Proceedings of the 1983
Winter Simulation Conference
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CLOSED LOOP METHODOLOGY APPLIED TO SIMULATION

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By using modern system theory to model physical systems, questions concerning the design, analysis, test, and evaluation of such systems can be answered. This tutorial will compare the traditional open loop methodology of simulation to the modern closed loop methodology. Advantages and disadvantages of each methodology will be discussed. Systems will be typed as being deterministic or stochastic. The use of partial differential equations, ordinary differential equations, and algebraic equations to describe the system will be discussed. In order to solve system questions, the need to solve the estimation, identi- fication, and control (EIC) problems will be motivated. The use of the EIC solutions with respect to the traditional open loop methodology (OLM) and the modern control loop methods (CLM) will be examined.

1. INTRODUCTION

The analysis of any physical system must begin by characterizing the specific process to be modeled. This characterization must consider (1) the system type, (2) governing equations of the process, and (3) system questions to be answered. Table 1 illustrates these notions.

o Types of Systems
Deterministic, i.e., noise-free system
Stochastic, i.e., systems with noise
o System Description
Ordinary differential equations
Partial differential equations
Algebraic Equations
o System Questions to be Answered
Solution to governing equations
Estimation
Identification
Control

Table 1: Process Characterization

Systems may be typed as being either deterministic or stochastic. A deterministic system is defined as one that incorporates no uncertainty and the stochastic type as one that includes uncertainty in the model. True systems may be represented by physical laws expressed by partial differential equations (PDE), ordinary differential equations (ODE), and algebraic equations (AE). Hence, the governing equations of the process must be given by PDE, ODE, AE, or a combination of the three. In the case of the deterministic type system, the solutions may be easily obtained. The stochastic system presents additional concerns.

The solution of a stochastic systems may be obtained by defining and solving the estimation, identification, and control (EIC) problems associated with the given process.

Open-loop methods (OLM) and closed-loop methods (CLM) are considered. OLMs are described generally by a system that lacks feedback from the operations that are occurring. An OLM is presented in Figure 1.

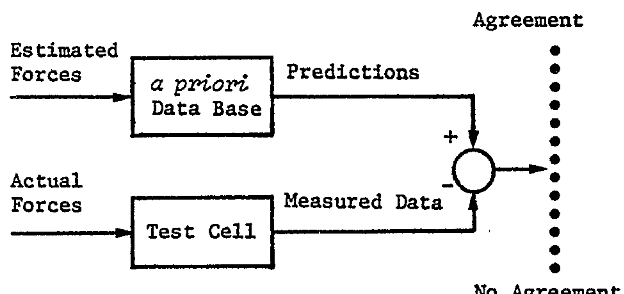

Figure 1: Open-Loop Method

The open-loop methodology in Figure 1 uses an a priori data base to provide predictions of outputs which are compared with measured data from the test cell. Results can range from complete agreement to no agreement at all. In addition, the confidence level of results of a given test is typically low until a large statistical data base indicates the attributes of the samples. Typically, as one begins to accumulate test data, the predictions are adjusted to accommodate these data. This "knob tweaking" is usually conducted by methods that are far from being mathematically rigorous.

Because OLMs have considerable shortcomings, the need for a more systematic approach has become evident in recent years. Modern system theory provides such an approach based on CLMs that form the bases for most applications where feedback is used to control the system. The CLM is shown in Figure 2 by using adaptive procedures that provide a model with quantified confidence levels. Modern system theory recognizes the potential differences in predictions and results that can be attributed to forcing function uncertainty, incorrect estimates of constituents in the governing equations, and the possibility that the model order is insufficient to describe these systems. The adaptive processor design attempts to accommodate these uncertainties and to systematically drive the difference between predictions and test data to a minimum.

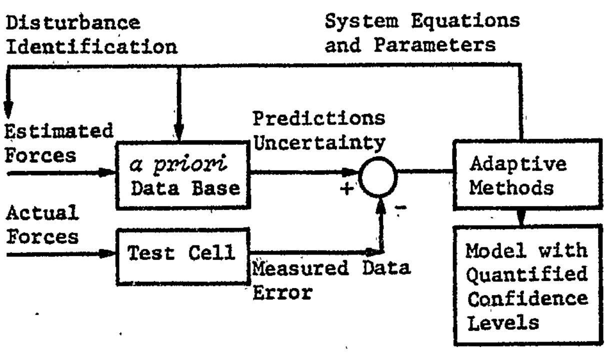

Figure 2: Closed-Loop Method

Having stated that OLM and CLM are essentially techniques for controlling the comparison error, the discussion will now turn toward the theoretical aspects required to obtain the systematic results of the CLM. This is introduced by presentation of the thesis:

> "Control of the error which results from the comparison of data obtained from the true system to that produced by an a priori data data base will result in:
>
> a) obtaining a classification set, which implies the system can be modeled. This in turn implies that the system equations can be solved, or
>
> b) denial of the classification set, which implies that the system cannot be modeled."

Acceptance of this thesis implies that the solution of the control problem is imperative to model the system.

In presenting the thesis, the term "classification set" has been introduced. Since the set has significant impact, it needs further clarification at this time.

The notion of a set is generally defined in mathematical terms. In particular, this set may be given as the set that contains the following elements: independent variables, dependent variables, coefficients of the governing equations, and the sensor output.

Since this definition for the classification set holds for the propagation of a signal through any system, the elements of this set will be obtained from the equations that govern the evolution of a specific phenomena. Prior to doing this, the relationships of estimation, identification, and control (EIC) to the classification set will be given. For example, a solution to the estimation problem will yield estimates of the independent and dependent variables. Similarly, the solution to control encompasses the solution of both estimation and identification, in addition to providing an estimate of sensor output.

Before further clarification of the classification set is feasible, the basic elements of the physical process must be considered. To do this for the OLM, a lower layer of Figure 1 must be produced. This second layer is shown in Figure 3. From this figure, two basic elements may be defined as being (1) the subsystem under evaluation, and (2) a measurement device. The subsystem under investigation produces a measurable set, M, and the measurement device acting on this set produces the output, z. In addition to illustrating the basic elements, Figure 3 shows the basis for their physical makeup.

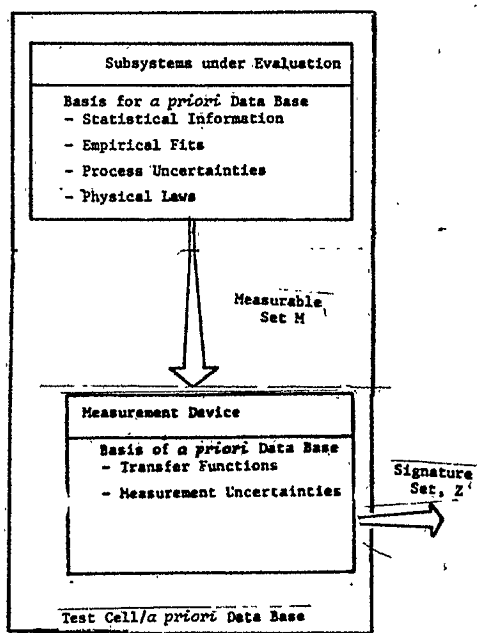

Figure 3: Second Layer of Physical Process

At this point, the subsystem under evaluation can be represented in state variable format by:

(1) $\underline{x}(k+1) = A(k+1, k)\,\underline{x}(k) + \underline{w}(k)$

where $\underline{x}$ is defined to be a n x 1 state vector; A is the n x n system dynamics matrix; and $\underline{w}$ accounts for random uncertainties in propagation of the state.*

In addition, the measurement device may be modeled by:

$$\underline{z}(k+1) = \underline{m}(k+1) + \underline{v}(k+1)$$

where the sensor output, $\underline{z}$, is given to be a m x 1 vector sum of the measurement set, $\underline{m}$, and the random sensor uncertainty, $\underline{v}$. Since the sensor output can be considered be a linear combination of the state, x, the expression:

(2) $\underline{z}(k+1) = H(k+1)\,\underline{x}(k+1) + \underline{v}(k+1)$

also may be used to represent the sensor output.

By mathematical representing the system via equations (1) and (2), further clarification of the classification set is possible. For example, the independent and dependent variables are defined by $\underline{x}$; the coefficients of the system are comprised of the system dynamics A, measurement matrix H, and variances on $\underline{w}$ and $\underline{v}$ respectively; the sensor output is given by the vector $\underline{z}$.

2. APPLICATION

As shown in Table 2, the application of the CLM technique can be used to address a wide range of problems. For example, consider the application of the methodology to the attitude determination problem. This problem may be considered to belong to a set of problems associated with the broad category of guidance and control. On the other hand, application of CLM to the combat vehicle support plan illustrates its use to a problem in operational research. This section will detail the first problem.

- o Attitude control systems
- o Attitude determination
- o Design of a ground control station
- o Control of large space structures
- o Robotics development
- o Magnetic dipole discrimination
- o Combat vehicle support plan
- o Calibration of system/sensor
- o Heat diffusion identification
- o IR signal propagation
- o Soil-structural dynamics
- o Acoustic damping
- o B-52 navigational system assessment

Table 2: CLM Applications

Consider the problem of attitude determination of a spinning satellite. The equations which govern the attitude of the spacecraft in the presence of external torque are given by the vector equation

(3) $\frac{d\vec{H}}{dt} = \vec{T}$

where $\vec{H}$ is the total angular momentum vector and $\vec{T}$ is the vector sum of the external torques. Since spacecraft orientation is concerned only with directional changes of H and not magnitude, equation (1) can be replaced by the two non-dimensional scalar equations

(4) $\frac{dx_1}{dt} = \frac{T_1}{H_o} \qquad \frac{dx_2}{dt} = \frac{T_2}{H_o}$

where the left hand side of the equations represent time translational changes of x_1 and x_2 respectively, T_1 and T_2 are respective components of $\vec{T}$ directed along x_1 and x_2, and H_o is the magnitude of the nominal momentum. Assume that the time derivative can be approximated by

$$\frac{dx}{dt} = \frac{x(k+1) - x(k)}{\Delta t},$$

where x(k+1) is the value of x at time k+1, x(k) the value of x at time k, and Δt the time interval between time k+1 and k. Then equation (4) can be written as

(5) $\underline{x}(k+1) = \phi(k+1,1)\,\underline{x}(k) + \theta(k+1,k)\,\underline{u}(k)$

where the transition matrix, ϕ, is of dimension 2x2; the matrix torque modifier, θ, is of dimension 2x2; and

$$\underline{x}^T = (x_1\ x_2);\ \underline{u}^T = (T_1\ T_2)$$

Having discussed the equations of motion, consider the measurement source, i.e., the measurement is given as a nonlinear function of the state plus noise

$$\underline{z}(k+1) = \underline{f}(\underline{x}(k+1)) + \underline{v}(k+1).$$

Expansion of the function $\underline{f}$ in a Taylor series about a nominal, $\underline{x}^*$, yields

$$\underline{z}(k+1) = \underline{z}^*(k+1) + H(k+1)\delta\underline{x}(k+1) + \underline{v}(k+1)$$

or

(6) $\underline{r}(k+1) = H(k+1)\delta\underline{x}(k+1) + \underline{v}(k+1)$

where the residuals, $\underline{r}$, are defined as $\underline{z}$ minus $\underline{z}^*$, and H is the partial of the vector function $\underline{f}$ with respect to $\underline{x}$.

Since propagation of the perturbed state must obey (5), the system to be considered is described by the residuals, (6), and the perturbed state x, i.e.,

(7) $\delta\underline{x}(k+1) = \phi(k+1,k)\delta\underline{x}(k) + \theta(k+1,k)[\underline{u}(k) + \underline{w}(k)]$

The sequence, $\underline{w}(k)$, describes the random uncertainties associated with T_1 and T_2. These uncertainties are characterized by the covariance matrix, Q(k). Errors in the torques are represented by $\underline{u}(k)$.

*The notation (k) implies that the equations are evaluated at time t_k.

At this point, one can apply the Kalman filter equations to the system described by (6) and (7), i.e., a sequential estimate of the perturbed state, $\delta\underline{x}$, can be obtained utilizing the residuals $\underline{r}$. Having obtained $\delta\underline{x}$, the absolute value of $\underline{x}$ is given by

$$(8)\quad \hat{\underline{x}}(k+1) = \underline{x}^*(k+1) + \delta\underline{x}(k+1)$$

The state obtained in (8) can now be compared to the state obtained by integration of the nominal torques, i.e., equation (5). If there is poor agreement between $\underline{x}$ and $\hat{\underline{x}}$, one would expect to see trends in the residual data. Trends in the residuals are illustrated in Figures 4 and 5. Since the residuals should have noise characteristics, these trends are undesirable.

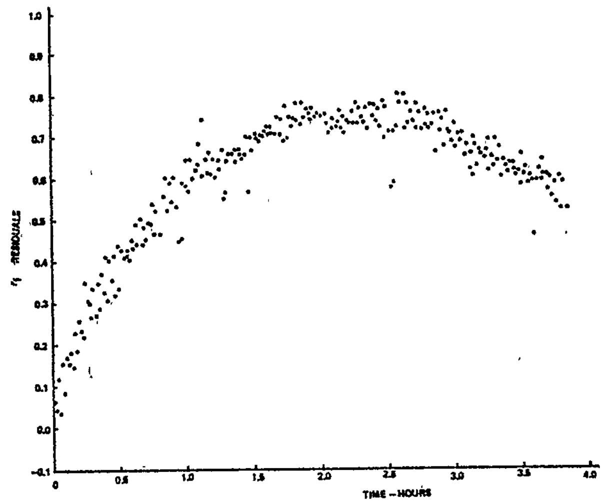

Figure 4: Residual r_1 Resulting From Nominal Attitude Vector

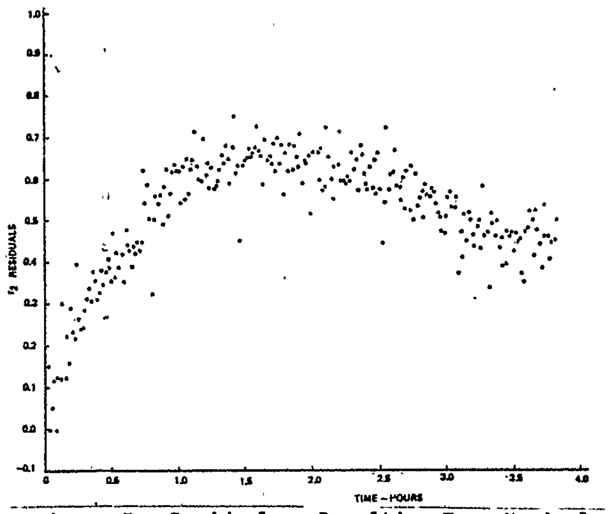

Figure 5: Residual r_2 Resulting From Nominal Attitude Vector

The reason for poor agreement between (5) and (8) is due to the absence of the perturbed torque, $\delta\underline{u}$, from (5), i.e., the nominal state given by (5) must be modified by (7) to yield the corected state given by

$$(9)\quad \underline{x}(k+1) = \phi(k+1)(\underline{x}^*(k)+\delta\underline{x}(k)) + \theta(k+1,k)(\underline{u}^*(k) + \delta\underline{u}(k) + \underline{w}(k))$$

where $\underline{x}^*$ and $\underline{u}^*$ are nominal values of the attitude and torques respectively. However, utilization of (9) requires that $\underline{u}$ be known.

The perturbed value of the torques, $\underline{u}$, can be otained from minimization of an error function, E. This cost function is given by

$$(10)\quad E = (\hat{\underline{x}}(k+1) - \underline{x}(k+1))^T P^{-1}(\hat{\underline{x}}(k+1) - x(k+1) + \underline{u}^T(k)Q^{-1}\,\underline{u}(k)$$

where the matrix, P, is the covariance matrix of $\underline{x}$ obtained via the Kalman filter algorithms, and $\underline{Q}$ is the covariance of the noise sequence, $\underline{w}$.

Minimization of (10) results in

$$(11)\quad \delta\underline{u}^T(k) = ((\delta\underline{x}^T(k+1) - \delta\underline{x}^T(k)\,\phi^T(k+1,k))P^{-1}\,\theta(k+1,k))(\theta^T(k+1,k)P^{-1}\,\theta(k+1,k) + Q^{-1})^{-1}$$

Having obtained a value for the bias torques, (11) can then be inserted into (9) to yield the new integrated state.

The previous discussion addressed torques of a general nature. Specifically, the main source of the perturbing torques which tend to alter the orientation of a satellite vehicle result from: 1) gravity gradient torques, 2) solar radiation pressure, and 3) interaction of the spacecraft's magnetic moments with the magnetic field of the earth. Torques resulting from atmospheric pressure are negligible compared to the above mentioned forcing functions.

The torques, T_1 and T_2, discussed previously are given explicitly as

$$(12)\quad T_i = T_{STi} + T_{MAGi} + T_{GGi} \qquad i = 1,2$$

where T_{ST} = linear combination of the solar torques
T_{MAG} = sum of magnetic torques
T_{GG} = gravity gradient torque.

The solar and magnetic torques can be written as

$$(13)\quad T_{ST1} = a \cdot T(1,1)+b \cdot T(2,1)$$
$$(14)\quad T_{ST2} = a \cdot T(1,2)+b \cdot T(2,2)$$
$$(15)\quad T_{MAG1} = c \cdot T(3,1)+d \cdot T(4,1)+e \cdot T(5,1)$$
$$(16)\quad T_{MAG2} = c \cdot T(3,2)+d \cdot T(4,2)+e \cdot T(5,2)$$

where the variables, a and b, are nominal parameters which modify the respective solar torques.

Magnetic torques are computed by

$$(17)\quad T = M \times B$$

where

$$M = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c \\ d \\ e \end{pmatrix}$$

The angle, θ, is the azimuth which relates a platform frame to the body reference frame. This platform reference frame is necessitated by the fact that the vehicle magnetic moments, c, d, and e, are measured in a platform frame. The magnetic field of the earth is represented by the vector, B. With some manipulation, (15) and (16) can be obtained from (17).

The gravity gradient torques, T_{GG1} and T_{GG2}, are obtained from

$$(18) \quad T = \frac{\mu \cdot 3 \vec{1}_r \times (I \cdot \vec{1}_r)}{r^3}$$

where μ = gravitational constant
r = magnitude of the position vector from the center of the earth to the spacecraft
$\vec{1}_r$ = unit vector directed along the position vector
I = inertia dyadic of the vehicle

Having defined the perturbing torques, the problem can be manipulated into the form given by (5).

Examination of Figures 4 and 5 imply that the nominal torque profiles currently employed are incorrect. These torques can be corrected utilizing an estimate of the attitude, and minimizing the weighted difference between the estimate and its corresponding nominal. These new torques can then be inserted to yield a modified attitude. Figures 6 and 7 yield a comparison of the estimated spacecraft orientation with its nominal. Figures 8 and 9 illustrate the comparison of the corrected attitude with its estimated orientation.

Nominal values for the solar torque modifiers and the magnetic components are given in Table 3. Utilizing these nominals, the error in x_1 was found to be primarily due to an error in T(1,1)--see Figure 10. The discrepancy in x_2 appears to be primarily a function of the error in T(1,2)--see Figure 11.

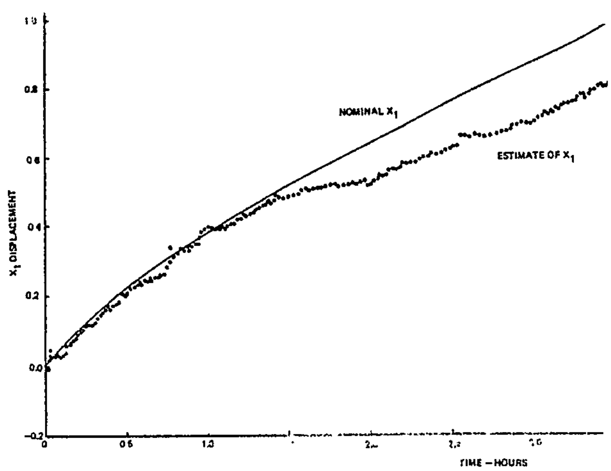

Figure 6: Comparison of Nominal X_1 with the Estimated Value of X_1

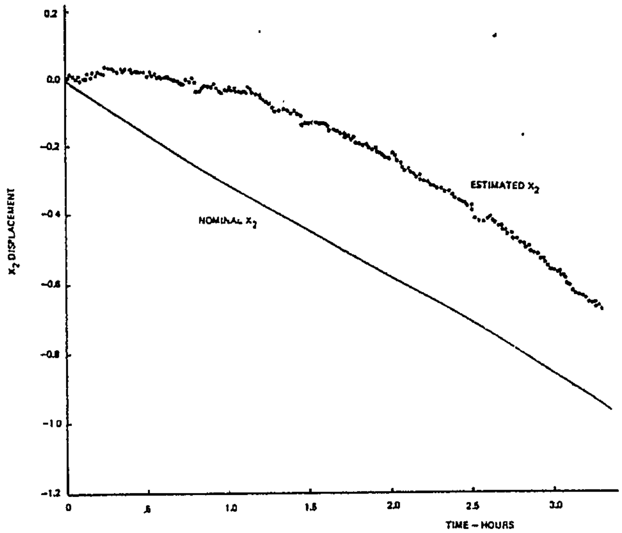

Figure 7: Comparison of Nominal X_2 with the Estimated Value of X_2

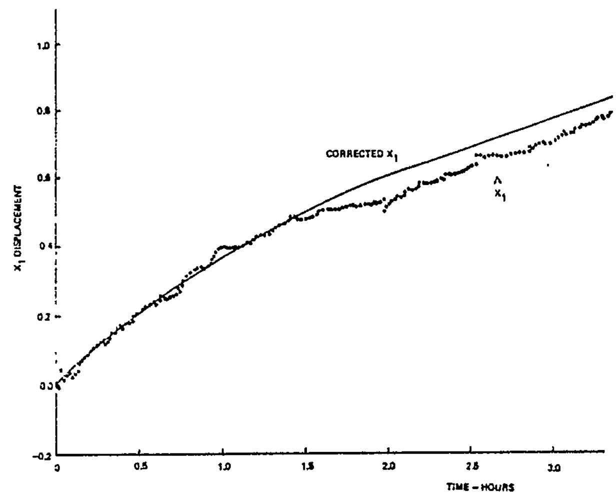

Figure 8: Comparison of X_1 and X_1 After Torque Correction has been Applied

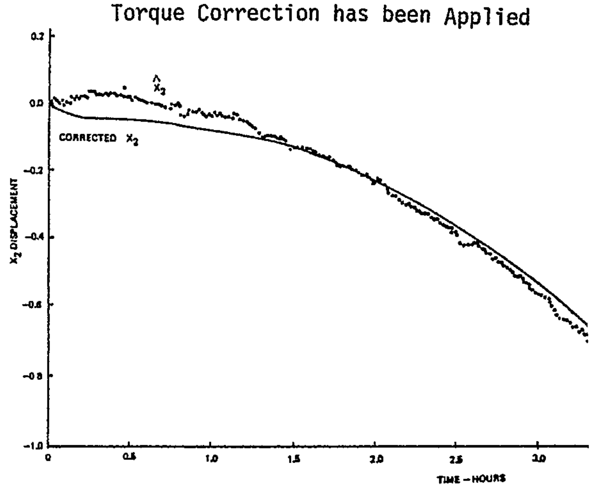

Figure 9: Comparison of X_2 and X_2 After Torque Correction has been Applied

PARAMETER	NOMINAL VALUE
a	1.0
b	1.0
c	200.0
d	-50.0
e	-200.0

Table 3: Nominal Torque Modifiers

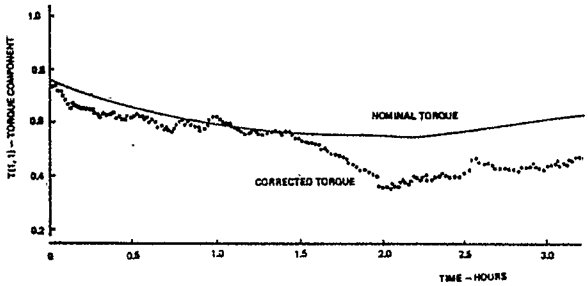

Figure 10: Comparison of Nominal Solar Torque Component T(1,1) with Corrected Value

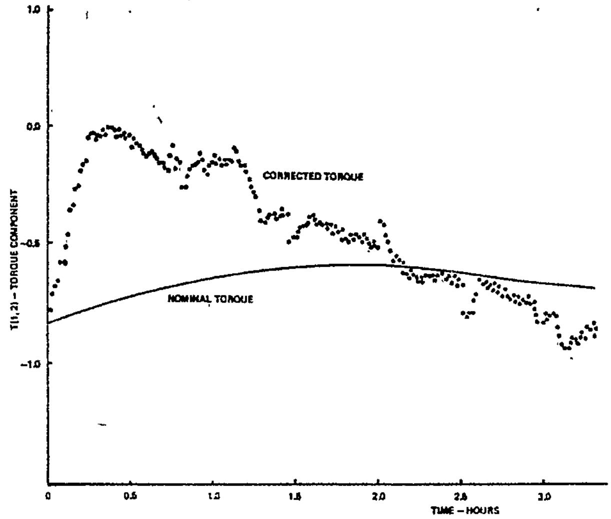

Figure 11: Comparison of Nominal Solar Torque Component T(1,2) with Corrected Value

In conclusion, it can be said that trends are indicative of modeling errors. In this case, trends are due to the modeling of the external torques. This paper illustrates the use of the Kalman filter algorithms to obtain an estimate of the attitude based on measurement information. Having an estimate of the state, the uncertainties in the torque models can be determined in order to minimize the difference between the estimate and the corresponding integrated state. These biases can then be added to the nominal attitude to produce a corrected spacecraft orientation.

REFERENCES

Doebein, EO (1980), System Modeling and Reponse, John Wiley and Sons.

Fermelia, A (1977a), Attitude determination based on erroneous torque models. In: First International Conference on Mathematical Modeling, X.J.R. Avula (ed.), Aug. 29-Sept. 1, 1977, Vol. III, pp. 1511-1524.

(1982b), Closed loop methodology applied to the combat vehicle support plan. In: Proceedings of the Fourth Meeting of the Coordinating Group on Modern Control Theory, H. Cohen, Oct. 27-28, 1982, Oakland University, Rochester, Michigan, pp. 87-104.

(1981c), Adaptive processor definition and design, Technical Report BDM/A-81-168-TR, BMD Corporation, Albuquerque, New Mexico, April 30, 1981, 154 pp.

Lee RCK (1964), Optimal estimation identification and control, M.I.T. Research Monograph 28, Cambridge, Mass.

Meditch JS (1969), Stochastic optimal linear estimation and control, McGraw Hill.