

QUALITY MODELLING OF ASSEMBLY SYSTEMS

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ABSTRACT

From the point of view of quality an assembly system can be viewed as consisting of two types of processes, those that create defects (assembly) and those that identify and remove defects (test). Thus in a general simulation model of an assembly system three types of blocks are required, assembly, test, and merge (lumping defects together). This approach was used to develop a simulation model of printed circuit board (PCB) assembly. An analytical model based on the same principles was developed in order to partly verify and validate the simulation. The paper also describes a comprehensive set of statistically designed experiments which was carried out in order to investigate the effect of various factors on the yield and outgoing quality of the system.

INTRODUCTION

The modeling of the quality aspects of production processes from a system perspective has received relatively little attention in the past. In the literature on the study of production quality [1][2][3][4], the most common approach is to construct a simplified model of an isolated process and to apply an optimization scheme to it so as to minimize production cost. This approach, although valuable in improving individual process quality, fails to take into account the interrelationship among the basic elements in a production system such as incoming inspection, assembly quality, test and rework quality.

In many situations, even if all the individual processes are performing satisfactorily, it does not imply that the overall quality of the product will be good. If the product is complex enough, a small number of defects occurring at each process may lead to poor overall product quality. Therefore, a different approach must be employed to investigate how processes such as assembly, test, inspection, rework and repair, combined with the incoming quality of component parts together determine the outgoing quality and the overall yield.

The simulation model to be presented in this paper is developed based on this idea. This model is constructed for a portion of an assembly line in a car radio manufacturing plant. From the point of view of quality, an assembly system consists of two types of processes: those that create defects (assembly) and those that identify and remove defects (test). A production system can thus be represented by the flow of defects with only those components that are defective specifically accounted for. Under this representation, it is assemblies of defects that are the entities that flow through the system. There

are some advantages in using this representation. Firstly, less programming effort and computational time will be required. Secondly, as will be shown later, the model can easily be extended to account for non-serial production system, i.e. those in which there are branching and merging.

This model also introduces the concept of test effectiveness and test efficiency. At each assembly process, various types of defects will be created. Each subsequent test will be designed to detect some, but not necessarily all of them. Further, even though a test may look for a specific type of defect, a defect may not be identified and removed due to the imperfection of the testing device. Thus it is important to distinguish, for a given defect type created at a particular assembly process, between test effectiveness, the probability that the test looks for the defect, and test efficiency, the probability that the defect is found given the test looks for it.

Extensive verification and validation procedures were employed for the model. This included both comparing analytical results for a simplified system with simulation results and comparing simulation results with the experience of the plant for which the simulation model was developed.

One thing that has often been overlooked by simulation practitioners is the experimentation with the simulation model after it is developed. Only through statistically designed experiments and statistical analysis of the results is one able to extract the most information out of the simulation model in the most economic way. In this study, statistically designed experiments were carried out with the model to evaluate the impact of various factors on the yield and outgoing quality. The combination of simulation modelling and statistically designed experiments provides an "objective" means of evaluating the system in a more systematic way.

GENERAL QUALITY SIMULATION APPROACH

An assembly can be simulated as an array of parameters in which records for the quantity of each defect type on that assembly are stored. As different processes are simulated, the quantity of each defect type will either be increased or decreased, an assembly process increases it, and a test process decreases it. A major part of the simulation program is to maintain the up-to-date status of all the defects residing in an assembly.

The basic elements in a production system can be simulated by three general blocks: assembly, test, and merge. At an assembly block the generation of defects in the product is simulated, at a test block the removal of defects is simulated, while a merge block may be necessary in order to combine defects created by different processes. Assembly may include such processes as soldering, component insertion, transport or painting, while test includes test, inspection, or adjustment.

At an assembly block, it is possible to have 0 or 1 or 2 or ... defects created for each defect type. The probability that one of these situations occurs is characterized by binomial distribution with n , the quantity of a component type, and p , the chance that a component of that type becomes defective, as parameters. Therefore, for a given set of n 's and p 's, one can calculate the cumulative probability functions associated with each defect type. During the simulation of an assembly, the number of defects created for a particular type of defects is obtained by generating a random number and comparing it to the corresponding cumulative probability function. For example, if the probability for 0 defect is 0.7, 1 defect is 0.2, and 2 defects is 0.1 (while that for others are 0), then the cumulative distribution for 0, 1, and 2 is 0.7, 0.9 and 1.0 respectively. For a random number which is less than or equal to 0.7, 0 is assigned to the appropriate parameter, otherwise, 1 is assigned if it is greater than 0.7 but less than or equal to 0.9, and 2 is assigned if it is greater than 0.9 but is less than or equal to 1.0.

At a test block, a defect is first classified as whether it is detectable or not. If it is detectable, there is a certain probability that it will be identified, otherwise, it will by-pass the test. In the simulation model, this is simulated as having two stages: In the first stage, a random number is generated for each defect that exits on an assembly, if the random number is greater than the corresponding test effectiveness, a defect is non-detectable, otherwise, it is detectable. All the non-detectable defects will be removed from the appropriate parameters and stored as a different set of parameters. In the second stage, a random number is generated for each detectable defect. If the random number is greater than the corresponding test efficiency, a defect is not identified, otherwise, it is identified and is eliminated from the record. The total number of defects found will be recorded as a separate parameter. A zero content of this parameter at the end of the test indicates that the assembly passed the test and simulation of the subsequent operations will be executed, otherwise, the assembly failed the test and the simulation of the retest will be carried out. Retest of a failed assembly is simulated by repeating the second stage of the test procedure. Simulation of the first stage is not needed since the non-detectable defects still remain non-detectable. The flows at a test block is shown in Fig. 1.

In a merge block, defect streams that are no longer distinguished in the subsequent processes are "combined" by transferring the defect records from one stream to another. The set of parameters from which records have been transferred elsewhere is available for subsequent use.

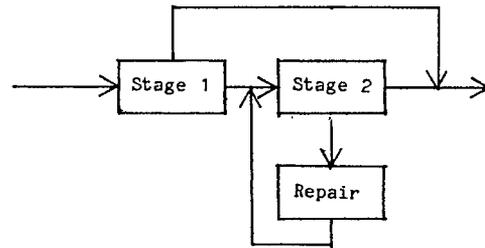


Figure 1: All Possible Paths at a Test Station

QUALITY SIMULATION MODEL OF PCB ASSEMBLY

The above approach has been applied to a printed circuit board assembly. The specific system consists of component incoming inspection, machine insertion of some components, transport, manual insertion of the remaining components, wave soldering, wire cropping, shorts/opens test, transport and oven drying, and HP3060 test [Fig. 2]. The HP3060 test consists of 4 phases: shorts/opens test, which is almost identical to the previous shorts/opens test, 1st component test, which tests a fraction of the components, functional test, and 2nd component test, which tests a fraction of the untested components when the functional test is failed. Any board that fails a test is sent to a repair and rework area. After the repair, it will be sent through the test again.

A block diagram for the present system, expressed in terms of the assembly, test, and merge blocks described earlier, is shown in Fig. 3. Several aspects of the model are worth mentioning: (1) the addition of inherent component defects to the assembly is represented by a single assembly block at the beginning of the system. In reality, inherent component defects are added onto the assembly at the time defective components are inserted onto the assembly. (2) Several processes such as inspection and drying processes which have little or no influence on the quality of the subassemblies are not included in the model. (3) The 3060 test is simulated as if it consists of only 3 stages: shorts/opens, first component, second component. It is assumed that if no defects are found on second component test, the PCB would have passed the functional test while if any defect is found on the second component test, the PCB would have failed the functional test. (4) The repaired assemblies that failed the 3060 test are looped back to the point of failure instead of to the beginning of the 3060 test. This would not affect the simulation result since the HP3060 is assumed to have 100% test efficiency.

Model Input and Output

Once the simulation model is developed, input data must be obtained for the model. Values for test effectiveness, test efficiency and defect probability are required. It is also necessary to specify the number of components of each type involved in each process. Furthermore, the defect categories must be selected since they dictate how the data is organized and used. For ease of data collection and/or estimation, defect categories were selected

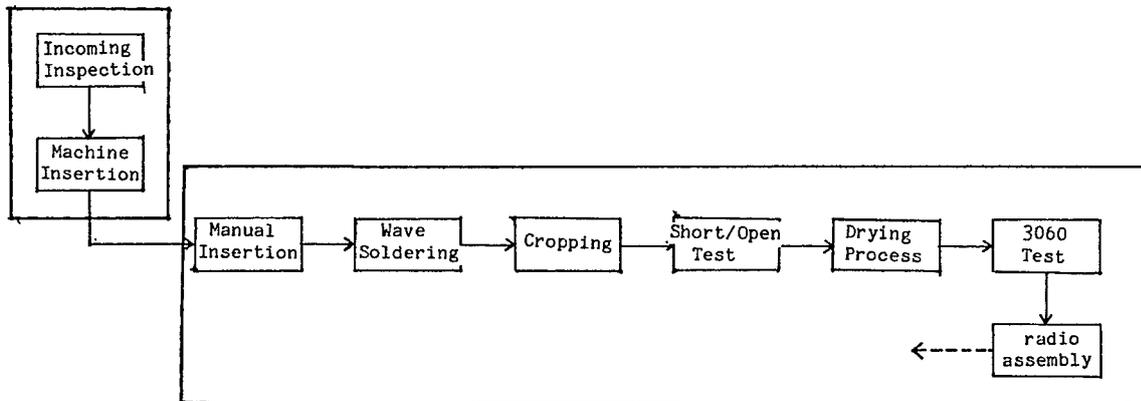


Figure 2: Block Diagram of the Present System

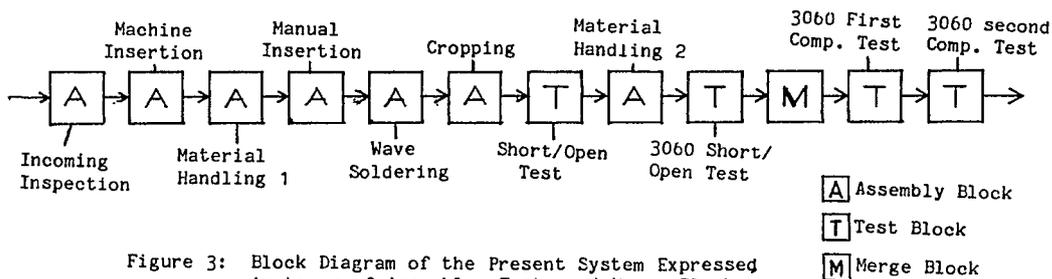


Figure 3: Block Diagram of the Present System Expressed in terms of Assembly, Test, and Merge Blocks

which were based on the combination of component type and process.

The components are classified into eleven types: (1) capacitor, (2) resistor, (3) coil, (4) diode, (5) transistor, (6) varactor, (7) I. C., (8) filter, (9) transformer, (10) jumper, (11) solder joint. Input parameters for quantity of components involved at each process and defect probabilities for each defect type are tabulated at Table 1. Most of these values are based on engineering judgement, the rest are obtained from records.

The output of the model consists of statistical tables showing the distribution of various defect types gathered at three different points in the system (1) after wire cropping, (2) prior to entering the 3060 test (3) at the end of the system. At each point, data is recorded on the total number of defects on the PCB and the total number of defects of each component type. At point (1) data is collected on the number of defects due to each process. Two summary statistics of particular interest are the outgoing quality, i.e. the fraction of PCB's leaving the system with at least one defect, and the yield, the ratio of the number of PCB's leaving the system having passed the 3060 test to the total number tested. In addition, aggregate statistics on the total number of defects entering the test, the number of these defects which are detectable and the number of defects actually identified are also observed for the shorts/opens test and the EP3060 test respectively (see Table 2).

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    *****
    *                               *
    * System Statistics *
    *                               *
    *****
    *
    The yield of the shorts/opens test is 63.2%. The outgoing quality after the shorts/opens test is 55.6%. Out of the 3549 defects entered the shorts/opens test, 2991 defects are detectable, out of which, 2413 defects were identified.
    *
    *
    The yield of the 3060 test is 74.3%. The outgoing quality after the 3060 test is 71.2%. Out of the 3277 defects entered the 3060 test, 1121 defects are detectable, out of which, 1121 defects were identified.
    
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Table 2: Simulation Summary

Simulation Duration

In this model, an assembly is generated at every time unit. To determine an adequate simulation length, the yield and outgoing quality produced by

PROCESS	INPUT PARAMETERS FOR ASSEMBLY BLOCKS										INPUT PARAMETERS FOR TEST BLOCKS																		
	INCOMING		MACHINE INSERTION		TRANSPORT 1		MANUAL INSERTION		WAVE SOLDERING		CROPPING		TRANSPORT 2		TEST EFFECTIVENESS DEFECTS	SHORT/OPEN TEST				3060 SHORT/OPEN				3060 1st COMP.			3060 2nd COMP.		
	M	P	M	P	M	P	M	P	M	P	M	P	M	P		IMMER DEFECT	WAVE SOLDERING	CROP PING	OTHER	IMMER DEF	W/S	CROP	OTHER	W/S	CROP	OTHER	W/S	CROP	OTHER
CAPACITOR	69	.3	31	.05	31	.5	38	.05	--	--	--	--	69	.5	CAPACITOR	100	0	0	0	100	0	0	0	0	0	59.4	0	0	7.1
RESISTOR	65	.28	40	.05	40	.125	25	.067	--	--	--	--	65	.125	RESISTOR	100	0	0	0	100	0	0	0	0	0	56.9	0	0	78.6
COIL	7	.2	5	.05	5	.125	2	.1	--	--	--	--	7	.125	COIL	10	0	0	10	10	0	0	10	0	0	0	0	0	0
DIODES	4	.375	1	.05	1	.125	3	.2	--	--	--	--	4	.125	DIODES	100	0	0	0	100	0	0	0	0	0	28.5	0	0	0
TRANSISTOR	11	.375	--	--	--	--	11	.1	--	--	--	--	11	.1	TRANSISTOR	5	0	0	5	5	0	0	5	0	0	0	0	0	0
VARIABLE	6	.2	--	--	--	--	6	.1	--	--	--	--	6	.1	VARIABLE	5	0	0	5	5	0	0	5	0	0	36.4	0	0	78.6
I.C.	3	.25	--	--	--	--	3	.1	--	--	--	--	3	.1	I.C.	0	0	0	0	0	0	0	0	0	0	0	0	0	0
FILTER	6	.3	--	--	--	--	6	.1	--	--	--	--	6	.1	FILTER	0	0	0	0	0	0	0	0	0	0	0	0	0	0
TRANSFORMER	10	.3	--	--	--	--	10	.5	--	--	--	--	10	.5	TRANSFORMER	40	0	0	40	40	0	0	40	0	0	0	0	0	0
JUMPERS	11	.2	--	--	--	--	11	.1	--	--	--	--	11	.1	JUMPERS	100	0	0	100	100	0	0	100	0	0	100	0	0	0
JOINTS	--	--	--	--	--	--	--	--	500	.048	500	.04	--	--	JOINTS	0	100	100	0	0	100	100	0	0	0	0	0	0	0
TEST EFFICIENCY															80%				100%				100%			100%			

Table 1. Input Parameters for the Simulation Model.

the simulation are observed at regular intervals. When these values "stabilized", the corresponding number of transactions generated is taken to be the adequate simulation length. For example, in this model, it is observed that the yields and outgoing quality "stabilized" when 2000 transactions are generated. In order to account for the fluctuation resulted from using different input parameters, another 1000 transactions are added. Hence, the simulation length is taken to be 3000 time units.

Model Verification

Model verification is the process of testing whether the model behaves as it is expected. This model is verified by means of a simplified analytical model in which the product consists of only one type of component. Capacitors, which are assembled to the board in almost every process, are chosen for this purpose.

The simplified analytical model is also based on three types of blocks, assembly, merge and test. An assembly block calculates the defect density due to a particular process, a test block calculates the defect density of the boards after passing the test for each type of defect, while the merge block calculates the "combined" defect density of the the merged defect streams. The notations defined in Table 3 are used in what follows.

(1) Assembly Block. As mentioned earlier, an assembly process is characterized by two parameters, the number of components involved in the operation, and the chance that a component will become defective. The probability that i defects are created at process m is then given by the binomial distribution, i.e.

$$d_i^m = \binom{n}{i} p_m^i (1-p_m)^{n-i}$$

(2) The Test Block. Three steps are involved in a test block calculation: (a) calculation of the conditional detectable defect density function given the quantity of the defects that enter a test, (b) calculation of the conditional identified defect density function given the quantity of the detectable defects that enter a test, and (c) calculation of the unconditional defect density function for the assemblies that leave a test. The calculation for (a) is straight forward since the variable has binomial distribution and is given by

$$u_{ij}^m = \binom{j}{i} (t_k^m)^i (1-t_k^m)^{j-i}$$

p_m - Probability that a type m defect is found in a component

$k d_i^m$ - Probability that there are i type m defects produced at process k

$k D_i^m$ - Probability that a total of i type m defects exist at process k

t_k^m - Test effectiveness of type m defect for type k test device

u_{ij}^m - Probability that i type m defects are detectable given j type m defects exist

e_k - Test efficiency of tester k

$k r_{ij}$ - Probability that i defects are identified given j detectable defects exist for a given test run at tester k

$k R_{ij}$ - Probability that a total of i defects have been identified given j detectable defects exist after the assembly has passed tester k

$k s_i^m$ - Probability that there are i type m defects left on the assembly after the assembly passed tester k

Table 3: Notation for the Simplified Analytical Model

To calculate (b), the probability that i defects are identified from j detectable defects for a single test needs to be calculated first. This is given by binomial distribution, i.e.

$$k r_{ij} = \binom{j}{i} e_k^i (1-e_k)^{j-i}$$

A defect free assembly must pass the test the first time it is being tested i.e. the probability that a total of 0 defects are identified given 0 detectable defect, R_{00} is equal to 1. For a defective assembly, if it passes the test the first time, R_{0k} is equal to r_{0k} (where k is greater than zero) since there is no retest involved. However, normally a defective assembly will not pass the first time. Every time when it is being retested after repair, more of the detectable defects will be discovered. Therefore the identified defect density at the time it passes differs from r_{ij} and is shown to have a recursive relation,

$$k^R_{ij} = \sum_{l=1}^i k^R_{lj} k^R_{(i-1)(j-1)}$$

Finally, the defect density for type m defect after test k is the uncondition probability

$$k^s_i^m = \sum_j \sum_l k^D_j^m u_{lj}^m k^R_{(1-i)j}$$

(3) Merge block. Suppose that n defect streams are merged into a single stream at process k, the defect density after process k is given by

$$k^D_i^m = \sum_{j_1=1}^i \sum_{j_2=1}^{i-j_1} \dots \sum_{j_n=1}^{i-j_1-j_2-\dots-j_{n-1}} 1^d_{j_1} 2^d_{j_2} \dots n^d_{j_n}$$

The simplified model has essentially the same structure as the original model except that this model only consists of those assembly or test processes which affect the capacitor defect density (i.e. the wave soldering and the cropping processes are missing). A merge block is added in front of the 3060 component tests because all defects created at the different processes are no longer distinguished by the component tests.

The observed variables are ECAP, the expected number of capacitor defects after wave soldering, ICAP, the expected number of capacitor defects before entering 3060 test, and TCAP, the expected number of capacitor defects leaving the system. The simulation model has been run twice using different random number generator seeds in order to obtain an estimate for the mean and variances for each variable. A t test, which tests the null hypothesis that the simulated mean is equal to the corresponding expected value against the hypothesis that the simulated mean and expected value are not equal, is employed. The result is tabulated in Table 4. It is observed that in all cases, the simulated result is equal to the calculated result at 10% significant level.

observed variables	observations			
	analytical result	simulation result		t value
		mean	std. dev.	
ECAP	.5370	.5310	.0042	2.0203
ICAP	.3716	.3750	.0113	0.4255
TCAP	.1354	.1355	.0007	0.2020

Table 4: Simulation Result and Analytical Result Comparison Summary

Model Validation

The validation of the present model has been largely dependent on subjective judgement since no appropriate real world data is available. The model has been run using data supplied by the plant personnel and the results appears to be in agreement with the experience of plant personnel. For example, the 3060 test is thought to be able to identify about 35% of all the defects. The simulated result indicates that about 38% of the defects are identified.

Also the model has been run for a variety of conditions and the results obtained are consistent with a

general understanding of the process. For example, low test effectiveness will give a high test yield but poor outgoing quality; 100% test effectiveness (and efficiency) will give an output which is independent of input component defect rate; allocating most incoming inspection effort to components which are tested by the 3060 will give a higher test yield.

User Interface

To facilitate the use of the model, user interface procedures programmed in Fortran have been developed for both input and output. The input user interface enables a user with no knowledge of GPSs to run the model and change the basic data on the number of components and the frequency of defects created at each assembly process or the effectiveness and efficiency of each test for each defect type. After all changes are made, the user interface program determines for each component type the cumulative probability distribution of number of defects created by each process. The program then generates automatically the revised GPSs statements resulting from the input data and inserts them in the GPSs model.

The output user interface enables the user to select any tables in the standard GPSs output that are of interest. A final report which consists of these tables with appropriate headings added is generated.

EXPERIMENTAL DESIGN

In order to identify the impact of various factors on the outgoing quality and yield, three designed experiments with each factor considered at 2 levels have been carried out with the model. The objective of the first experiment is to identify the factors that significantly affect the yield and outgoing quality of the shorts/opens test and the HP3060 test respectively. Because the interactions are believed to be not significant, a Resolution III Plackett Burman design [5] which studies (N-1) factors in N runs (provided that N is a multiple of 4) is used. The second experiment uses a fractional factorial design developed especially for the current system since some two factor interactions are believed to be significant. The design is developed in such a way that those two factor interactions that are thought to be significant do not confound with any other main effects and with each other (please refer to the appendix for the development of this model). The third experiment uses a full factorial design [6] to investigate the interactions among the factors identified as significant in the earlier experiments.

The values of the low and high levels for the selected 52 factors are tabulated in Table 5. For assembly processes, the data have a range of 5 or 10 to 1 corresponding to pessimistic and optimistic estimates of the defect rate. For the tests, the data corresponds to a reasonable level of effectiveness and no testing or minimally effective testing.

Incoming component defects, manual insertion defects, second component defects, wave soldering defects and cropping defects, totalling 32 factors are studied in the Plackett Burman design, while the 20 component test effectivenesses are studied in the 2¹⁵⁻⁴ fractional factorial design especially designed for this system. Four recorded response variables for each run are: (1) Y11 - the

Defect Type	Incoming Comp Defect			Manual Insertion Defect			Wave Soldering			Wave Soldering			Second Material Handling Defect			First Comp. Test Effectiveness			Second Comp. Test Effectiveness		
	Factor No.	+	-	Factor No.	+	-	Factor No.	+	-	Factor No.	+	-	Factor No.	+	-	Factor No.	+	-	Factor No.	+	-
Capacitor	1	.4	.04	11	.2	.02							23	.05	.01	33	80	40	43	30	0
Resistor	2	.4	.04	12	.2	.02							24	.05	.01	34	80	40	44	100	60
Coil	3	.4	.04	13	.4	.1							25	.2	.05	35	10	0	45	5	0
Diodes	4	.4	.04	14	.4	.1							26	.2	.05	36	40	0	46	5	0
Transistor	5	.5	.1	15	.4	.1							27	.2	.05	37	10	0	47	5	0
Varactor	6	.4	.04	16	.4	.1							28	.2	.05	38	60	30	48	100	50
I.C.	7	.5	.1	17	.4	.1							29	.2	.05	39	10	0	49	5	0
Filter	8	.4	.04	18	.4	.1							30	.2	.05	40	10	0	50	5	0
Transformer	9	.4	.04	19	1.0	.2							31	.2	.05	41	10	0	51	5	0
Jumpers	10	.4	.05	20	.4	.1							32	.2	.05	42	100	0	52	100	0
Joints	← NA →			← NA →			21	.1	.01	22	.1	.01	← NA →			← NA →			← NA →		

(+) high level
(-) low level

Table 5. Values of low and high levels for the J2 factors that are studied in the experiments.

shorts/opens test yield, (2) OGQ1 - the outgoing quality of the board after the shorts/opens test, (3) YIL2 - the HP3060 test yield, and (4) OGQ2 - the outgoing quality of the board after the HP3060.

The design and the result of each experiment are displayed in Table 6 and Table 7 respectively. For each response variable, the main effect for factor j is estimated using the following equation

$$l_j = \frac{2}{N} \sum_{i=1}^N Y_i d_{ij} \quad (j=1,2,\dots,k)$$

where d_{ij} is the i^{th} element of the j^{th} column of the design matrix, while Y_i is the observation corresponding to the i^{th} simulation run and k is the number of factors. The result of these two experiment showed that 6 factors are significant. Therefore, the third experiment is a 2^6 factorial design. The design and the result of the third experiment is shown in Table 8.

The results of each experiment are analyzed in two steps: (1) Normal plots are used to help identify the effects that appear to differ from the random error. Since linear combinations of random variable are statistically tending to have a normal distribution [7], those estimates that have values which are primarily due to the errors of observation should have the appearance of events from a normal distribution. This implies that if the values have true value zero, then the ordered estimated effects will, when plotted against the quantile of a standard $N(0, \sigma)$ distribution, tend to fall along a "straight line" which is determined by the points in the inner quantiles. (2) Analysis of Variance (ANOVA) technique is used to compare the sum of squares of the effects with the estimated variance. If the ratio of the sum of square of the effects and the estimated variance is larger than the tabulated F statistics with the appropriate degrees of freedom,

the effect is significant, otherwise, it is not significant. In the first experiment, Plackett Burman design allows 35 factors to be studied in 36 runs. Since only 32 factors are investigated, 3 degrees of freedom are available for estimating the required variance (or residual mean square) [5] in the ANOVA. The second and the third experiment has no degrees of freedom for the residual mean square, therefore the estimated variances obtained in the first experiment are used.

For the first experiment, the YIL1 normal plot indicated that only the effects corresponding to the incoming capacitor defect rate, the incoming resistor defect rate, the wave soldering defect rate and the cropping defect rate deviate from the straight line constructed by the data belong to the inner quantiles. Other normal plots showed that no effects have a significant effect on YIL2, OGQ1 and OGQ2. This result is consistent with the result obtained from the ANOVA (see Table 9). This finding is not surprising since capacitors and resistors made up approximately 70% of the components on the PCB and there are roughly 500 solder joints on a PCB. The processes and components which account for the majority of defects would be expected to have the dominant effect on yield because yield is determined by the number of defects found in the test. This implies that when the incoming capacitor and resistor defect rate, the cropping defect rate and the wave soldering defect rate are changed from the pessimistic level to the optimistic level, YIL1 can be improved significantly; while YIL2, OGQ1 and OGQ2 will not change significantly by changing any of the factors.

For the second experiment, the YIL2 and OGQ2 normal plots indicated that the test effectiveness for capacitors in the first and second component test appear to be significant; other normal plots showed that no test effectiveness affects YIL1 and OGQ1.

FACTOR RUN	EXPERIMENTAL DESIGN (I)																																EXPERIMENTAL RESULT					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	R.V. RUN	YIL 1	OGQ1	YIL 2	OGQ2	
1	-	-	-	-	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	70.9	37.1	62.3	62.9
2	+	-	-	-	-	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	60.2	38.2	64.3	64.5
3	-	+	-	-	-	-	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3	58.5	38.5	64.5	64.2
4	+	+	-	-	-	-	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	63.6	38.7	63.0	66.9
5	+	+	+	-	-	-	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	66.3	40.6	56.6	66.3
6	+	+	+	+	-	-	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	6	60.4	38.3	56.8	64.0
7	-	+	+	+	+	-	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	7	65.7	38.7	64.1	65.8
8	-	+	+	+	+	-	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	8	84.8	41.0	66.6	65.9
9	-	-	-	+	+	+	-	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	9	85.7	38.2	64.7	62.0
10	+	-	-	-	+	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	10	74.9	37.4	64.8	65.1
11	+	+	-	-	-	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	11	59.8	39.5	65.3	68.4
12	+	+	+	-	-	-	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	12	60.2	38.2	64.3	64.5
13	+	+	+	+	-	-	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	13	58.9	35.9	63.3	63.1
14	+	+	+	+	-	-	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	14	59.0	36.7	63.3	64.9
15	-	+	+	+	+	-	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	15	64.4	37.8	63.1	66.9
16	+	-	+	+	+	+	-	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	16	58.3	33.8	62.0	64.1
17	+	+	-	+	+	+	+	-	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	17	59.4	38.5	62.4	68.7
18	+	+	+	-	+	+	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	18	67.8	38.3	65.1	63.7
19	-	+	+	+	+	+	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	19	63.5	38.7	65.1	64.9
20	-	-	+	+	+	+	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	20	68.7	34.1	65.5	58.8
21	+	-	-	+	+	+	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	21	73.2	36.6	63.5	62.9
22	-	+	-	-	+	+	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	22	64.2	40.5	66.4	64.0
23	-	-	-	-	+	+	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	23	68.3	34.7	63.6	63.2
24	-	-	-	-	+	+	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	24	68.6	39.9	64.9	67.6
25	-	-	-	-	+	-	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	25	61.5	39.5	65.6	67.1
26	+	-	-	-	-	+	-	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	26	58.0	36.8	62.5	66.1
27	-	+	-	-	-	-	-	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	27	65.1	36.3	62.0	66.4
28	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	28	75.2	36.5	63.5	61.7
29	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	29	74.5	35.9	63.5	63.9
30	+	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	30	63.9	37.2	65.9	60.3
31	+	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	31	53.7	36.0	63.1	64.8
32	-	+	+	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	32	58.2	38.4	63.2	66.4
33	-	-	+	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	33	67.3	39.9	62.8	69.2
34	+	-	-	+	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	34	57.7	39.3	65.2	62.9
35	-	+	-	-	+	+	-	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	35	63.6	38.2	63.9	61.4
36	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	36	88.8	48.4	69.6	76.8

TABLE 6. DESIGN AND RESULT OF EXPERIMENT 1.

ANOVA is performed based on the variances estimated in the first experiment. The result showed that the changes in YIL2 resulting from the change in capacitor test effectiveness of the first and second component test are actually not significant when compared to the variance at 5% significant level. However, changing the capacitor test effectiveness in the first component test has a significant effect on OGQ2 at 5% level. The fact that no factor affects YIL1 and OGQ1 is expected since the 3C60 test is located after the point where YIL1 and OGQ1 are observed. The results also indicate that as the number of defects for each defect type becomes very low the outgoing quality of the assembly may not be improved much by a higher test effectiveness. This is reflected by the fact that among 20 factors for the test effectiveness, only those corresponding to the capacitors are observed to be significant. Those corresponding to the resistors are not significant even though the number of resistors on the PCB is also high and its defect rate is almost the same as that of the capacitor at almost all processes. However the material handling defect rate for capacitors after machine insertion is 4 times that of resistors and the material handling defects are not detectable by the shorts/opens test. This implies that the number of capacitor defects entering 3C60

test would be higher. Hence, a change in the test effectiveness for capacitors from high to low level will have a higher impact on the outgoing quality. Generalizing, in order to optimize the outgoing quality, test effort must be allocated in proportion to the expected number of defects for each defect type that enter the test. Therefore in this case, the outgoing quality at the 3CC test station can further be improved by raising the capacitor test effectiveness at the first component test to a level higher than the current high level.

The result of the third experiment showed that all of the interactions are not significant except that corresponding to the cropping and wave soldering defect rate. The interaction is found to have a positive effect on YIL1. This means that when both factors are set at their low level, the observed YIL1 is higher than the one which would have obtained when there is no interaction between the two factors [Fig. 4].

From the above experiments it is also observed that, for a given test, the yield depends on both the defects density entering the test and the test effectiveness of the test. Therefore yield can either be improved by reducing the number of defects

Factor Run	Experimental Design (II)														Experimental Result (Set 1)				Experimental Result (Set 2)			
	1	2	3	4	12 =5	14 =7	13 =9	24 =8	23 =6	34 =10	123 =16	124 =27	154 =49	234 =38	1234 =5(10)	YIL 1	OGQ1	YIL 2	OGQ2	YIL 1	OGQ1	YIL 2
1	+	+	+	+	+	+	+	+	+	+	+	+	+	+	62.9	56.8	71.3	78.3	62.2	57.6	73.8	74.7
2	+	+	+	-	+	-	+	-	+	-	+	-	-	-	63.2	57.1	71.8	76.5	62.5	55.6	73.5	72.1
3	+	+	-	+	+	+	-	+	-	-	-	+	-	-	63.1	56.7	72.2	76.8	62.7	57.0	74.3	72.8
4	+	+	-	-	+	-	-	-	-	+	-	-	+	+	63.1	56.8	72.4	75.8	62.8	56.4	74.6	71.3
5	+	-	+	+	-	+	+	-	-	+	-	-	+	-	63.6	55.9	71.3	76.6	62.8	56.4	73.7	72.9
6	+	-	+	-	-	-	+	+	-	-	-	+	-	+	62.7	55.8	72.4	73.7	62.7	56.0	74.0	72.6
7	+	-	-	+	-	+	+	-	+	-	+	-	-	+	62.9	56.9	71.8	76.9	62.0	57.6	73.9	73.8
8	+	-	-	-	-	-	+	+	+	+	+	+	+	-	62.6	55.6	71.6	75.0	62.8	55.5	73.7	71.4
9	-	+	+	+	-	-	-	+	+	+	+	-	-	+	63.3	55.3	73.3	73.4	62.2	57.6	74.0	74.3
10	-	+	+	-	-	+	-	-	+	-	+	+	+	-	63.3	54.9	73.7	72.1	62.5	55.6	73.7	71.8
11	-	+	-	+	-	-	+	+	-	-	+	-	+	-	63.2	55.8	75.3	70.5	62.4	56.5	73.8	71.4
12	-	+	-	-	-	+	+	-	-	+	+	+	-	+	63.2	54.6	75.4	69.0	63.2	55.9	74.2	71.9
13	-	-	+	+	+	-	-	-	-	+	+	-	-	-	63.2	55.8	76.0	69.8	62.7	56.3	74.3	71.4
14	-	-	+	-	+	+	-	+	-	-	+	+	+	+	63.4	55.1	75.5	68.9	62.7	55.8	74.2	72.2
15	-	-	-	+	+	-	+	-	+	-	+	-	+	-	63.0	55.9	73.0	73.9	62.0	57.5	73.8	73.9
16	-	-	-	-	+	+	+	+	+	+	+	+	-	+	62.8	55.6	73.7	73.0	62.8	55.5	73.4	71.8

Table 7. Design and Result of Experiment 2.

De- sign	Experimental Result (III)									
	Factor	YIL 1	OGQ1	YIL 2	OGQ2	Factor	YIL 1	OGQ1	YIL 2	OGQ2
+	1	52.9	56.8	71.5	76.3	33	56.1	55.7	70.8	76.4
+	2	52.9	56.8	72.0	75.4	34	56.1	55.7	71.4	75.8
+	3	52.4	55.7	74.0	71.4	35	55.2	56.3	73.1	72.1
+	4	52.4	55.7	75.3	69.5	36	55.2	56.3	74.5	70.1
+	5	57.8	56.0	71.3	75.8	37	62.2	56.2	71.5	75.9
+	6	57.8	56.0	71.8	75.2	38	62.2	56.2	72.0	75.1
+	7	58.2	55.5	73.9	72.0	39	62.4	55.9	73.5	72.4
+	8	58.2	55.5	75.7	69.6	40	62.4	55.9	74.7	70.8
+	9	58.0	55.4	70.8	77.2	41	62.5	56.7	71.4	76.8
+	10	58.0	55.4	71.3	76.3	42	62.5	56.7	71.8	76.3
+	11	58.3	55.4	72.8	72.9	43	63.1	56.7	73.4	73.0
+	12	58.3	55.4	74.4	70.8	44	63.1	56.7	74.7	71.0
+	13	66.8	57.2	72.3	76.6	45	74.2	57.6	72.7	76.6
+	14	66.8	57.2	72.9	75.6	46	74.2	57.6	73.4	75.6
+	15	66.9	56.0	75.6	69.8	47	73.6	58.0	76.1	72.6
+	16	66.9	56.0	75.6	69.8	48	73.6	58.0	77.4	71.0
-	17	54.9	55.4	71.4	76.6	49	58.7	56.7	71.3	76.6
-	18	54.9	55.4	72.0	75.6	50	58.7	56.7	71.9	75.5
-	19	55.1	56.1	74.3	71.3	51	58.5	56.2	73.2	71.8
-	20	55.1	56.1	75.7	69.4	52	58.5	56.2	74.8	69.5
-	21	62.7	55.2	70.3	76.1	53	67.2	57.0	71.5	76.5
-	22	62.7	55.2	70.9	75.2	54	67.2	57.0	72.3	75.2
-	23	62.7	55.9	73.3	71.3	55	67.9	57.7	74.8	72.4
-	24	62.7	55.9	75.0	69.0	56	67.9	57.7	76.5	70.2
-	25	62.2	57.9	71.8	77.0	57	67.4	58.4	72.5	76.7
-	26	62.2	57.9	72.2	76.5	58	67.4	58.4	73.1	75.8
-	27	62.2	55.1	73.7	72.2	59	67.2	57.4	74.5	70.4
-	28	62.2	55.1	75.2	70.0	60	67.2	57.4	75.3	69.5
-	29	73.4	57.6	73.0	76.0	61	85.1	58.5	74.9	75.4
-	30	73.4	57.6	73.5	75.1	62	85.1	58.5	75.3	74.6
-	31	73.6	57.6	74.7	72.2	63	84.8	58.6	77.4	72.1
-	32	73.6	57.6	76.1	70.7	64	84.8	58.6	78.9	70.1

Table 8. Result of Experiment 3.

SOURCE OF VARIATION	MEAN SQUARE				DEGREE OF FREEDOM	F TEST			
	YIL1	OGQ1	YIL2	OGQ2		YIL1	OGQ1	YIL2	OGQ2
FACTOR 1	328.818	10.347	19.507	4.340	1	57.890	2.012	5.22	1.899
FACTOR 2	422.988	0.147	16.402	1.103	1	74.470	0.029	4.39	0.483
FACTOR 3	10.671	16.402	2.402	21.623	1	4.879	3.189	0.058	9.459
FACTOR 4	0.000	4.480	1.914	1.734	1	0.000	0.871	0.046	0.759
FACTOR 5	0.054	1.647	2.102	35.403	1	0.0095	0.320	0.051	15.487
FACTOR 6	3.484	5.062	0.022	0.380	1	0.613	0.984	0.00	0.166
FACTOR 7	4.000	0.302	0.234	0.902	1	0.704	0.059	0.006	0.395
FACTOR 8	5.601	2.200	0.100	9.507	1	0.986	0.428	0.002	4.158
FACTOR 9	26.694	3.180	8.507	10.780	1	4.700	0.618	0.204	4.716
FACTOR 10	31.734	2.947	0.203	9.302	1	5.587	0.573	0.005	4.069
FACTOR 11	3.004	26.867	26.867	21.314	1	0.529	5.224	7.191	9.323
FACTOR 12	7.654	12.367	7.380	1.103	1	1.348	2.405	0.177	0.483
FACTOR 13	6.760	0.014	0.023	1.734	1	1.190	0.003	0.00	0.759
FACTOR 14	0.360	0.903	0.063	0.203	1	0.063	0.176	0.002	0.089
FACTOR 15	4.410	15.080	0.614	20.400	1	0.776	2.932	0.015	8.924
FACTOR 16	0.360	8.900	4.480	0.100	1	0.063	1.731	0.108	0.044
FACTOR 17	0.090	3.674	5.062	12.840	1	0.016	0.714	0.122	5.617
FACTOR 18	0.490	14.823	3.802	38.647	1	0.086	2.882	0.091	16.906
FACTOR 19	11.111	13.080	2.614	33.063	1	1.956	2.543	0.063	14.463
FACTOR 20	17.084	3.423	2.103	1.400	1	3.008	0.666	0.051	0.612
FACTOR 21	727.201	0.080	0.380	0.562	1	128.028	0.0156	0.009	0.246
FACTOR 22	745.290	15.080	0.902	0.062	1	131.213	2.932	0.022	0.027
FACTOR 23	6.084	0.514	10.563	0.467	1	1.071	0.100	0.253	0.204
FACTOR 24	2.890	0.234	16.134	7.747	1	0.509	0.045	0.387	3.389
FACTOR 25	8.604	10.347	12.367	34.222	1	1.515	2.012	0.297	14.970
FACTOR 26	0.751	10.78	0.380	23.847	1	0.132	2.096	0.009	10.432
FACTOR 27	2.560	2.723	0.100	6.002	1	0.451	0.529	0.002	2.626
FACTOR 28	4.134	2.507	6.847	9.100	1	0.728	0.487	0.164	3.981
FACTOR 29	9.404	0.234	8.702	0.563	1	1.656	0.045	0.209	0.246
FACTOR 30	7.654	7.023	0.780	0.003	1	1.348	1.366	0.019	0.001
FACTOR 31	1.068	3.300	13.814	11.674	1	0.188	0.642	0.332	5.107
FACTOR 32	0.090	0.080	0.902	2.507	1	0.016	0.016	0.022	1.097
R. ERROR	5.680	5.143	3.736	2.286	3				
	2418.152	214.176	187.483	329.490	35				

Table 9. ANOVA for Plackett Bauman Design.

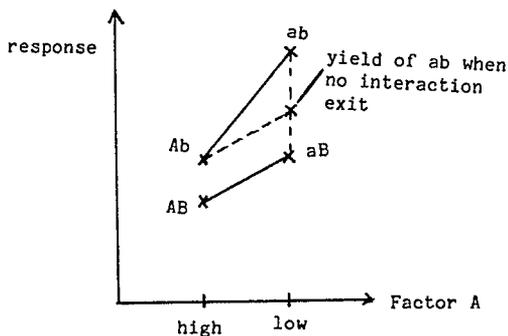


Figure 4: An illustration of the Positive Two Factor Interaction

set at high level. The result showed that YIL1 is improved from 64% to 84%; OGQ1 is improved from 57% to 60%; YIL2 is improved from 74% to 77%; while OGQ2 is improved from 70% to 73%. Note that while YIL1 is substantially improved, the other performance measures are not much improved.

GENERALIZATION OF THE MODEL

One of the advantages of a simulation model that only focuses on quality is that the model can be easily generalized to account for non-serial production system i.e. a system which is constructed of merging and diverging branches. Since the quality records for each process are stored at different parameters in a transaction, the order that defects are assigned to the transaction at different blocks in a simulation model is not very important. For example, in the present system machine insertion defects can be assigned after manual insertion, handling defects can be assigned at the beginning of the system and so on. The order of assignment will not affect the statistics which are gathered at the end of wave soldering processes. This implies that processes in parallel can be represented by blocks in series and a merging and branching system can be simulated by sequences of serial subsystems. This generalization is not possible if it is necessary to take the transit time into account in a simulation model.

entering a test or by making the test less effective. Since the latter is not recommended the only solution to improving yield is thus to control incoming quality and process quality. Otherwise, better quality requires that one should sacrifice yield.

Based on the above findings, the simulation model is run with the incoming capacitors and resistors defect rate, cropping and wave soldering defect rate set at their low level; and with the capacitor test effectiveness at the first and second component test

CONCLUSION

A system approach to the production quality enables one to answer questions such as: Where to carry out test and inspection? How much inspection and test? What defects should the test look for? What incoming quality should be required of component? What quality level should be sought of individual process? Should quality be created in the product from the beginning or should quality be inspected into the product? With a good user interface procedure and statistically well designed experiments, a quality simulation model such as the one outlined in this paper can serve as a powerful tool for the users to gain insight about the system and to explore the optimal process parameters of each process as well as the structure of the system.

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APPENDIXDevelopment of the 2^{15-11} Fractional Factorial Design

In this experiment, 20 factors for component test effectiveness are under investigation. Of these 20 factors, 10 are for HP3060 first component test effectiveness and the other 10 are for HP3060 second component test effectiveness. Since the two component tests detect the same defect type, the interactions among these factors are first examined.

In the model, the first component test detects a fraction of the components on each board while the second component test detects a fraction of the remaining components that are not addressed by the first component test. Suppose that the effectiveness of the first component test is 90% and the second component test is 80% effective, then the actual effectiveness of the second component test will be $10\% \times 80\% = 8\%$. If the first component test is 50%, and the second component test effectiveness remains at 80%, then the actual effectiveness of the second component test becomes $50\% \times 80\% = 40\%$. Therefore the actual effectiveness of the second component test depends on the corresponding first component test effectiveness. According to the definition defined in the previous section, there are "interactions" between the first component test and the second component test procedures which detect the same defect type.

The 20 factors can thus be paired in such a way that the factors which detect the same type of defect belong to a single pair. The problem is then to develop a design which studies the main effects as well as the 10 interactions between the factors within a pair. In the literature, such a design does not exist, and it will be very time consuming to develop this design manually. A compromise is to construct a smaller design for half of the factors (i.e. five pairs of the factors) and perform the experiment twice. By doing so, the construction process for the required design will be greatly simplified. No information will be lost since the interactions among pairs of factors are insignificant.

Suppose for the 5 pairs of factors that are to be studied in the same design, the factors that belong to the first component test are numbered from 1 to 5 and those belong to the second component test are numbered from 6 to 10 respectively, then the interactions that are of interest are 16, 27, 38, 49 and 5(10). Since 15 estimates are needed, therefore 16 runs, i.e. a 2^{15-11} fractional factorial design is required.

Using the procedure to construct a 2^{k-p} fractional factorial design [6] a matrix of independent variables is constructed for a 2^4 full factorial design first. The first 4 columns are designated to factor 1 to 4 respectively. The next 6 columns, which are the 2 factor interactions among the first four factors, need to be associated with factor 5 to 10 in such a way that the required interactions between factors within a pair do not confound with any of the main effects. This can be done as follow:

- (1) List the six 2 factor interactions of the first four factors:

12, 13, 14, 23, 24, 34

- (2) Add to each of these a third factor that does not interact with any of the factors in the pair. These are the potential generators:

125	236	137	128	129	12(10)
135	246	147	148	139	13(10)
145	346	347	248	239	14(10)
235					23(10)
345					24(10)
					34(10)

- (3) Among these generators, six are chosen such that

- (a) Each of the two factor interactions listed in (1) can only appear once in the generators.

- (b) No two generators belong to the same column in (2) can be chosen.

There are more than one feasible combinations, one chosen is

$I=125=236=147=248=139=34(10)$

Therefore factor 5 is assigned to column 12, factor 6 is assigned to column 23, ... etc..

- (4) By multiplying the six generators with each other in pairs, triplets, etc., the 2^6 members or "words" of the defining relation obtained are:

(the generators)

$I=125=139=147=236=248=34(10)$

(pairs)

$=2359=2457=1356=1458=12345(10)=3479$
 $=1269=123489=149(10)=123467=1278=137(10)$
 $=3468=246(10)=238(10)$

(triplets)

$=1234579=569=34589=2459(10)=34567=578$
 $=2357(10)=1234568=1456(10)=1358(10)$
 $=24679=23789=79(10)=14689=123469(10)$
 $=1289(10)=13678=1267(10)=123478(10)$
 $=68(10)$

(4-tuples)

$=145679=135789=1257(10)=245689=34569(10)$
 $=589(10)=235678=567(10)=34578(10)$
 $=12568(10)=6789=23679(10)=24789(10)$
 $=13689(10)=14678(10)$

(5-tuples)
 =1256789=135679(10)=145789(10)
 =235689(10)=245678(10)=346789(10)
 (6-tuples)
 =123456789(10)

- (5) Find the two factor interactions which are aliases of 123, 124, 134, 234, and 1234 by multiplying the effect with the defining relation. Ignoring those aliases that contain more than two factors, the result is:

	effects				
	123	124	134	234	1234
	35	45	49	46	5(10)
2 factor aliases	29	27	37	38	89
	16	18	1(10)	2(10)	67

Note that each of the required interactions is confounding with only one effect, namely, 16 with 123, 27 with 124, 49 with 134, 38 with 234 and 5(10) with 1234 respectively. Therefore interaction 16 is assigned to column 123, interaction 27 is assigned to column 124, ..., etc.

In the first part of the experiment, factor 33 to 37 are associated with factor 1 to 5 and factor 43 to 47 are associated with factor 6 to 10 in the design. While in the second part of the experiment, factor 38 to 42 are associated with factor 1 to 5 and factor 48 to 52 are associated with factor 6 to 10 in the design.

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