Proceedings of the 1985 Winter Simulation Conference
D. Gantz, G. Blais, S. Solomon (eds.)

ANALYSIS OF SIMULATION OUTPUT DATA

W. David Kelton
Department of Industrial and Operations Engineering
The University of Michigan

ABSTRACT

The problem of analysis of simulation output data is motivated and put in the context of related methodological problems. Examples are given to indicate why this problem is important, and why it is difficult.

INTRODUCTION

Simulation has evolved into an extraordinarily powerful tool for systems analysis, and enjoys a correspondingly high frequency of application. We now have available several sophisticated languages for a wide variety of types of simulation, together with accompanying capabilities for model specification, control of experimental conditions, output formatting (not only final reports but also in data bases for ease of postprocessing), and animation.

Further, the acceptance of simulation as an appropriate analysis tool seems to be gaining even wider acceptance, both in the industrial application community and among researchers. The ability to model a system "as it is" rather than "as it must be" to admit an analytic solution is very appealing and less open to the (nearly unanswerable) criticism of "looking where the light is." As a matter of practicality, one of the main historical drawbacks of simulation, the sometimes great cost in terms of computer resources, is becoming less severe as technological advances in hardware make inexpensive computing widely available.

It might seem, then, as if all the real difficulties in using simulation have evaporated. There are, however, difficulties of another type that will probably never be overcome purely by advances in software or hardware. These are **methodological** difficulties, and are present regardless of which language or computer is chosen; also, every user of simulation faces these difficulties with every project, although there may not be recognition of this or appreciation of how important and intractable these difficulties can be. Examples of methodological problems in simulation are:

1. Determining whether the model assumed (and presumably programmed correctly in the simulation) is an accurate representation of the real system of interest; this is often called a question of model **validity**.

2. Choosing the appropriate constants or probability distributions to use in driving the simulation. This is sometimes referred to as **input modeling,** and is related to the question of model validity, since it is, in a way, a part of the modeling process.

3. Summarizing, evaluating, and interpreting the output from a simulation model or from several models of potentially competing systems designs. This is usually called **output analysis,** and leads naturally into related questions concerning the design of the course of simulation experimentation, such as specification of the initial conditions for the run(s), determining the length of the run(s), and deciding how many runs to make of a given system configuration.

The focus here will be on this final difficulty of the analysis of simulation output data.

Recently, comprehensive surveys of this problem have appeared giving a great deal of detail and many references to the original works; see Kelton (1983), and especially Law (1982). In addition, recent books on simulation have devoted considerable space to discussion of various methodological questions, including output analysis; see, for example, Banks and Carson (1984) or Law and Kelton (1982). Thus, the focus here will be to motivate the reader as to the importance and difficulty of this problem.

IS THIS PROBLEM REALLY THERE?

Yes. Perhaps the best way to see this is by an example, taken from Chapter 9 of Law and Kelton (1982). Consider designing some sort of service facility in which "customers" arrive one at a time in a Poisson process at rate 1 per minute, and we have a choice of installing either:

(a) A single "fast" server that can serve customers with exponential service times having a mean of 0.9 minute, or

(b) Two "slow" servers, each of which supplies service with times being exponentially distributed with mean 1.8 minutes. (In this case, a single queue feeds both servers.)

The fast server is exactly twice as expensive to install, maintain, and operate as a single slow server, so the two possibilities would incur the same cost. Thus, we would prefer to install the alternative that provides the better customer service, as measured by the expected average amount of time the first 100 customers have to wait in line, assuming empty and idle initial conditions.

The answer is (b). This was determined from some rather involved queueing-theoretic considerations (which, incidentally, would not be applicable if the arrival pattern, for example, were not Poisson). Without access to these analytical tools, a reasonable approach to this problem would be to make a simulation run of each system (independently), and choose the system giving the lower average queueing time of the 100 customers. There is a chance, however, that because of the random nature of the inputs driving the simulation (specifically in this case, the interarrival times and the service times) that (a) could come out looking better, and we would erroneously make this choice. To see how likely this mistake might be, 100 **pairs** of simulations of (a) and (b) were made, and 44 of these pairs resulted in (a)'s looking better! In other words, there is only a little better than an even chance that this scheme would lead us to the correct decision.

Admittedly, this example is oversimplified and somewhat contrived. However, its general outline (make one run of each alternative and choose the best) probably sees more application than most appropriate output analysis techniques. The remedy here seems fairly clear: make **several** runs of each alternative and use the **averages** across these runs to make a decision. Less clear, perhaps, is the question of **how many** runs are enough; the reader is referred to the papers or books mentioned above for discussion of this and related problems.

IS THIS PROBLEM DIFFICULT?

Yes. The remedy in the above example, basically, is to make several runs of the simulation. However, this is not always the solution to a given output analysis problem.

The above example led to trouble basically because the output from a simulation is subject to variability, and we needed to take this into account; the methods for deciding on how many runs to make all attempt to estimate this variation and use the result to prescribe the required number. In other contexts (specifically, in **steady-state** simulation aimed at learning about long-run system behavior), we need to get an estimate of the variability of the output from a single run of the simulation; again we turn to an example from Law and Kelton (1982).

Consider a queueing system exactly as in (a) in the above example, except that it is initialized in steady-state conditions. That is, the first customer arrives to find some random number of other customers already present, whose distribution is known from elementary queueing theory. Make a simulation run of just 10 customers, and let D_i be the observed delay in queue of the i^{th} arriving customer, and let

$$\bar{D}(10) = \sum_{i=1}^{10} D_i \;/\; 10$$

be the observed average delay in queue of these customers; it turns out that the expected value of $\bar{D}(10)$ is 8.1 minutes. From a single simulation run, we obtain the data $D_1,\ldots,D_{10}$ from which $\bar{D}(10)$ was determined, but we realize that this average is subject to some uncertainty, and that if we were to make another run, a different value of $\bar{D}(10)$ would result; thus we need an estimate of $\mathrm{Var}[\bar{D}(10)]$. The most natural thing to do is to estimate this variance by

$$s^2(10) \;/\; 10 \;=\sum_{i=1}^{10} [D_i - \bar{D}(10)]^2 \;/\; [10\ (10 - 1)],$$

the usual unbiased variance estimator from classical statistics. The difficulty in using such a formula for a variance estimator is that it assumes that the basic data (in this case, the D_i's) are independent. However, the successive delays in queue may naturally be expected to be correlated with one another; if D_i is long, then it is likely that D_{i+1} will also be long. As a result, $s^2(10)/10$ may be a biased estimator of $\mathrm{Var}[\bar{D}(10)]$; in this case it turns out that the expectation of this variance estimator is only about 3.4% as large as the actual value of $\mathrm{Var}[\bar{D}(10)]$, the quantity being estimated. In other words, the "usual" variance estimator is nearly 97% smaller, in expectation, than the quantity it is supposed to be estimating! Thus, we see that direct application of the familiar tools of classical statistics can lead us badly astray in simulation output analysis. (As a footnote, quantities such as $s^2(10)$ above from a single simulation run often appear in the standard output of simulation languages, and they are probably best ignored.)

This example provides just one instance of the difficulties inherent in the statistical analysis of simulation output. Again, the reader is referred to the survey papers or books mentioned above for much more comprehensive treatment of both the pitfalls and suggested methods for analysis.

CONCLUSIONS

The power of the simulation technique is by now generally appreciated, especially with modern software and hardware tools. Less appreciated, however, is the need to carry out careful design and analysis to address methodological problems in general, and output analysis problems in particular. Ignoring these problems, however, can lead to very serious errors in interpretation and conclusion, to the point that it might have been better not to do the simulation at all. One of the purposes of this paper has been to point out by example the kinds of errors that might easily be made, and to direct the reader to sources of information concerning the appropriate methodologies for use in simulation output analysis.

REFERENCES

Banks, J. and J.S. Carson, II. Discrete-Event System Simulation, Prentice-Hall, Englewood Cliffs, New Jersey, 1984.

Kelton, W.D., "Simulation Analysis," Proceedings of the 1983 Winter Simulation Conference, 159-168, 1983.

Law, A.M., "Statistical Analysis of Simulation Output Data," Operations Research 31, 983-1029, 1983.

Law, A.M. and W.D. Kelton, Simulation Modeling and Analysis, McGraw-Hill, New York, 1982.

W. DAVID KELTON is Assistant Professor of Industrial and Operations Engineering at The University of Michigan, in Ann Arbor. He received a B.A. in Mathematics in 1972 from the University of Wisconsin-Madison, an M.S. in Mathematics in 1975 from Ohio University, and M.S. and Ph.D. degrees in Industrial Engineering from Wisconsin in 1979 and 1980, respectively. His research interests include statistical and probabilistic aspects of simulation, inference on stochastic models, and statistical quality control.

Department of Industrial and Operations Engineering
IOE Building, 1205 Beal Avenue
The University of Michigan
Ann Arbor, Michigan 48109-2117
(313) 763-1412