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WALSH FUNCTIONS FOR SPECTRAL ANALYSIS

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The identification of significant parameters or factors is an important area of operations research and statistics. Until recently it has been too expensive for many computer simulation studies, which Hillier and Lieberman state is one of the major shortcomings of simulation. The traditional approach to this problem is to make separate computer runs for each of many different factor values. The basic experimental unit is the computer run. Although the number of runs required may be reduced using screening designs, it increases as a product of the number of factors. This approach becomes prohibitively expensive in terms of both user and computer time for all but the simplest of models.

Schruben and Cogliano addressed this problem utilizing an entirely different approach. Normally one views a parameter as a fixed, possibly unknown, attribute of the system. However, in a computer simulation the experimenter has complete control of the model and can alter parameter values during the run, hence the terms parameter and factor are used interchangeably. Schruben and Cogliano proposed varying the parameters sinusoidally during a run. Each parameter is assigned a unique frequency, and spectral estimators are used to analyze the system output at the different frequencies. After performing a suitable statistical test, if the power spectrum is not significantly different from zero at a given frequency it is concluded that the system is insensitive to the parameter which was assigned that frequency. By analyzing the spectrum instead of just the assigned frequencies, one can detect non-linear effects through a relatively simple set of relations.

The advantage of their approach is that analysis is moved to the frequency domain. The output time series is represented using trigonometric functions as a linear algebraic basis. The experimental unit becomes a frequency band, and a single run of the simulation contains many almost independent frequency bands. The number of runs required is greatly reduced.

One limitation of the Schruben/Cogliano procedure is that it can only be used to evaluate continuous parameters. This problem can be removed by using the same basic procedure, but choosing a different set of functions as a basis. We have investigated several sets of discrete functions, of which the most promising were Walsh functions, which are named after their inventor. Walsh functions are two-valued functions which have the desirable property of forming a complete orthogonal basis. The orthogonality allows us to perform a spectral decomposition, and gives independence to the resulting spectral estimators. The completeness property means that Walsh functions form a legitimate basis for any set of functions in general.

Walsh analysis has several nice properties aside from allowing the experimenter to work with discrete parameter spaces. There is a Fast Walsh Transform (FWT) which is analogous to the Fast Fourier Transform (FFT) used in more traditional spectral computations. The FWT is like the FFT in that it requires $n \log_2 n$ operations to evaluate, where n is the number of observations available. However, in the case of the FWT those operations are additions or subtractions, which are substantially faster for digital computers than the complex multiplications required by the FFT. Furthermore, addition and subtraction are computationally stable operations for finite bit arithmetic, while multiplication and division are not.

We will show in the oral presentation that if the error term of a stochastic system is additive discrete white noise then the Walsh spectral estimator has a χ^2 distribution with 2 degrees of freedom. This enables the experimenter to perform statistical tests to accept or reject the hypothesis that a spike in the spectrum is due to random fluctuation.

One problem remaining in the use of Walsh analysis is that it is sensitive to time lags in the model. We have not yet determined an analytical method for predicting the effect that time lags have on the spectral output. Further research is needed in this area.

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