Proceedings of the 1985 Winter Simulation Conference
D. Gantz, G. Blais, S. Solomon (eds.)

COMPARISON OF TWO STATIONARY STOCHASTIC PROCESSES USING STANDARDIZED TIME SERIES

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ABSTRACT

References to confidence interval estimations for the difference between the means and for the ratio of the variances of two independent strictly stationary phi-mixing processes using standardized time series are given. Results of a limited empirical investigation on one set of the estimators for the difference between the two means are presented after they are briefly discussed. These estimators performed well in this investigation.

1. INTRODUCTION

A common statistical problem is that of comparing two populations with respect to some characteristic. The characteristic may be the means or variances of the two populations. Comparisons are usually made by comparing samples from the two populations. This is often done by computing the sample mean or variance from each population and combining them by taking their difference (means) or ratio (variances). Comparisons can be made more meaningfully if the distributions of comparison statistics are known. This is generally simple for independent observations in each sample. For observations from a strictly stationary phi-mixing stochastic process (Billingsley [1968]), a standardized time series approach introduced by Schruben [1983] can be used to obtain asymptotically the distributions of comparison statistics.

Recently, four new types of confidence interval estimators were developed by Chen and Sargent [1984a, 1984b] for the comparison of the means and variances of two independent, strictly stationary phi-mixing stochastic processes. In this paper we discuss some limited empirical experiments for the difference between the means for the area estimators given in Chen and Sargent [1984a]. The area estimators were chosen because initial research into the convergence rate of these four different types of estimators by Goldsman [1984] tended to show for the simple cases studied that the area estimator may converge more rapidly to the asymptotic results than the other types of estimators. The purpose of the experiments was to determine that the estimators perform as expected and not for a definitive empirical investigation of the estimators' behavior.

This paper is organized as follows. Section 2 presents a brief discussion of the area interval estimators for the difference between the means for three cases, namely, (1) equal sample sizes with unknown variances, (2) unequal sample sizes with unknown common variance, and (3) unequal sample sizes with unknown unequal variances. Section 3 discusses the results of a limited empirical investigation of the estimators. Section 4 is the conclusion.

2. CONFIDENCE INTERVAL ESTIMATIONS FOR THREE CASES

For s=1,2, let $\{X_{sj}\}_{j=1}^{n_s}$ be the sample of the n_s observations from two independent strictly stationary phi-mixing processes with means μ_s and variances $\sigma_s^2 = \lim_{n_s\to\infty} n_s \operatorname{Var}(\bar{\bar{X}}_s)$, where $\bar{\bar{X}}_s = \frac{1}{n_s}\sum_{j=1}^{n_s} X_{sj}$. These sets of data are divided into b_s adjacent, nonoverlapping, equal-sized batches with each of size m_s, i.e., $n_s = m_s b_s$, s=1,2, where m_s and b_s are assumed to be integers. Chen and Sargent [1984a] developed confidence interval estimators for $\mu=\mu_1-\mu_2$ for the three different cases. These cases depend on the equalities of the sample sizes and variances.

(1) Equal sample sizes with unknown variances. In this case, $n_1=n_2=n$, $b_1=b_2=b$, and $m_1=m_2=m$, let $Z_j=X_{1j}-X_{2j}$ for j=1,2,...,n. Then, the standardized time series approach can be applied to $\{Z_j\}_{j=1}^{n}$ to obtain the area confidence interval estimator for μ with significant level α:

$$CI_1 = \bar{\bar{Z}} \pm t_{b,\alpha/2}\sqrt{H/nb},$$

where $\bar{\bar{Z}} = \bar{\bar{X}}_1 - \bar{\bar{X}}_2$ and

$$H = \frac{3}{m^3-m}\sum_{i=1}^{b}\left[\sum_{k=1}^{m}(2k-m-1)\ Z_{(k+(i-1)m)}\right]^2 .$$

(2) Unequal sample sizes with unknown common variance. In this case, $\sigma_1^2=\sigma_2^2=\sigma^2$. The area confidence interval estimator is given by

$$CI_2 = (\bar{\bar{X}}_1-\bar{\bar{X}}_2)\quad t_{b_1+b_2,\alpha/2}\sqrt{\frac{H_1+H_2}{b_1+b_2}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)},$$

where

$$H_s = \frac{3}{m_s^3-m_s}\sum_{i=1}^{b_s}\left[\sum_{k=1}^{m_s}(2k-m_s-1)\ X_{s,(k+(i-1)m_s)}\right]^2,\ s=1,2.$$

(3) Unequal sample sizes with unknown unequal variances. The approximate area confidence interval estimator for this case is given by

$$CI_3 = (\bar{\bar{X}}_1-\bar{\bar{X}}_2) \pm t_{f,\alpha/2}\sqrt{\frac{H_1}{n_1b_1}+\frac{H_2}{n_2b_2}},$$

where

$$f = \frac{\left[\dfrac{H_1}{n_1b_1}+\dfrac{H_2}{n_2b_2}\right]^2}{\dfrac{H_1^2}{(b_1+2)n_1^2b_1^2}+\dfrac{H_2^2}{(b_2+2)n_2^2b_2^2}} - 2 \text{ and}$$

$$H_s = \frac{3}{m_s^3 - m_s} \sum_{i=1}^{b_s} \left[\sum_{k=1}^{m_s} (2k-m_s-1)\ X_{s,(k+(i-1)m_s)} \right]^2, s=1,2.$$

3. LIMITED EMPIRICAL EXPERIMENTS

A set of limited experiments applying the confidence interval estimators given in the previous section is discussed in this section.

The experiments were conducted on a M/M/1 queueing model in steady state. Observations were collected on the customers' waiting time for three different examples of parameter settings for ρ equal to 0.2 and 0.6. Table I contains the parameter settings used and the theoretical values of the means for each example. Estimators CI_1 and CI_3 were used to estimate the difference between the means of examples 1 and 3 and CI_2 was used between examples 1 and 2.

TABLE I

M/M/1 Model Data

ρ	Example	1/λ	1/μ	Mean Waiting Time
0.6	1	1	0.6	0.9
	2	1	0.6	0.9
	3	2	1.2	1.8
0.2	1	1	0.2	0.05
	2	1	0.2	0.05
	3	2	0.4	0.10

A variety of measures of the effectiveness of confidence interval procedures are commonly used in empirical investigations (Schriber and Andrews [1981]). The most popular measure is the observed frequency C with which, for example, 90% confidence intervals actually includes the true mean value. The expected value of C should be 0.90 for an effective estimation procedure. However, the deviation of C from 0.90 can be reduced by increasing or decreasing the widths of the confidence intervals. If the value of C is near 0.90, a small average value for the half-widths is desirable for an efficient procedure. Therefore, in the empirical experimentation, we also compute the average half-widths HW of the various confidence intervals.

Another measure of effectiveness addresses the variation in the half-widths of confidence intervals. It provides a measure of the relative stability of a confidence interval procedure. If other things are equal, one prefers a confidence interval procedure having a small standard deviation of the half-widths. A smaller standard deviation, S, provides less variation of the lengths of confidence intervals produced by the confidence interval procedure.

Another measure of effectiveness is the behavior of the coverage function introduced and discussed by Schruben [1980]. The coverage function, F(η), is the frequency that an interval estimator contains the true mean value as a function of the confidence coefficient, η. Ideally, F(η) is the distribution function of uniformly distributed random variable η on the [0,1] interval. The use of the coverage function to investigate the behavior of a proposed confidence interval estimation procedure consequently involves determining η for each output sequence in a collection of output sequences, then determining the extent to which the resulting empirical cumulative distribution function $F^*(\eta)$ for η approximates the distribution function of a uniform random variable on the interval [0,1]. Let $D^+ = \max(F^*(\eta)-\eta,\ 0)$ and $D^- = \max(\eta-F^*(\eta),0)$. The Kolmogorov-Smirnov statistic for testing the uniformity of $F^*(\eta)$ is given by $D_{max} = \max\ \{D^+,\ D^-\}$.

Other considerations are the validity and efficiency of a confidence interval procedure. A large value of D^+ indicates that the widths of the confidence intervals generated by a procedure are larger than necessary and the procedure is not efficient. A large value of D^- indicates that the coverage frequency is lower than intended and the intervals are not valid. A single measure of the discrepancy in both validity and efficiency is $D=D^+ + D^-$. Small values for D are desirable. Ideally, D=0 (Schruben [1983]).

The results of the experiments are summarized in Table II. The computations were based on 100 and 50 independent replications for ρ=0.2 and 50 independent replications for ρ=0.6. A single batch was used for all experiments, i.e., $b=b_1=b_2=1$. Each experiment was computed using the same data sets, where possible. For example, the same data sets were used for CI_1, and CI_3, the first 10,000 observations were taken from the 20,000 observations, and for ρ=0.2, the 50 replications were taken from the first half of the 100 replications.

Table II shows that the observed frequency C of the converges of the confidence intervals is near 0.90. The values of HW and S are small. The Kolmogorov-Smirnov statistic D_{max} demonstrates that we can't reject the uniformity of $F^*(\eta)$ at significant level 0.1 except for CI_1 in the case of 100 replications of =0.2. This set of limited experiments demonstrate that the confidence interval estimators developed in Chen and Sargent [1984a] work well.

4. CONCLUSION

The objective of this paper is to show that the confidence interval estimation for the difference between two means from two independent strictly stationary phi-mixing processes using standardized time series approach worked well in a limited empirical investigation. The results also show that confidence interval estimator CI_2 gives shorter half-length and smaller standard deviation, S, of the half-length (also see Chen and Sargent [1984a]). Therefore, research on the comparison between two variances of stationary stochastic processes is an important area. (see Chen and Sargent [1984b]).

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Table II

Performance of Confidence Interval Estimators for the Mean Waiting Time from an M/M/1 Queue Model with $b=b_1=b_2=1$.

ρ	Estimator	n_1	n_2	n_3	C	HW	S	D^+	D^-	D_{max}	D
0.6 (50 replications)	CI_1	20000		20000	0.90	0.38355	0.33814	0.13116	0.04009	0.13116	0.17124
	CI_2	20000	20000		0.96	0.16274	0.07805	0.08731	0.12949	0.12949	0.21680
	CI_2	10000	20000		0.86	0.19087	0.10759	0.08015	0.06202	0.08015	0.14217
	CI_3	20000		20000	0.88	0.27153	0.25520	0.04927	0.07302	0.07302	0.12229
	CI_3	10000		20000	0.90	0.31205	0.24088	0.11013	0.03773	0.11013	0.14786
0.2 (50 replications)	CI_1	20000		20000	0.90	0.01677	0.01293	0.10310	0.02661	0.10310	0.12972
	CI_2	20000	20000		0.90	0.00589	0.00318	0.06916	0.10262	0.10262	0.17177
	CI_2	10000	20000		0.88	0.00697	0.00403	0.10365	0.03901	0.10365	0.14266
	CI_3	20000		20000	0.88	0.01220	0.00983	0.10070	0.02154	0.10070	0.12224
	CI_3	10000		20000	0.88	0.01234	0.00832	0.15541	0.03517	0.15541	0.19057
0.2 (100 replications)	CI_1	20000		20000	0.90	0.01747	0.01520	0.14242	0.02060	0.14242*	0.16302
	CI_2	20000	20000		0.88	0.00588	0.00321	0.07007	0.05229	0.07007	0.12236
	CI_2	10000	20000		0.87	0.00727	0.00381	0.07842	0.01551	0.07842	0.09392
	CI_3	20000		20000	0.88	0.01310	0.01121	0.11295	0.02066	0.11295	0.13361
	CI_3	10000		20000	0.88	0.01347	0.00958	0.10854	0.02502	0.10854	0.13356

* The coverage function is significant from a uniform distribution at significant level 0.10 by the Kolmogorov-Smirnov Test.

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