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ANALYTIC REPRESENTATIONS OF SIMULATION

Moderator: Bruce W. Schmeiser
School of Industrial Engineering
Purdue University
West Lafayette, Indiana 47907

ABSTRACT

In much the same way that a mathematical model of a real or conceptual system is used to improve the design and operation of the system, mathematical models of simulation are used to improve the design and operation of simulation experiments. The participants in this discussion are advocates of various analytic representations of simulation. Contained in the paper that follows are each participant's responses to a set of questions concerning the representation of simulation they represent.

PARTICIPANTS

"Simulation Models as Graphs"
Lee Schruben
School of Operations Research and Industrial Engineering
Cornell University
Ithica, New York 14853

"Hybrid Simulation/Analytic Models"
J.G. Shanthikumar
School of Business Administration
University of California
Berkeley, California 94720

Robert G. Sargent
Department of Industrial Engineering and Operations Research
Syracuse University
Syracuse, New York 13210

"Modular Hierarchical Simulation Model Representation"
Bernard Zeigler
Department of Computer Science
Wayne State University
Detroit, Michigan 48202

"Generalized Semi-Markov Processes"
Peter Glynn
Department of Industrial Engineering
University of Wisconsin - Madison
1513 University Avenue, Room 385
Madison, Wisconsin 53706

"Sample Space Definition"
Barry L. Nelson
Department of Industrial and Systems Engineering
The Ohio State University
1971 Neil Avenue
Columbus, Ohio 43210

Bruce W. Schmeiser
School of Industrial Engineering
Purdue University
West Lafayette, Indiana 47907

QUESTIONS AND RESPONSES

1. Why should there be mathematical representations of simulation when simulation is often considered to be the approach taken when analytic solutions are not possible?

SHANTHIKUMAR AND SARGENT: Normally one resorts to simulation only when an efficiently implementable analytic solution is not available for the problem at hand. However, often a portion of the problem at hand (in the sense of the physical system which contributed the problem or the behavior of the system over time) may have an efficient solution approach. Hence without the hybrid simulation/analytic characterization one cannot make use of such a situation.

ZEIGLER: Simulation has outlived the early days of scarce computer resources and exaltation of analytic solutions (to relatively simple models). It is increasingly an oft used tool integrated with other computerized problem solving support. Models are complex and quality assurance is a crucial issue. Thus a mathematical, i.e. formal, computer compatible, disciplined, framework is imperative. The advent of structured programming methodology with respect to software engineering is a telling analogy.

GLYNN: When analytic solutions are not available, simulation can often be used to estimate parameters of interest. For complex systems, however, the question of "statistically efficient" design of the simulation experiment is important. The generalized semi-Markov process (GSMP) framework provides significant insight into this latter design component of the simulation.

NELSON AND SCHMEISER: Mathematical representations of simulation are useful for the development of new simulation methodology. The abstract study of simulation experimentation as an application area is at a higher level than the use of simulation to solve specific problems, and thus the results of such study are more general.

2. There are many aspects of simulation. What aspects are highlighted by your representation?

SHANTHIKUMAR AND SARGENT: Reducing computational time while maintaining a desired accuracy of the final solution is an important aspect of the simulation model set up. Hybrid simulation/analytic modelling allows one to either provide a variance reduction (Sargent and Shanthikumar, 1982, Shanthikumar, 1983, Shanthikumar and Sargent, 1983) or an approximate solution at a reduced computational time compared to the traditional simulation modelling (Shanthikumar and Sargent, 1983).

ZEIGLER: Aspects highlighted: Methodology for model construction, especially in relation to creation of model bases and development of new simulation media. Characteristic of the approach is its search for opportunities for computer support of the model development and recycling process. Thus it emphasizes formal, algorithmic, and structural considerations.

GLYNN: The generalized semi-Markov process (GSMP) is a mathematical tool for analyzing discrete-event simulations. Given a discrete-event simulation, one can define an equivalent GSMP. For example, the GSMP uses as basic building blocks a set of physical states and a set of clocks. The process evolves as follows: When a clock runs down to zero, the process jumps from the current (physical) state to a new state. In the new state, some clocks may be reset, independently of the past, to new values, whereas other clocks may continue to run down from their values in the previous state visited. As should be clear from this description, the family of GSMP's is in correspondence with a large class of discrete-event simulations. For a more complete discussion of the relationship between GSMP's and discrete-event simulations, see Glynn (1983).

In any case, the GSMP description of a discrete-event simulation emphasizes the stochastic nature of such a simulation. The precise mathematical nature of the probabilistic dependencies between all the variables driving a discrete-event simulation is readily available via a GSMP description of the process in question.

NELSON AND SCHMEISER: The sample space definition of simulation experiments characterizes a simulation experiment in terms of the known sample space and probability distribution of the inputs, an output function defined on the inputs, a sampling plan, a statistics function defined on the outputs, and parameters of interest that the statistics estimate. It is particularly useful for studing variance reduction.

3. What can be done because of your representation that would be difficult or impossible without it?

SHANTHIKUMAR AND SARGENT: Without the hybrid simulation/analytic modelling characterization one often simulates a complex system completely even when a portion of it (in the sense of physical part or in the sense of its behavior over time) is simple enough to be analytically solved. In hybrid simulation/analytic modelling this partial analytic solution could be used to one's benefit.

ZEIGLER: The characterization makes possible a unification of the various means of model specification (world views), the automatic mapping of such specifications onto alternative media, such as either to conventional simulation languages or to distributed simulators, the modular specification of model and experiment, and on this basis, a general methodology for creation of model base environments.

GLYNN: The GSMP description of a discrete-event simulation is ideally suited to the analysis of questions related to "statistically efficient" design of simulation experiments. Powerful tools from the statistics and probability literature can be applied, in a reasonably direct way, to GSMP's.

Now, much of the current literature on simulation output analysis requires that the output process satisfy assumptions which are clearly invalid in "real-world" situations. For example, the time-series approach to steady-state simulations requires that the output process be stationary, in some sense. Because of the "initial bias" problem, this is essentially never true in practice. On the other hand, since the GSMP is a description of the discrete-event simulation, it has precisely the properties of the simulation in question. Thus, any conclusions which one reaches for the GSMP under consideration hold, <u>without further simplifying assumptions</u>, for the discrete-event simulation corresponding to the GSMP.

NELSON AND SCHMEISER: The sample space definition makes possible the definition of six classes of transformations of simulation experiments. These classes exhaust the set of all possible variance reduction techniques under composition and are mutually exclusive classes. This unified view of variance reduction provides four benefits:

a. Applications -- Since there are only six classes of transformations, the practitioner has a simple, yet complete and nonoverlapping checklist of ideas to consider. Thus, generation of variance reduction techniques is easier.

b. Research -- The sample space definition provides a framework within which an interactive guidance program may be possible. Such a program would examine a simulation experiment and suggest variance reduction strategies based on whatever additional knowledge the experimenter could supply beyond what was required to build the simulation model. The sample space definition also provides a new level for theoretical insight into variance reduction techniques, such as establishing conditions under which a variance reduction is guaranteed.

c. Communication -- Reporting variance reduction research and practice is easier because it can be expressed in terms of a well-defined framework.

d. Teaching -- By viewing the classical variance reduction techniques as compositions of the six classes of transformations, variance reduction is seen by the student as a structured discipline rather than merely a bag of tricks. (However, we have found that only a quick overview of the six classes of transformations can be presented before specific techniques are introduced. Following these examples, the sample space definition provides the structure needed for deeper study.)

4. How is your representation useful for practitioners, researchers, language designers, teachers, and/or others?

SHANTHIKUMAR AND SARGENT: Certain types of hybrid simulation/analytic modelling (specifically the uniformization approach, Shanthikumar, 1983) lend themselves very easily for a language design. This would then allow the practitioners to use the hybrid modelling even without having to identify the analytic characterization of the problem being solved. On the other hand, several other forms of hybrid simulation/ analytic approaches are very much problem oriented and pose several interesting research problems. For example, an efficient implementation of such an approach may require the computer generation of random variables with the knowledge of their Laplace transforms only.

Teaching in simulation has been often separated from analytic solution approaches (and vice versa). This often leaves the students with a confused feeling that either all practical problems should be simulated or solved analytically. This leads the student having to make a choice between simulation and analytic solution approaches. (A student's decision to accept either one of these philosophies may depend on which instructor has been most impressive!) Hopefully teaching hybrid simulation/analytic modeling will bring out the much needed amalgamation of the solution approaches (simulation and analytic) they have learned.

ZEIGLER: Usefulness: To practitioners as a conceptual framework for understanding the activities involved in simulation; to researchers as a basis for further development (see question 7); to language and system designers, concepts and structures with which to implement computer-based (hardware, software) simulation environments; to teachers an intellectually valid framework for teaching an integrated view of simulation as opposed to the case study, language, or statistical methodology approaches; to computer system designers a handle on the simulation component of larger decision support systems.

GLYNN: For researchers in the simulation area, GSMP's provide a framework in which they can apply powerful techniques from the statistics and probability literature. From a teaching standpoint, these processes provide a nice "bridge" between the stochastic processes literature and the simulation area. For example, for students that have been previously exposed to the basic theory of Markov chains, GSMP's provide a natural means of introduction to the discrete-event simulation. Finally, it is our belief that certain output analysis methods, based on the probability structure of GSMP's, can be incorporated into simulation software. This would have obvious benefits to practitioners.

NELSON AND SCHMEISER: See question 2.

5. Given a particular simulation model (experiment), is there a unique correspondence between elements in the simulation and elements in your representation, or are there multiple interpretations?

SHANTHIKUMAR AND SARGENT: For a given problem, one may adopt a host of different hybrid simulation/analytic characterizations. The best choice among such characterizations will depend heavily on the knowledge of analytic solutions for problems similar to the problem at hand.

ZEIGLER: As in any model, no one-one correspondence exists with reality, nor is it desirable that it should.

GLYNN: No. As is well known by simulators, a given stochastic model can give rise to more than one discrete-event simulation. For example, if a simulation's clocks are always reset by exponential random variables, it is known that one can replace the multiple clocks present in each state by a single clock; this replacement is justified by the memorylessness of the exponential. Thus, such a stochastic model can be generated by two different discrete-event simulations, one having multiple clocks and the other single clocks in each state. Since there is a correspondence between discrete-event simulations and GSMP's, this implies that at least two GSMP's can be associated with such a stochastic model.

NELSON AND SCHMEISER: Yes and no. An unusual feature of the sample space definition is that it includes the knowledge the experimenter used to build the model as part of the definition of the experiment. For a particular experimenter there is a unique correspondence, but two different experimenters might view the same computer code differently in terms of the sample space definition.

6. How inclusive is your representation? Does it include discrete event, process, and continuous time simulation? Are procedures not normally considered simulation covered by your representation?

SHANTHIKUMAR AND SARGENT: The hybrid simulation/analytic approach is a marriage between the simulation and analytic solution approaches. Such a procedure is thus not normally covered by simulation. Furthermore, this procedure is not restricted to discrete event processes only. One may apply the hybrid simulation/analytic technique to continuous state, continuous time processes as well.

ZEIGLER: The characterization is based on system theoretic concepts hence is inclusive of both discrete and continuous constructs (world views). Transformations have been given between various discrete event formalisms and between discrete event and differential equation formalisms. Simulation model formalisms are the raison d'etre, but other modelling and design formalisms can be accomodated as well.

GLYNN: As discussed above, the class of GSMP's corresponds to the family of discrete-event simulations. Consequently, GSMP's do not appear to be suitable for dealing with continuous or hybrid systems (except to the extent that discretizations of such systems often lead to discrete-event processes).

NELSON AND SCHMEISER: The sample space definition is very inclusive in that any input/output system that can be coded and executed is included. Monte Carlo experimentation and survey sampling are natural special cases.

7. What direction should simulation research take with your representation as a basis?

SHANTHIKUMAR AND SARGENT: Research on simulation has been very much disjoint from the research on analytic modelling. Thus, several simulation models and analytic solution procedures are available (in the form of software packages as well) for "standard practical problems" with varying degrees of assumptions. It is possible to develop hybrid simulation/analytic models for such "standard practical problems" (Shanthikumar and Sargent, 1981). Furthermore, as pointed out earlier, languages may be developed to automate hybrid simulation/analytic modelling. Several open questions such as generating random variables with the knowledge of their Laplace transforms, or obtaining bounds for the error in approximations from the confidence intervals of the input parameters can be found in the field of hybrid simulation/analytic modelling.

ZEIGLER: Research in a variety of directions is possible based on this approach:

a. Design of simulation environments centered on model base concepts.

b. Design of distributed simulation architecture based on model formalisms and decompositions.

c. Deeper investigation into model representation constructs (e.g., parameter correspondence structures) and their integration with the knowledge representation mechanisms of artificial intelligence, towards intelligent simulation support systems.

GLYNN: As emphasized above, the GSMP provides a means of applying powerful tools from probability and statistics to the problem of designing "statistically efficient" simulation experiments. For example, GSMP tools can be used to determine whether a discrete-event system has a steady-state or not, and whether the steady-state is independent of the initial condition. Clearly, if the steady-state depends on the initial condition, this has important implications for the practitioner and modeller.

A second application is the use of analytical tools from the general theory of Markov processes to determine a priori bounds on certain parameters of interest to a simulator. For example, in certain systems, to determine the "relaxation time" of the process from the building blocks of the simulation; i.e. at what time T is the initial bias small. These types of a priori bounds can also be used to verify, prior to the simulation, that the moment conditions needed for certain limit theorems to hold are, in fact, valid.

As a final example, it is to be anticipated that the probability structure of GSMP's can be exploited to obtain new variance reduction techniques for broad families of discrete-event simulations. This is in contrast to much of the current variance reduction literature, in which a great deal of structure must normally be imposed on the discrete-event system in order to justify (or sometimes even apply) the procedures. The types of results just described are ones for which further research should yield important benefits to the simulation community.

NELSON AND SCHMEISER: See 2b.

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ADDENDUM

1. Why should there be mathematical representations of simulation when simulation is often considered to be the approach taken when analytic solutions are not possible?

SCHRUBEN: Simulation is a mathematical modeling technique.

2. There are many aspects of simulation. What aspects are highlighted by your representation?

SCHRUBEN: The approach I use in developing a simulation program is to model the system as a graph. The vertices or nodes of the graph represent state changes and the edges or arcs represent time flow. Specifically, after the change in state at node i, if there is an arc from i to j in the graph, then there might be a subsequent state change at node j. The details of the approach are presented in an article in the Communications of the A.C.M.,Vol. 26, No. 11, pg. 957. Basically the graph is an index set for state transition functions and the conditions and times in the dynamics of the system. The graph can be easily translated into computer code.

3. What can be done because of your representation that would be difficult or impossible without it?

SCHRUBEN: The event graph approach allows the modeler to structure a system, determine which events must be initially scheduled, select a priority between possible simultaneous events, minimize the number of distinct event procedures that must be coded, develop part of a simulation program while colleagues work on other parts, and know which state variables are necessary for the program to behave correctly. Programs developed from event graphs can be easily altered or enriched.

4. How is your representation useful for practitioners, researchers, language designers, teachers, and/or others?

SCHRUBEN: Event graphs are useful both in structuring simulation programs and in teaching the concepts of discrete event systems. The abstract concept of an event seems easier to understand when presented in the context of an event graph.

5. Given a particular simulation model (experiment), is there a unique correspondence between elements in the simulation and elements in your representation, or are there multiple interpretations?

SCHRUBEN: Programs can be coded in various ways, likewise there can be several quite different but equivalent (same input implies same output) event graphs for the same system. This is one of the real benefits of event graphs; the modeler can represent a system in a variety of equivalent ways and select that which is easiest to code, or easiest to understand, or easiest to modify.

6. How inclusive is your representation? Does it include discrete event, process, and continuous time simulation? Are procedures not normally condidered simulation covered by your representation?

SCHRUBEN: Continuous and discrete event models can be represented by event graphs as pointed out in the above mentioned reference.

7. What direction should simulation research take with your representation as a basis?

SCHRUBEN: Event graphs are an ideal candidate for a graphical based model building language. Some work on this was done with the General Electric Labs in Schenectady, New York this past year. A prototype Pascal program that allows event graphs to be constructed and translates them into SLAM code was developed on an Apple IIe computer (Reference: Fitzpatrick, D., T. Capatosto, and J. Glasso, Masters of Engineering Project, S.O.R.I.E., Cornell, 1984). An approach to modeling that I call event synthesis (where elementary state transitions are condensed into events using graph analysis techniques) is currently being developed.