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EVALUATION AND COMPARISON METHODS FOR CONFIDENCE INTERVAL PROCEDURES

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RESEARCH SUMMARY

The major concern of this research is to develop concepts and methods to evaluate and compare confidence interval procedures (CIP). A CIP is a function parameterized by a scalar constant $\alpha \in [0,1]$, mapping a realization of continuous or discrete stochastic process $\{X(t);\ t \in T\}$ to the real-valued random interval $(L_\alpha,\ U_\alpha)$.

The primary criterion to evaluate the CIP's is the comparison between the nominal coverage, $1 - \alpha$, and the actual coverage of a known parameter θ_0. One minus this probability is analogous to type I error in hypothesis testing. This coverage can be generalized to a coverage function, $\beta(\alpha,\theta) = \Pr\ \{L_\alpha \leq \theta \leq U_\alpha\}$, $0 \leq \alpha \leq 1,\ -\infty < \theta < \infty$. When $\theta \neq \theta_0$, this probability is analogous to type II error; the lower the probability the better the procedure.

In addition to the coverage, the mean and the variance of the interval width are measures of the accuracy and variability of the point estimator of θ_0, respectively.

Both coverage probabilities and properties of interval width are special cases of expected loss. The loss function $\mathcal{L}[\theta_0,\ (L_\alpha,\ U_\alpha)]$ represents the loss incurred by the interval (ℓ,u) including the points other than θ_0. The expected loss is defined by,

$$E\{\mathcal{L}[\theta_0,(L_\alpha,\ U_\alpha)]\} =$$
$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{L}[\theta_0,(\ell,u)]\ f_\alpha(\ell,u)\ d\ell\ du$$

The coverage function, $\beta(\alpha,\theta)$, is obtained from the expected loss of the 0 - 1 loss function. The mean and variance of the intervals are obtained analogously by considering the loss functions $(u - \ell)$ and $[(u - \ell) - E(U_\alpha - L_\alpha]^2$, respectively.

Coverage contours are the pictorial representation of the coverage function, $\beta(\alpha,\theta)$. The expected interval width and the coverage appear on one figure. Bounds on the variance of interval widths can be obtained from the coverage function.

The joint density function, $f_\alpha(\ell,u)$, is fundamental to the study of the properties of confidence intervals. The performance of a CIP on a single realization is soley a function of the interval (ℓ,u) arising from its application. Since the realization is random, fundamental evaluation of a CIP, which requires the knowledge of $f_\alpha(\ell,u)$, must be based on performance over many replications. A natural way of estimating $f_\alpha(\ell,u)$ is scatter diagrams. Various scatter diagrams, which exhibit the variability of confidence intervals, are developed to supplement the coverage contours.

Reference

K. Kang and B. Schmeiser, "Evaluation and Comparison Methods for Confidence Interval Procedures", Research Memorandum 82-5, School of Industrial Engineering, Purdue University, 1982.