery repair shop. Machines arrive for repair at the rate of 21/day. Three alternate system configurations are to be tested: (I)four servers (repairmen, automated equipment, etc.), each able to repair an average of 6 machines per day, (II) three servers, each able to repair an average of 8 machines per day, or (III) two servers, each able to repair an average of 12 machines per day. In addition, the firm must decide between employing a first-in-first-out (FIFO) or a last-in-first-out (LIFO) queue discipline for machines waiting to be serviced.

An experiment was set up in order to determine what effect, if any, queue discipline and/or system configuration has on three measures of performance: the average number of machines waiting to be serviced (LQ), the average time spent in the shop per machine (W), and the proportion of machines which had to wait longer than 10 minutes to be repaired (PROP). In addition, the firm wished to test for the existence of an interaction between queue discipline and system configuration. Ten independent replications were run for each queue-discipline by system-configuration combination. Thus, there were a total of 60 simulation runs.

In experimental design terminology, this is a 3 by 2 factorial design with 10 replications per cell. Table 1 presents the cell means for this experiment.

	QUEUE DISCIPLINE					
	LIFO			FIFO		
SYSTEM CONFIG.	LQ	W	PROP	LQ	W	PROP
I	5.266	.417	.1776	5.266	.418	.4491
II	5.510	.387	.1863	5.510	.388	.4717
III	5.813	.360	.1966	5.813	.361	.4979

Table 1: Cell Means

Table 2, which presents the results of the MANOVA, indicates no significant interaction effect present.

Null Hypothesis	Wilks' lambda	F	d.f.	p
no overall QuDIS effect	.0093	1840.01	3, 52	<.01
no overall SYST effect	.0043	247.01	6, 104	<.01
no overall interaction effect	.8376	1.61	6, 104	>.15

Table 2: Multivariate Analysis of Variance

However, both main effects were significant at the p=.01 level. Thus, both the queue discipline and the system configuration significantly affected the values of the measures of performance, on a multivariate (experimentwise) level. The test statistic used in the MANOVA is Wilks' lambda (sometimes called a U-statistic), which computes the ratio of two determinants in order to compare the error sum-of-squares-and-cross-products (SSCP) matrix to the total SSCP matrix. The mathematics of this statistic are adequately explained in (14, 1, 10), and a researcher using a statistical package such as SAS (16) or SPSS^X (19) will be automatically provided with a transformation of this statistic which either follows or approximates an F distribution.

As explained in (9), only if the multivariate tests are significant are the univariate ANOVA's examined. Table 3 contains these results for the three measures of performance. The results indicate that the only measure which contributed significantly to the rejection of the null hypothesis was PROP. LQ and W were not affected by differences in queue discipline, and only W was significantly affected by the differences among the three system configurations.

Dependent Variable	Effect	F-value	d.f.	p
LQ	Q-DIS	0.0	1, 54	>.9
	SYST	1.86	2, 54	>.15
	Q-DIS*SYST	0.0	2, 54	>.9
W	Q-DIS	0.0	1, 54	>.9
	SYST	9.15	2, 54	<.01
	Q-DIS*SYST	0.0	2, 54	>.9
PROP	Q-DIS	2030.84	1, 54	<.01
	SYST	9.53	2, 54	<.01
	Q-DIS*SYST	1.84	2, 54	>.15

Table 3: Univariate ANOVA's

This small (and, admittedly, contrived) example serves to illustrate the application of multivariate analysis of variance to data generated by multiple response simulation experiments. In general, any univariate statistical model of experimental design has a multivariate counterpart. A reference textbook in multivariate analysis should be consulted.

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LINDA WEISER FRIEDMAN is an Assistant Professor of Statistics and Computer Information Systems at Baruch College of the City University of New York. She received her B.A. from Baruch, and her M.S. and Ph.D. (Operations Research) degrees from Polytechnic Institute of New York. Her major research interests are the statistical analysis of simulation output data, applications of multivariate statistical analysis, methods for promoting ethics and professionalism in computer science students, and marketing research. Dr. Friedman has published in several scholarly journals, including the *Journal of Statistical Computation and Simulation*, *Interface: the Computer Education Quarterly*, the *Journal of Advertising Research*, and *Simulation*. She also writes fiction and poetry. Dr. Friedman lives in Brooklyn, New York with her husband and five little girls, some of whom will be of marriageable age in a few years. Interested parties may respond to:

Prof. Linda Weiser Friedman
Department of Statistics and Computer Information Systems
Baruch College - Box 513
17 Lexington Avenue
New York, New York 10010
(212) 725-3168