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INTRODUCTION TO SIMULATION

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ABSTRACT

This paper offers an introduction to the fundamental concepts of system modeling with emphasis on the application of digital simulation. The topics presented include system modeling, model classification, a discussion of mathematical and simulation models, their distinction and relative advantages, an overview of systems analysis, an example simulation model, and brief discussions of random numbers, random variable generation and simulation languages. The material presented is largely conceptual and requires no prior background in modeling.

INTRODUCTION

Managers today find that effective execution of their responsibilities is a perplexing task of ever increasing complexity. In part, this is the result of a changing and often unpredictable world, national and local economies, the pervasive impact of federal and state regulation, changing attitudes of the consuming public and the relentless and accelerating growth of modern technology. The contribution of each of these elements of the business environment has led to larger and more complicated organizational structures to effectively carry on business activity. Consequently the manager is forced to recognize and understand the interactive behavior of an increasing number of components within the organization and its environment, at least to the extent that they affect and are affected by his/her decisions.

Fundamentally a manager is a decision maker. The process of making a decision involves the identification, evaluation and comparison of alternative courses of action in light of a variety of conditions which may prevail during the period for which the decision will be in effect. Because of the number of factors which must be considered and the complexity of their interaction, the manager often turns to a system model for quantitative analysis of the impact of each alternative decision under each set of conditions anticipated.

SYSTEM MODELING

In a broad sense a model may be described as the representation of some aspect of reality without the presence of that reality. In this sense models have been used by man throughout recorded history. A photograph, painting or drawing is a two-dimensional representation of the visual aspects of the reality portrayed. A sound recording is anauditory representation and a scale model is a three-dimensional representation. However, managerial decisions often require a level of abstraction which can only be provided by a mathematical or simulation model.

Model Classification

Models may be classified in a variety of ways. Considering the manner in which a model represents a system, a model may be iconic, analog or symbolic. The common property of iconic models is reproduction of a physical characteristic of the entity modeled. Hence, an iconic model looks like the reality modeled. The common feature of analog models is replacement of a property of the physical system by a substitute property in the model. The distinguishing characteristic of symbolic models is the replacement of properties of the physical system by symbols, and include mathematical and simulation models.

A model may be classified by the purpose for which it is developed. In this context a model may be descriptive or normative. A descriptive model is one which describes the behavior of properties of the system modeled. The output of such a model is not intended to recommend a course of action but rather simply describes what happens. A model which is intended to recommend a course of action is called a normative model. More often than not a normative model is the result of a manipulation of or operation on a descriptive model.

Models may be further categorized acording to whether or not they portray the behavior of the system modeled over time. A model which describes the behavior of a system through a given time interval is called a dynamic model. A model which portrays the behavior of a system at a single point in time is called a static model. As an illustration consider a system model which describes the mean cost of production per unit manufactured. If the model portrays the fluctuation in the mean throughout the period of production then the model is dynamic. If the model yields only the mean for the entire production period then the model is static.

The fourth dimension of model classification deals with whether or not the model explicitly recognizes the presence of random variation in the system modeled. Very few real world systems, if any, are free of the influence of the unpredictable or random behavior of the elements of the system or its environment. A deterministic model is one which does not recognize the randomness of the behavior of the system. While a system may be influenced by random behavior, the impact of that behavior may be sufficiently slight that the random component may be ignored for practical purposes. In such cases a deterministic model is entirely appropriate. A model which explicitly captures the random components of system behavior is called a probabilistic or stochastic model.

The final dimension of model classification treats the manner in which the model represents change within the system modeled. If a model describes change in the status of the system as occurring only at isolated points in time then the model is called discrete. On the other hand, if the model treats change as a continually occurring phenomenon then the model is called continuous.

By their nature mathematical and simulation models are symbolic. While both types of models may be either descriptive or normative, more often than not simulation models are descriptive. A reveiw of the literature on modeling would indicate that static mathematical models are more prevalent than their dynamic counterpart. Conversely dynamic simulation models are reported more frequently than are static models. A wide variety of deterministic and stochastic mathematical models are reported in the literature, the type of model being dependent upon the nature of the system modeled. However, simulation models are more often stochastic than deterministic. Finally, change is treated as a discrete phenomenon more often than a continuous phenomenon in the case of both mathematical and simulation modeling. While continuous mathematical and simulation models are certainly not uncommon, continuous change systems are frequently approximated by discrete models.

Mathematical and simulation models are used to describe the interactive behavior of the organization and its business environment under prescribed conditions of operation. The input to either type of model usually defines the operating conditions assumed and the decision alternatives considered. The output of the model describes the resulting response of the organization and its environment. Model output usually includes measures of system or organization performance such as profit, cost, level of service, sales volume, product quality, etc. Whatever the measure of performance, a common feature of most mathematical and simulation models is quantitative measurement providing a basis for comparison of alternative decision strategies.

Simulation and/vs. Mathematical Models

Both mathematical and simulation models have as their intended purpose estimation of the value(s) of one or more measures of system performance. This is accomplished by relating each measure of performance to the interactive behavior of the system studied and in turn relating that behavior to governing operating conditions and decision alternatives.

Mathematical Models. Mathematical models are characterized by one or a series of equations relating the measure(s) of system performance to the variables which affect system performance and equations or inequalities which define constraints on the range of values which those variables may assume. The variables of the system may be classified as decision variables, variables under direct control from within the system, and variables which cannot be directly controlled. Uncontrollable variables may be further classified as those which are not influenced by the values assumed by other variables, independent variables, and dependent variables whose values are determined by the values of the decision variables, the independent variables and other dependent variables.

The equations defining a mathematical model usually attempt to describe system behavior in aggregate form. To illustrate, suppose that units of product are manufactured at a uniform rate and p_i is the probability that ith unit will be defective. If each defective unit costs C_D and M units are produced then the mean cost of defective units per unit manufactured, C_μ, is given by

$$C_\mu = \frac{C_D}{M} \sum_{i=1}^{M} p_i \tag{1}$$

If the probability distribution of p_i is known for the manufacturing period then equation (1) may be simplified to

$$C_\mu = C_D \mu_p \tag{2}$$

where μ_p is the mean probability of a defective item occurring during the period of production. Thus to estimate the mean cost of defective items the analyst need not calculate the probability that each item will be defective, but rather through equation (2), he/she may deal directly with the mean probability of a defective item occurring, simplifying the computational effort. In other words, equation (2) deals with the aggregate behavior of manufactured units rather than the behavior of each item.

Mathematical models have long been a basic tool of the physical sciences and engineering. During World War II such models were applied to the analysis of organizational systems, leading to the emergence of the discipline of operations research. This discipline is also referred to as management science, systems analysis and systems engineering, although the latter two terms are also applied to other disciplines as well. More recently, mathematical models have been successfully applied in the social sciences.

Simulation Models. There are similarities between mathematical and simulation models. Both have the same purpose and both utilize mathematical relationships. However, rather than attempt to deal with aggregate system behavior directly, a simulation model focuses on the behavior of individual components of the system and attempts to capture each change in the status of the system as it occurs over time. Returning to the example where the model is to estimate the mean cost of defective units per unit produced, a simulation model would approach this problem by defining a different value for p_i for each of the M units produced, determining whether each is good or bad, summing the costs of those identified as defective and dividing the result by M. However, the value assigned to each p_i must be drawn at random from the governing probability distribution of p_i at the time the ith unit is produced. In this sense the analyst must synthetically sample from the probability distribution of p_i and the sampling technique applied is called the Monte Carlo method.

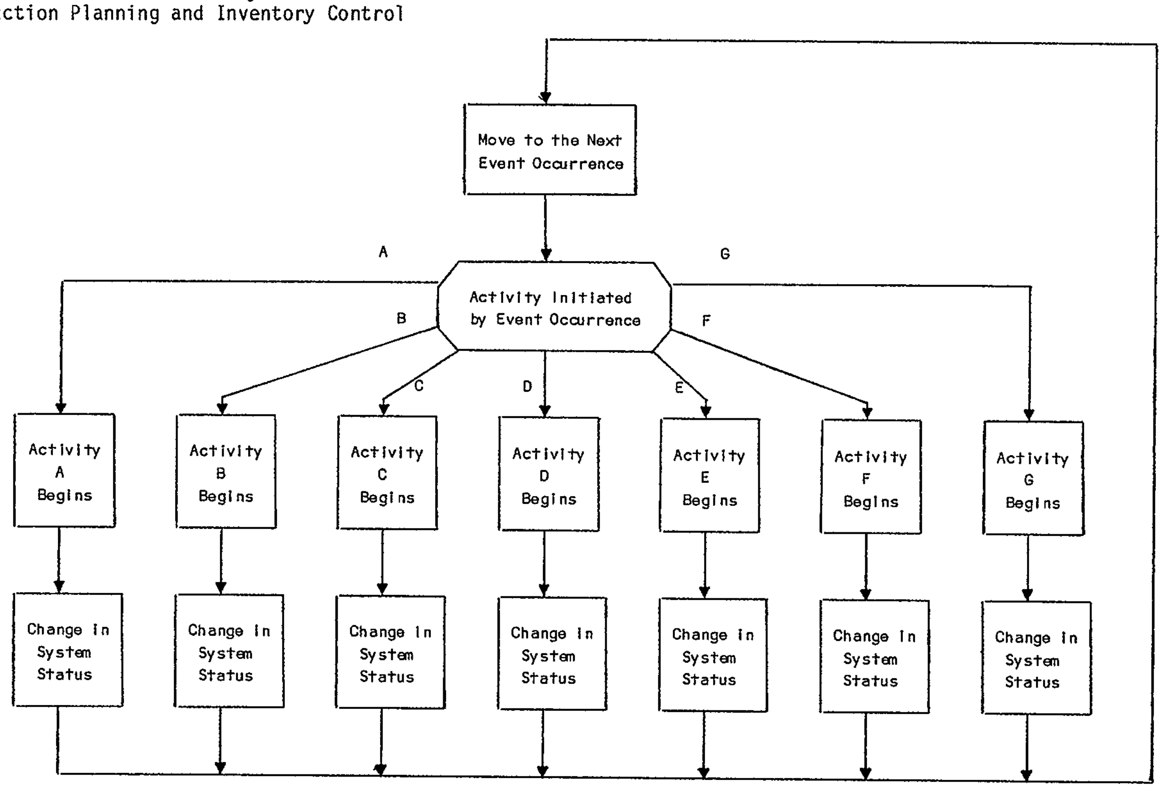
As the foregoing discussion suggests, a simulation model may be viewed as a vehicle for observing the time dependent random behavior of a system where time is artificially accelerated. The system observed is composed of entities such as human beings, materials and equipment which are intended to achieve specified objectives through coordination of the entities of the system and the activities which result from that coordination. The activities which take place within the system are initiated and terminated by events. Events may be caused by human intervention within the system, by human intervention external to the system, by natural phenomena which are beyond human control, and by natural phenomena which may be influenced but not controlled by the human. Frequently the events which lead to a change in system status occur at random points in time. The Monte Carlo method is used to define random event times as well as other random phenomena which influence system behavior.

The fundamental operation of a time dependent system simulation model is shown graphically in Figure 1. As Figure 1 illustrates each event leads to a reaction by the system. The system reaction may include a variety of activities some of which take place immediately and some of which are delayed. In turn the activities resulting from an event occurrence lead to changes in system status. The model then moves forward in time to the next event and the process repeats until the simulation experiment is completed.

Some authors trace the origin of simulation to the early sampling experiments of W.S. Gosset, who published under the name Student (24). However, the foundations of modern simulation methodology are usually traced to the works of von Neumann (26) and Ulam (25). Their work, conducted in the late 1940's, involved the analysis of nuclear-shielding problems through a technique which they termed "Monte Carlo Analysis." However, it was not until the early 1950's, with the arrival of high-speed computing equipment, that the horizons for application of simulation were broadened to the point where it became available and practical for the analysis of engineering, business, and behavioral systems. Since that time simulation has been applied in such diverse areas as:

The Analysis of Air Traffic Control Systems
The Analysis of Large-Scale Military Operations
Communication Systems Analysis
Job-Shop Scheduling
Analysis of the U.S. Economy
Production Planning and Inventory Control
Determination of Manpower Requirements
Instructional Modeling for Higher Education
Energy Supply and Demand Analysis
Competitive Market Analysis
Housing Market Analysis
Transportation Planning
Financial Investment Analysis
Man-Machine Interface
Corporate Planning

Advantages and Disadvantages of Mathematical and Simulation Modeling. Since simulation modeling and mathematical modeling are the most frequently employed modeling techniques applied to support the managerial decision process, the advantages and disadvantages of each will be expressed in relation to the other. Perhaps the primary advantage of simulation over mathematical modeling is its relative simplicity. First the system may be sufficiently complex to defy a complete mathematical description while being amenable to representation by a simulation model. Second, the level of mathematical sophistication and training required for the development of complex mathematical models is generally greater than that required for development of a corresponding simulation model. Thus, while the system may be amenable to mathematical analysis, the level of sophistication required for the analysis may be beyond that of the analyst while he/she may well have the background and training required for development of a simulation model. In summary then, the relative advantages of simulation are versatility and simplicity.



Fundamental Operation of a Time Dependent System Simulation Model

Figure 1

While simulation is a relatively simple and versatile technique for the analysis of complex systems, it is not without its disadvantages. Today most simulation models are executed on digital computers. As the complexity of the system grows the execution time required to obtain meaningful results can increase rapidly. Hence, systems analysis through simulation can become expensive. On the other hand, a corresponding mathematical model, given that it can be developed, can generally be executed on a digital computer much more rapidly and therefore at less expense. In addition, when system optimization is called for it can often be achieved directly through analytic techniques where such solutions are not available in the case of simulation.

Since most simulation models attempt to capture at least some of the random variation present in the real world system, the output of the simulation model will include random components. The random nature of the output of the model frequently clouds the precision with which the results can be interpreted, leading to inconclusive analyses. On the other hand, the output of a mathematical model is deterministic providing the analyst with a more precise basis for the interpretation of results.

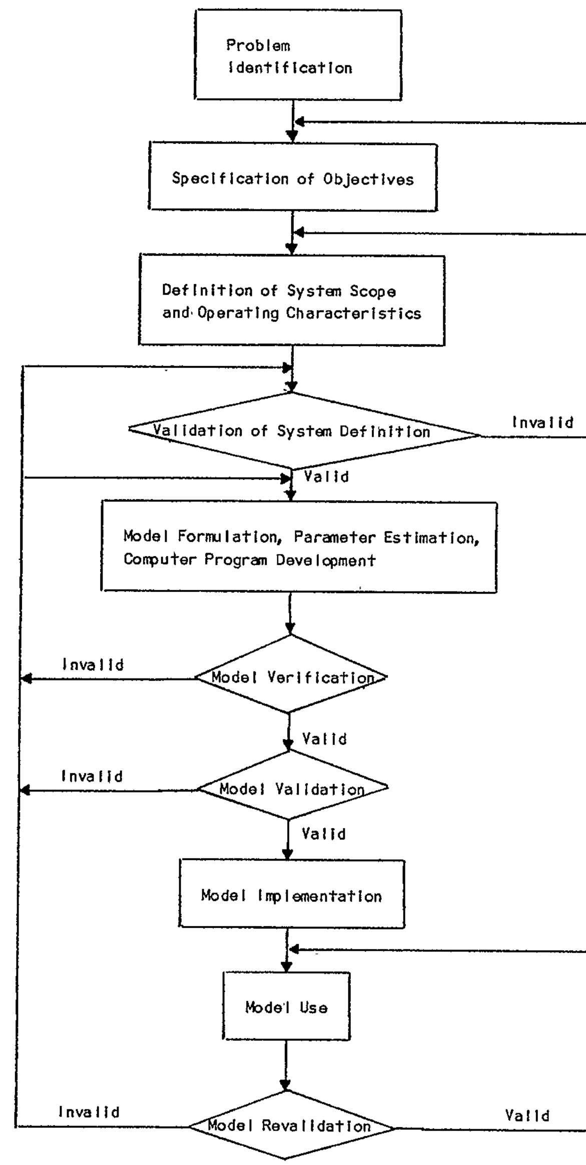
SYSTEMS ANALYSIS THROUGH SIMULATION

In many respects the steps necessary for the analysis of a system through simulation are the same as those taken when using any other modeling approach. These steps can be summarized as follows:

1. Identification of the problem
2. Specification of the objectives of the analysis
3. Definition of the scope of the system analyzed
4. Definition of the operational characteristics of the system and its interaction with its environment
5. Validation of the definition of the scope and operation of the system
6. Formulation of a system model
7. Estimation of the parameters of the model
8. Development of required computer programs
9. Preliminary model validation (verification)
10. Model validation (comparison of model and system results)
11. Model implementation and use
12. Periodic model revalidation

A procedural summary of the steps usually taken in analyzing a system are summarized in the flowchart in Figure 2. While the sequence of steps indicated may not be completely exhaustive nor in the proper chronological order in all cases, this outline may be used as a rough guide for the analysis of most systems problems.

While a complete treatment of systems analysis is beyond the scope of this discussion, a brief discussion of the steps outlined above as they relate to simulation analysis is in order. To resolve any problem one must first understand the nature of the problem. Problem situations are recognized by the symptoms which they display. While in some cases treatment of the symptoms may suitably arrest the problem, this solution is often unsatisfactory. Thus the analyst is likely to be more interested in identification of the root cause of the problem and the discovery of means to resolve the problem itself rather than simply treating its symptoms.



Flowchart for Systems Analysis
Figure 2

Once the problem has been identified the analyst must outline a procedure which will hopefully lead to its resolution. To properly analyze the problem and develop procedures for its resolution, the objectives to be achieved as a result of the analysis must be specifically identified. In addition criteria should be defined whereby the degree to which those objectives are achieved can be determined. For example, the objective of the analysis of a production facility might be to reduce downtime at a given machine center. However, a solution which purports to achieve this objective carries little credibility unless the degree of achievement can be measured by projecting the reduction in downtime which will result from implementation of the recommendations resulting from the analysis.

The development of a model must be founded upon specific definition of the system modeled, a thorough understanding of the interaction of the elements of the system and the interaction of the system with its environment. To this end the analyst should pursue a thorough investigation of the system to identify its purpose, operational characteristics, and obtain data describing the behavior of the system under current and past operating conditions. This exercise should be followed by validation of his/her understanding of system operation. This usually requires the participation of others more familiar with the system and its operation than the analyst. At this point the analyst is usually in a position to initiate model development. The model developed at this point must be considered preliminary since it represents the analyst's initial perception of system behavior.

Inherent in every simulation model are certain parameters whose values must be estimated from real world data. For example, to simulate the behavior of an inventory control system one must first know something about the shape and values of the parameters of the demand distribution. Following parameter estimation, and in some cases concurrent with it, is the development of a computer program for execution of the simulation model.

Validation of the simulation model at this point is a two stage process. The first step in this process is to make sure that the simulation model is functioning in the manner intended by the analyst and is often referred to as model verification. This exercise consists largely of a logic check of the computer program developed for execution of the simulation model. The second stage of validation presents a much more difficult problem. Having ascertained that the model is functioning in its intended manner, the analyst must determine whether or not the intended functioning of the model conforms to reality. Where the simulation model is used to represent the operation of an on-going system, the results of the simulation model can often be compared with those achieved by the real world system. However, even when this comparison is favorable there is no guarantee that the simulation model will function in a manner representative of the real world system under conditions which have not yet been experienced. The problem inherent at this stage of validation is compounded when the simulation model is intended to describe the functioning of a system which is not in existence at the present time but which is contemplated at some time in the future. In this case there is no system available which can be used to check the results of the model. In the final analysis then, complete validation of a simulation model is usually not possible.

The final stage in the analysis of a system is model implementation which leads to the specification and execution of the simulation experiments to be carried out and the analysis of the results of those experiments. The design of simulation experiments consists of specifying the conditions under which the simulation will be executed and the number of simulation runs, replicates, to be executed under each of the conditions specified. The set of conditions under which the simulation will be carried out will be dictated, to a large extent, by the objectives of the analysis. Ideally the analyst would hope to simulate the operation of the system under investigation under every condition which might be anticipated in the future. However time and budget limitations may require a compromise in this regard.

To obtain a measure of the variability and assess the precision of the results achieved under any set of conditions, the analyst would normally choose to replicate the simulation experiment under each set of conditions. The essential question to be answered here is how many replicates are necessary to achieve acceptable precision. In general, the precision of the estimate of a measure of system performance is improved by increasing the number of replicates of the simulation experiment. However, as the number of replicates is increased the cost of executing the experiment will increase proportionately.

The success of validation efforts is predicted on the conformance of model behavior to system performance. In the course of model development the most convincing basis for validation is data describing recent system behavior. However, since close conformance of model and system behavior on this basis offers no guarantee of continuing conformance in the future, validation should be viewed as an on-going process throughout the life of the model. With this in mind, when the model is implemented it may be useful to schedule validation exercises at specified time intervals in the future; at the same time specifying the system data required for the validation effort and the manner in which those data should be collected. One consequence of anticipating the need for continued revalidation is that the system data collected usually provide the basis for a more complete validation effort than was possible during model development.

SIMULATION MODEL DEVELOPMENT

Perhaps the simplest way to present the basic concepts of simulation modeling is by example. Consider a sampling system for quality control by attributes. Manufacturing lots containing L items are submitted for inspection. The inspection procedure consists of drawing a sample of size n, inspecting each item in the sample, identifying each as either good or bad, recording the total number of defects found, x, and comparing the number of defects identified with a criterion variable called the acceptance number, c. If the number of defects found in the sample is less than or equal to the acceptance number the lot is accepted. Otherwise the lot is rejected. Assume that the objective of the analysis is to determine the proportion of lots which one might expect to be rejected as a result of implementation of the quality control system.

First let us examine how one might determine the proportion of lots rejected by experimenting with the physical system. This could be accomplished by implementing the inspection system defined above and using it for the inspection of M manufacturing lots. Associated with each lot selected is the number of items contained in the lot and the proportion of those items which are defective. For simplicity we will assume that the lot size is constant from one lot to another. However a similar assumption with respect to the proportion of defective items contained in each lot would be unrealistic. Assume that an unknown proportion of defective items will be contained in each manufacturing lot, and that the proportion defective will vary from lot to lot. From each lot a sample of size n is drawn and the sampling procedure already defined is carried out. Repeating this process for a total of M lots record the total number of lots rejected and divide this number by the total number of lots inspected, M, to obtain an estimate of the proportion of lots rejected. A schematic representation of the experiment with the physical system is shown in Figure 3.a.

Now consider the development of a simulation model to conduct a similar analysis. It is a relatively simple task to develop a computer program to execute the steps indicated in Figure 3.a with the exception of definition of the proportion defective for each lot and the execution of inspection of each item included in the sample. Thus to completely define the simulation model a method must be developed for assinging a value to the proportion of defective items included in the lot such that the simulated variation in proportion defective from one lot to another is representative of the variability which exists in the real world system. In addition, complete specification of the simulation model will require the development of a technique whereby each item in the sample is classified as good or bad in a manner descriptive of actual conditions.

The problem of simulating the inspection of individual items in the sample will be treated first. It should be obvious that the proportion of defective items in the lot will influence the number of defective items detected in the sample. Assume at this point that a value has been assigned to the proportion of defective items in the lot, P. If an item is drawn at random from a lot having a proportion of defective items P, then the probability that the item is defective is P. Hence the methodology developed for simulation of the inspection process should have the property that the probability that any item selected is defective is P as it is in the case of the real world system. This is accomplished by using what are known as random numbers. A random number is a random variable which is uniformly distributed on the interval (0,1). Hence, in drawing or generating a random number, each number between 0 and 1 has an equal and independent chance of occurring.

To examine how one would use a random number to determine whether or not an individual item of product is defective, suppose that the proportion of defective items in a lot is 0.05, that is $P = 0.05$. If a succession of items is drawn from the lot one would expect to find that approximately 5% were defective. Now consider drawing a sequence of random numbers. Since these numbers have the property that each value between 0 and 1 has an equal and independent chance of occurrence, one would expect approximately 5% of the numbers drawn to lie on the interval (0.0, 0.05). Thus to simulate the inspection of items, a sequence of n random numbers is drawn and compared to the proportion defective, P. If the random number, r, is less than or equal to P, the item is classified as defective. On the other hand, if r is greater than P the item is considered good. To put the process in probabilistic terms, a 0 will correspond to a bad item and 1 to a good item. The probability of a 0 occurring (bad item) is then P and the probability of a 1 (good item) is 1-P. The cumulative probability distribution for this random variable is shown in Figure 4. By choosing a random number, r, the analyst is actually designating a value of the distribution function. Entering the y axis in Figure 4 at the point designated by the random number, proceed horizontally until the distribution function is intersected. Dropping vertically from the point of intersection yields the value assigned to the random variable. As indicated previously, if r is less than or equal to P, a 0 is generated and if r is greater than P, a 1 is generated. The procedure just described provides a synthetic method of categorizing items as good or bad which has the property that the probability that any item is categorized as defective is the same as the probability that the item would be found defective in the physical inspection process.

The fundamental methodology used to generate all random variables is similar to that described for the inspection process discussed above. Let us apply this approach to the generation of values of proportion defective for those lots for which the inspection process is to be simulated. Proportion defective is a random variable which can assume values between 0 and 1. A typical cumulative distribution function for proportion defective is shown graphically in Figure 5. To generate values of proportion defective one selects a random number, r, and enters the y axis of the distribution function at the point defined by r. The value of proportion defective is obtained by dropping vertically from the intersection of the distribution function to the x-axis.

The reader will recall that the two central problems which existed in developing a simulator for the quality control system discussed here were to identify synthetic means of generating proportion defective in a manner such that the variability in values of proportion defective generated would correspond to those which exist in the real world system and to devise a method of designating sampled items as good or bad. Having developed these two techniques the analyst is in a position to complete the logic for preparation of the final simulation model. A flowchart for the simulation model for this quality control system is shown in Figure 3.b.

In discussing the development of the simulator for the sampling inspection system described above, it was assumed that the analyst had available a source of random numbers and that he/she is able to construct the cumulative probability distribution functions required for generation of the random variables included in the system. Random numbers can be obtained from standard tables or can be generated on a digital computer. In order to construct the cumulative distribution function of a random variable data must be collected from the system under study which indicates the variation which exists in the random variable of interest. While the data collection effort required may be expensive and time consuming, it is a necessary prerequisite to the development of a valid simulation model or any system model for that matter.

RANDOM NUMBERS

As the example cited above indicates, random number generation is an important component of every stochastic simulation model. An essential property of every random number generator is the ability to generate random variables which are uniformly distributed on the interval (0,1). Actually digital methods for generating random numbers are algorithmic and therefore the numbers resulting are generally termed pseudo-random numbers. That is, since the numbers are generated algorithmically they are not actually random. However, were one to compare a set of numbers derived from a reliable digital generator with numbers which were truly random, the distinction between the two sets of numbers would not be apparent. A discussion of algorithmic methods for the generation of random numbers is beyond the scope of this article. However, random number generators are available for most digital computers.

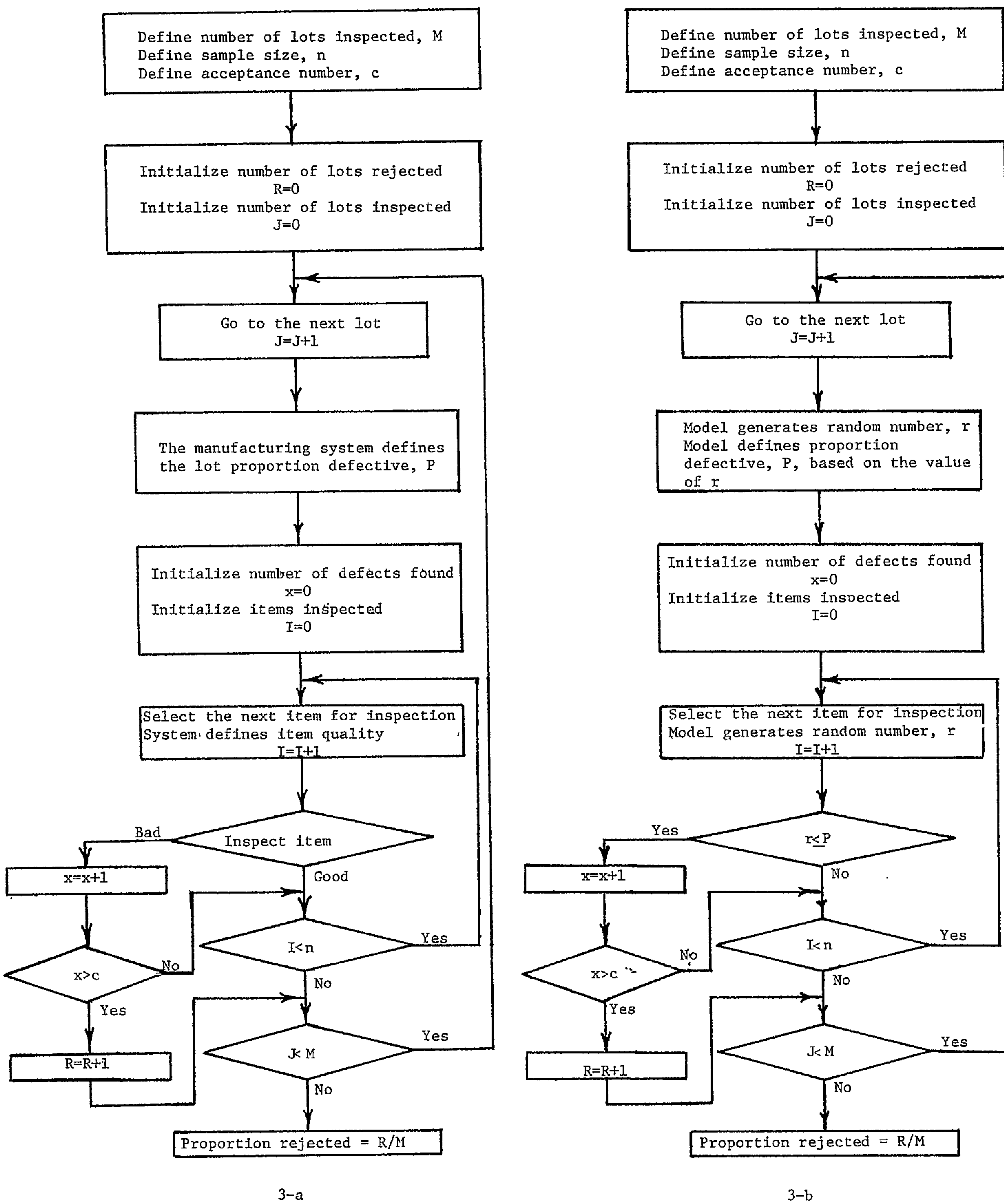
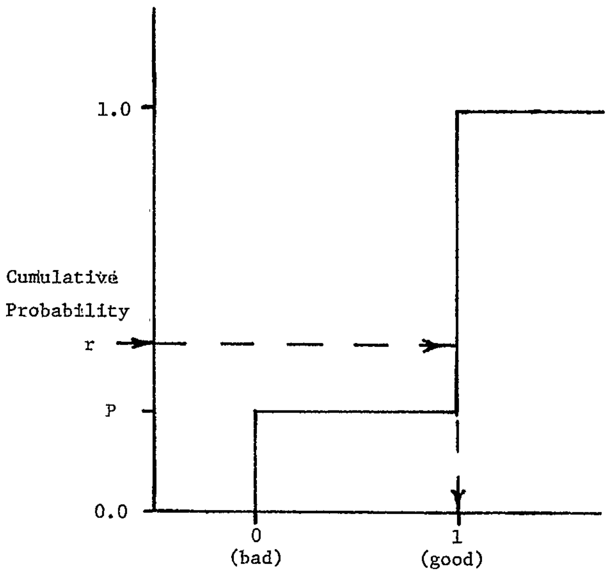


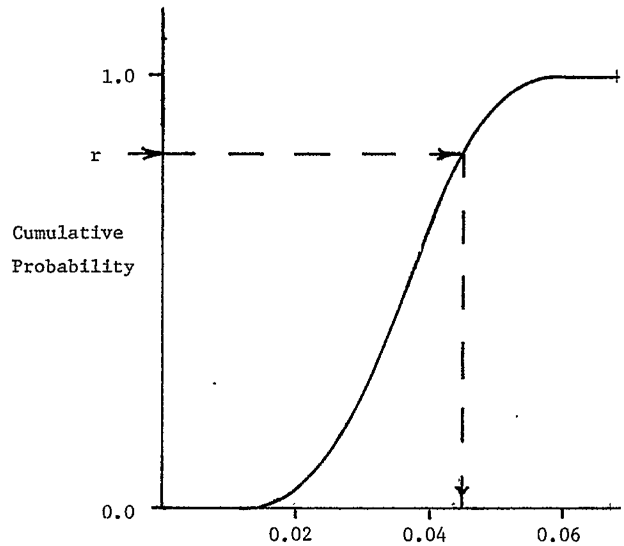
FIGURE 3

Flowcharts for Physical (a) and Simulated (b) Sampling Inspection Systems



Simulation of an insepction process

Figure 4



Generation of the random variable proportion defective

Figure 5

GENERATION OF RANDOM VARIABLES

At the heart of every simulation model is a mechanism for generating values of those random variables which influence the behavior of the system analyzed. The method used to generate values of a random variable is often referred to as a process generator. Fundamentally, a process generator defines a relationship between each possible value of the random variable considered and values of a uniformly distributed random number. The principle underlying the generation of random variables was discussed in the preceeding section and is illustrated graphically in Figures 4 and 5.

While the graphical approach to process generation is acceptable in some cases, it is often useful to define mathematical relationships which simplify the process. The reader will recall that in the graphical approach we selected the value of a uniformly distributed random number which was in turn used to define a specific value of cumulative distribution function of the random variable to be generated. Let r be the value of the random number and let F(x) be the value of the distribution function such that

$$F(x) = r \tag{3}$$

where r and F(x) lie in the interval (0,1). The problem at hand is to find the value of the random variable x which satisfies (3). That is, x is related to r through (3) and we must find an inverse relationship of the form

$$x = h(r) \tag{4}$$

To illustrate how this is accomplished, assume that x is exponentially distributed with the probability density function given by

$$f(x) = \lambda e^{-\lambda x}, \; 0 < x < \infty \tag{5}$$

The distribution function of x, F(x), is given by

$$F(x) = 1 - e^{-\lambda x} \tag{6}$$

By generating a value of r the analyst is simply specifying a particular value of F(x). That is

$$\begin{aligned} r &= F(x) \\ &= 1 - e^{-\lambda x} \end{aligned} \tag{7}$$

Solving for x in terms of r

$$x = -\frac{1}{\lambda} \ln(1-r) \tag{8}$$

Thus we have a simple mathematical expression which will define a unique value of the exponential random variable, x, for each value of r generated.

The technique described here and that used in the quality control example are applications of the inverse transformation method. Although conceptually simple, for some random variables its application may prove tedious. Alternatives to the inverse transformation method include

Composition
Convolution
Acceptance/Rejection

SIMULATION LANGUAGES

The complexity of the systems usually analyzed through simulation generally requires that the analysis be carried out on a digital computer. In such cases the analyst must translate the simulation model into a medium which can be interpreted by the computer. This translation is accomplished through a programming language. Translation of the simulation model can be accomplished through a general purpose language or a special purpose language. General purpose languages such as FORTRAN, BASIC and PL/I provide the programmer with a tool for the analysis of a virtually limitless number of problems, of which simulation is only one. On the other hand, special purpose simulation languages are designed to address problems to be analyzed through simulation although the variety of simulation problems which can be handled by these languages is quite broad. Included in the category of special purpose simulation languages are GPSS, SIMSCRIPT, SLAM, DYNAMO and SIMULA.

Perhaps the principle advantage of general purpose languages lies in the fact that one of these languages is probably already known to the programmer. In addition these languages provide the analyst with a maximum of flexibility in the design of his/her analysis. However, because special purpose languages are oriented toward the specific application of simulation, the programming time required for translation of the model is generally less than that required in the case of general purpose languages since the time keeping mechanism and many of the subroutines normally required in any simulation model are built in. In addition, the structure of special purpose languages will often help the analyst to formulate the model. However, in using a special purpose language the analyst is restricted to a prescribed output format and increased computer running time.

SUMMARY

Simulation has proved to be an effective and versatile modeling technique for the analysis of complex interactive systems in both the private and public sectors. In addition simulation offers the advantage of relative simplicity in model construction as compared with mathematical modeling.

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