SIMULATION OF THE HUMAN MITRAL VALVE

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Introduction

The model discussed here is a mathematical representation of the anatomical and physiological features of the human mitral valve and associated cardiac structures. This model describes mitral valve position and profile during systole, the blood ejection phase of the cardiac cycle. It is based on published data relating to geometry and mechanical properties of the valve components as well as physiological data relating to timing events during the cardiac cycle. Therefore, this model is limited by deficiencies in this data base and by the assumption of a quasi-static nature of force analysis that serves as a basis for this model. This simulation is intended to clarify the intricate geometrical and mechanical interrelationships which exist between the mitral valve and associated cardiac structures. Ultimately, this model may improve our understanding of the many factors associated with mitral valve pathologies and may provide information relating mitral valve leaflet profiles to specific etiologies.

Physiology

The human heart is a four chambered dual pump. The right (pulmonary) pump propels blood to the lungs in order to oxygenate blood. The enriched blood returns to the left (systemic) pump to be propelled throughout the remainder of the body. The atria serve to collect blood from the vein and to force this blood into the main pumping chambers, the ventricles. The left atrium sends oxygenated blood into the left ventricle, which in turn pumps the oxygenated blood through the systemic circulation to all regions of the body. The right atrium sends deoxygenated blood (collected by venus return from the above regions) into the right ventricle, from which it is pumped into the pulmonary circulation of the lungs. Each ventricular pump acts in a rhythmic beating fashion. During diastole, the ventricles relax, allowing the atria to contract and fill the ventricles with blood. During systole, the ventricles contract, expelling the collected blood into the circulation. This relaxation/ejection sequence is repeated for each heart beat and makes up the cardiac cycle.

Four heart valves control blood flow within the heart. Atrio-ventricular valves, located between each atrium and associated ventricle, control blood flow during diastole, when the ventricles are relaxed and filled by the atria. Semilunar valves located at the outflow tract of each ventricle control blood flow during systole, as the ventricles contract and expel blood into their respective circulations.

The mitral valve is the atrio-ventricular valve located between the left atrium and left ventricle. In the closed position, it's function is to prevent back flow of blood into the atrium from the ventricle during systole. It is a passive valve in that it opens and closes in response to a pressure gradient in either direction. The thin filmy mitral valve requires almost no back flow to cause closure. This is quite different from the much heavier semilunar aortic valve which requires strong back flow for a few milliseconds to produce closure.

This regurgitation (back flow) through the mitral valve is not a normal situation and represents a pathological state. Another pathalogical of interest is valve prolapse, a bulging of the valve leaflets into the atrial cavity during systole. This can result from weakened components of the mitral valve structure. Prolonged prolapse can lead to valvular regurgitation. These pathologies, along with mitral stenosis, a narrowing of the mitral valve orifice, are so common, that mitral valve replacement (with a prosthetic valve) is now routine procedure.

Anatomy

The anatomy of the mitral valve and associated structures is shown in Figure 1. The valve leaflets are in the form of two elastic cusps which are unequal in size. The longer anterior cusp (24mm height) hangs down like a curtain between the mitral and aortic orifices, while the shorter posterior cusp (14 mm height) originates from the lateral portions of the mitral ring. The combined surface area of the two valve cusps is nearly twice as great as the area of the mitral orifice which they must occlude.
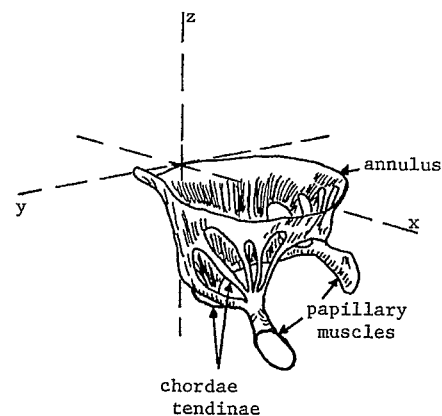

Figure 1. Anatomy of the Mitral Valve

The mitral valve orifice is actually much smaller than the mitral ring because the valve cusps are joined at the commissures so that the upper portion of the valve resembles a funnel. A coaptation line or plane exists below the plane of the valve orifice, such that normal valve closure is within the left ventricle. Mitral valve prolapse exists when the valves join so that they bulge into the left atrium (see Figure 2).

Chordae tendinae radiate from the ventricular border of the valve leaflets and extend to the papillary muscles. These correspond to multiple guy lines. Chordae from both valve cusps insert upon both papillary muscles. Thus, tension is extended through the chordea to draw the two valve cusps together. If the papillary muscles begin their contraction early in ventricular systole, traction on the valve leaflets should facilitate apposition of the valves. The chordae are elastic in nature and are relatively passive structures.
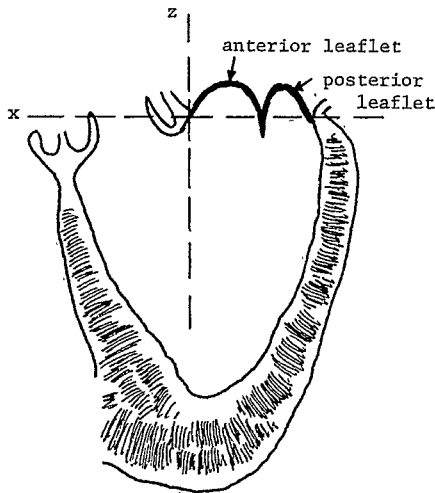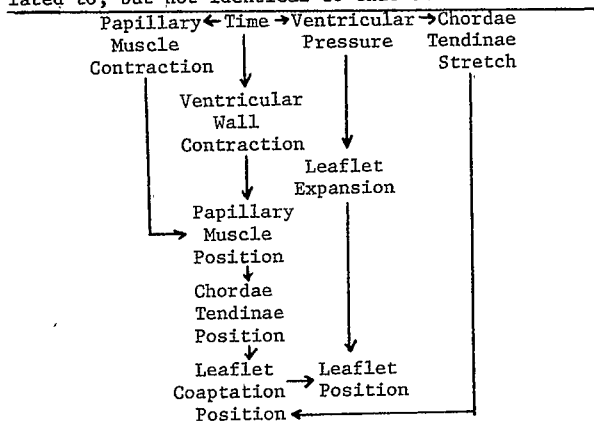
Model Formulation

The model inputs are time and ventricular pressure. These inputs are utilized in a four pronged calculation as is shown in figure 3. The time input is used to compute the papillary muscle and ventricular wall contraction. These are then utilized to produce the top papillary muscle position which represents the anchor point of the mitral valve complex. This also represents the connection point of the bottom of the chordae tendinae.

The time input is utilized to produce a ventricular pressure table which is further utilized to produce stress-strain data on both the chordae tendinae and mitral valve leaflets. The chordae stretch are incorporated with chordae position (as calculated from above) to produce leaflet coaptation position. The latter also requires input from leaflet expansion data. A diagrammatic representation of the valve structures as modeled is shown in Figure 4.

Figure 2. Prolapse of the Mitral Valve

Figure 4. Schematic View of Valve.

The papillary muscles are extensions of the ventricular wall and serve as anchor points for the chordae tendinae/valve leaflets structures to the ventricular wall. These muscles are active elements which can contract in a timing sequence, which is related to, but not identical to that of the ventricle.
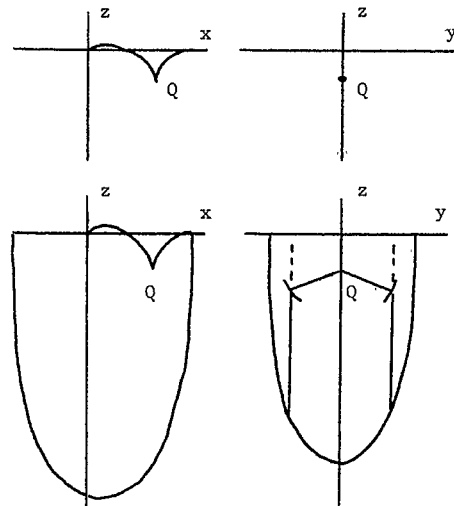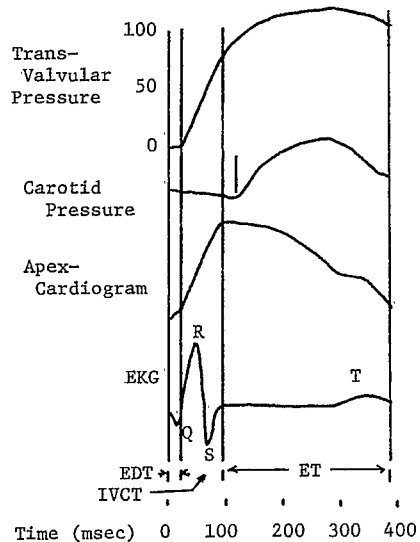

Timing events are shown in Figure 5.

Assumptions and operating conditions utilized in the model, include the following:

1) During electromechanical delay, (EDT), only the mitral annulus contracts. Mitral valve leaflets are in equilibrium and are coapted during this interval. (EDT is 22 milliseconds from onset of systole).

2) Left ventricular wall and papillary muscle contractions begin with the onset of intraventricular contractions time (IVCT). Transvalvular pressure begins to increase. (IVCT is 71 milliseconds). Transvalvular pressure equals 80 millmeters mercury at the end of IVCT.

3) Annulus, left ventricular wall and papillary muscle contractions continue during the ejection time (ET), which lasts another 292 milliseconds. Transvalvular pressure can be related to ejection by $P = 80 + 40 \sin (0.009*ET)$ where pressure (P) is in mmHg and ET is in milliseconds.

4) The anterior leaflet height, representing a prestressed dimension, is 2.4 cm., while the posterior height is 1.4 cm. The leaflets are characterized by a two phase stess-strain model as seen in Figure 6.
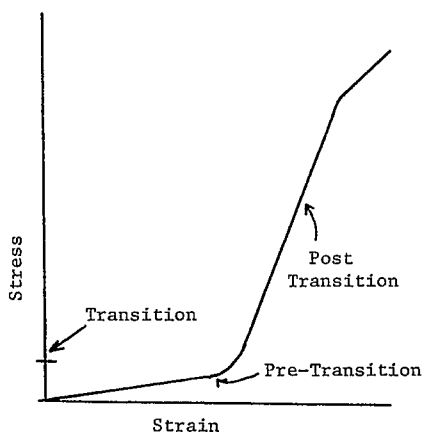

Figure 6. Stress-Strain Model of the Valve Leaflets.

The pretransition modulus of elasticity is 1.14×10^5 dynes/cm^2, while the posttransition modulus is 9.0×10^7 dynes/cm^2. Transition occurs at 3.5×10^4 dynes/cm^2 which is created by a transvalvular pressure below 5 mmHg.

5) The leaflets are uniform, thin membranes which can only support internal stress in tension. The leaflets assume an elliptical shape which has as its major axis an imaginary line joining the free edge of the leaflet to the point of attachment of the leaflet to the annulus. The extent of cooptation is determined by the natural intersection of the two elliptical segments drawn to represent the anterior and posterior leaflets.

6) The forces exerted by the chordae on the free edges of the leaflet act as a distributed tension along the entire free edge. This attachment acts as an ideal hinge with no added resistance to rotation.

7) Internal forces are neglected and the mechanical forms generated by transvalvular pressure are treated in a piecewise, quasi-static manner. The leaflets are prestressed and are represented by their past transition modulus as are the chordae. The chordae modulus is 2×10^9 dynes/cm^2. The initial length for all chordae is 1.5 cm.

8) The papillary muscles are attached to the ventricular wall at points two thirds of the distance from annulus to ventricular apex. The ventricle is a truncated ellipsoid. Contractions along the major and minor axes are linear functions of time.

Program Operation

Mitral valve motion is predicted by a computer program which describes the dynamic alterations in the various components of the mitral apparatus. Following input of clinically obtained and/or assumed data, the program initializes the geometry of the model as seen in Figure 1. The anterior portion of the annulus is located at x = 0, y = 0, and z = 0. The posterior portion is located at x = ANEDL (annulus diameter at end-diastole), y = 0, z = 0. The leaflets are represented by elliptical segments in the x-z plane, extending from the annulus to the point of coaptation (Q) of the leaflet free edges. In the y-z plane, the leaflets are represented as a single point, Q. A truncated ellipsoid is used to model the left ventricle. Its major axis is parallel to the z axis and located within the x-z plane. The minor axis is parallel to the x-y plane and separated from this plane by a distance specified by the extent of truncation. The ellipse representing the ventricle is assumed to pass through the posterior portion of the annulus (x = ANEDL, y = 0, z = 0), but can be positioned anywhere along the x axis by specifying a separation between the annulus and ventricular wall (GAPIL) as an input condition.

The papillary muscles are represented as straight lines oriented parallel to the z axis in the y-z plane, and attaching to the ventricular wall at a point two-thirds from the distance from annulus to apex. The lengths of the papillary muscles are determined by geometrical considerations, specifically, the intersection of lines representing chordae tendinae with vertical lines from the papillary muscle attachment sites toward the annulus.

The computer program is designed so that this initial geometry can be easily altered to describe a wide range of clinical or hypothetical configurations of the mitral apparatus. Typical input values are listed below:

Electromechanical Delay Time	22 msec.
Isovolumetric Contraction Time	71 msec.
Ejection Time	292 msec.
Anterior Leaflet Height (Initial)	2.4 cm.
Posterior Leaflet Height (Initial)	1.4 cm.
Leaflet Elastic Modulus	9×10^7 dynes/cm^2
X coordinate of Leaflet Free Edges	2.0 cm.
Y coordinate of Leaflet Free Edge	0.0 cm.
Z coordinate of Leaflet Free Edge	-0.9 cm.
Annulus Diameter at End Diastole	2.7 cm.
Annulus Diameter at End Systole	2.56 cm.
Chordae Tendinae Length (Initial)	1.5 cm.
Chordae Tendinae Modulus	2.9×10^9 dynes/cm^2
Papillary Muscle Percent Shortening	16.4%
Annulus to Apex - End Diastole	7.3 cm.
Annulus to Apex - End Systole	6.75 cm.
Minor Axis - End Diastole	4.86 cm.
Minor Axis - End Systole	3.34 cm.

Truncation Percent   60%
Annular Attachment to Ventricular Wall   0.0 cm.
Time Increment Steps   10 msec.

Program Results

The mitral valve model program output produces a time series description of mitral valve components throughout systole. At ten millisecond intervals, the following numeric output is listed:

1) anterior and posterior coordinates of the annular ring, (AAX, AAZ; APX, APZ).
2) coordinates of the coaptation point, Q. (QX, QZ).
3) anterior and posterior leaflet lengths (ARC1, ARC2).
4) maximum leaflet deflection in the Z direction for each leaflet (M1X, M1Z; M2X, M2Z).
5) papillary muscle attachment points to the chordae tendinae (P1Y, P1Z; P2Y, P2Z).
6) annulus to lower ventricular apex distance.
7) annulus to upper hypothetical ventricular ellipse.
8) ventricular minor axis.
9) chordae tendinae length.
10) papillary muscle length.
11) stress and strain for anterior and posterior leaflets.
12) major and minor axes for the anterior leaflet ellipse (C1, D1).
13) major and minor axes for the posterior leaflet ellipse (C2, D2).
14) QZ', ARC1' and ARC2' representing values obtained from a zone region of coaptation.
15) elapsed time of systole.

Typical outputs at 0, 100 and 385 milliseconds are shown in Figure 7. Plots of leaflet profiles are shown at these times in Figure 8. These plots represent movement of the leaflets which would better correlate to such clinical standards as cine-angiography. Sensitivity studies have been performed and are reported in Hunter, et al. (1). This program is run on a main frame system in Fortran or on a microprocessor system in Basic. Model validation can be accomplished by comparison of predicted results to two dimensional real time ultrasound or biplane cine-angiographic data of the left ventricle.

References

1. Hunter, J.F., Seaton, D.P, Lively, W.M., Miller, G.E. and Stoner, D.L. Texas J. Science, Vol. 35, No. 1, pp. 5-36, 1983.

Time = 100 msec
Pressure = 80.392 mm Hg
Stress = 1192464   Strain = 1.302738E-02
Stress = 632976.4   Strain = 6.810849E-03

Annulus to Lower Apex = 7.29 cm
Annulus to Upper Apex = 4.86 cm
Minor Axis   = 4.82 cm
Chordae Length   = 1.51 cm
Papillary Length   = 2.74 cm

AAX	AAZ	APX	APZ	QX	QZ	ARC1	ARC2	M1X
0.00	0.00	2.66	0.00	2.00	-0.97	2.43	1.41	0.20

M1Z	SLOPE1	M2X	M2Z	SLOPE2	P1Y	P1Z	P2Y	P2Z
0.08	2.1	2.57	0.03	-0.7	0.95	-2.14	-0.95	-2.14

C1	D1	C2	D2
1.112	0.313	0.586	0.284

Time = 385 msec
Pressure = 95.90034 mm Hg
Stress = 1440761   Strain = 1.678624E-02
Stress = 981388.8   Strain = 6.237653E-03

AAX	AAZ	APX	APZ	QX	QZ	ARC1	ARC2	M1X
0.00	0.00	2.56	0.00	2.11	-0.79	2.44	1.41	0.24

M1Z	SLOPE1	M2X	M2Z	SLOPE2	P1Y	P1Z	P2Y	P2Z
0.08	2.7	2.34	0.06	-0.6	0.66	-2.15	-0.66	-2.15

C1	D1	C2	D2
1.12	0.286	0.453	0.444

Figure 7 (cont) Mitral Valve Model Output

Time = 0 msec
Pressure = 0 mm Hg
Stress = 0   Strain = 0
Stress = 0   Strain = 0

Annulus to Lower Apex = 7.30 cm
Annulus to Upper Apex = 4.87 cm
Minor Axis   = 4.86 cm
Chordae Length   = 1.50 cm
Papillary Length   = 2.84 cm

AAX	AAZ	APX	APZ	QX	QZ	ARC1	ARC2	M1X
0.00	0.00	2.70	0.00	2.00	-0.90	2.40	1.40	0.22

M1Z	SLOPE1	M2X	M2Z	SLOPE2	P1Y	P1Z	P2Y	P2Z
0.08	2.2	2.58	0.04	-0.8	0.96	-2.05	-0.96	-2.05

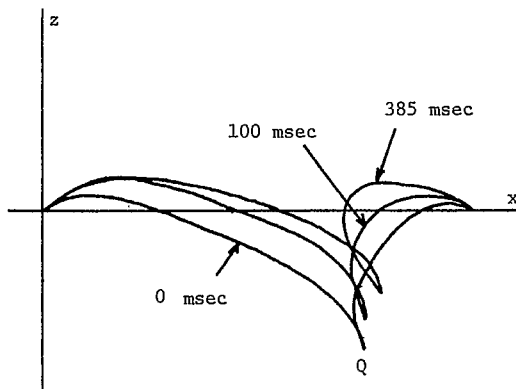| C1	D1	C2	D2
1.097	0.312	0.570	0.300

Figure 7. Mitral Valve Model Output

Figure 8. Mitral Leaflet Profiles from Model Output.