AN EXPANDED OPTIMUM DESIGN OF P CHARTS

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Abstract

Many articles have studied the problem of optimum design of variable control charts (1,3,4,6,7,9,10,11,13), but few authors focused attention on the attribute charts problem (2,8,12).

In this study, an expanded model for the optimum design of the percent defective control chart is approached. Nineteen independent parameters that affect the design are encountered in the model.

The size and frequency of sampling, and the position of the control limits that maximize the long run average net income, are approached by solving the model by the computer.

In order to investigate the effect of each of the independent parameters on the optimum design, 25 examples are solved and conclusions are drawn.

INTRODUCTION

The central line in the p chart is set at p", and the control limits are taken as $p"\pm k\sqrt{p"(1-p")/n}$, where p" is the standard value for the central line taken from past experience, and k is a constant. Samples of size n are taken from the process at intervals h. The various values of p are entered in the chart.

In the presented study, a model giving the long run average net income per unit time is approached. This model is a function of the three parameters, sample size n and its frequency h and multiple k of standard deviations determining the control limits.

The developed model involves 19 independent parameters that affect the net income function.

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FEATURES OF THE MODEL

The main features of the model are:

1. A single assignable cause occurs at random and results in a shift in the process average from p" to $p"\pm\delta\sigma"$. The standard deviation is assumed to be constant throughout.

2. When the chart signals an out of control state, the process is shut down while a search for trouble is carried out and repair is made.

3. In accordance with the waiting time analysis (5), the average time required for the assignable cause to occur will be $1/\lambda$.

4. The probability P that the assignable cause will be detected when it has occurred is $P=\phi+\eta$, where ϕ is that probability for a sample point that falls outside the lower limit, and η is that probability for upper limit.

$$\phi=\sum_{r=0}^{r=np"-k\sqrt{np"(1-p")}-\delta\sqrt{np"(1-p")}} {}^{n}c_{r}\, p^{r}(1-p)^{n-r}$$

$$\eta=\sum_{r=np"+k\sqrt{np"(1-p")}-\delta\sqrt{np"(1-p")}}^{r=n} {}^{n}c_{r}\, p^{r}(1-p)^{n-r}$$

5. When the process is in control, the probability of a sample point falling outside the upper control limit is α_1, and for the lower limit it is α_2.

$$\alpha_1=\sum_{r=np"+k\sqrt{np"(1-p")}}^{r=n} {}^{n}c_{r}\, p^{r}(1-p)^{n-r}$$

$$\alpha_2=\sum_{r=0}^{r=np"-k\sqrt{np"(1-p")}} {}^{n}c_{r}\, p^{r}(1-p)^{n-r}$$

It is assumed that the rate of production is sufficiently high such that the probability of change in the process average during the taking of a sample is negligible.

6. The average time the process will be out of control before a sample point falls outside the control limits is:
$(1/p)-(0.5)+(\lambda h/12)$

7. T the time required to take and inspect a sample and compute the results (the delay in plotting a point) is given by (en+l), where e is the time for sampling and charting which is directly related to the sample size n, and l is a constant term independent of the sample size.

8. D is the average time teken to look for an assignable cause after a point has been found to fall outside the upper control limit, and d is that for the lower limit.

9. G is the average time required for adjustment and repair of the process to bring it back to a state of control when a sample point falls outside the upper control limit. g is the average time required to put the process in a state of better quality than the level of control, when a sample point falls outside the lower limit.

10. The proportion of time, β, the process will be in control is:
$\beta=(1/\lambda)/((1/\lambda)+B)$ where,

$B=ah+(en+l)+(D+G)(\eta/P)+(d+g)(\phi/P)$ and,

$a=(1/P)-(0.5)+(\lambda h/12)$

hence the time proportion during which the process is out of control equals:
$\delta=1-\beta$

11. The average expected number of false alarms per unit time is:

$\beta(\alpha_1+\alpha_2)/h$

12. If T is the cost of looking for an assignable cause when none exists for a point outside the upper control limit, the expected loss per unit time (of operations) due to false alarms will be:

$(\beta\alpha_1 T/h)$

And if t is that cost for lower limit, the expected loss per unit time of operation due to false alarms will be:

$(\beta\alpha_2 t/h)$

13. The average number of times per unit time the process actually goes out of control is:

$\varepsilon=1/((1/\lambda)+B)$

14. If W is the average cost for finding the assignable cause when it occurs for a point outside the upper limit, the average cost per unit time will be:

$(\varepsilon W)(\eta/P)$

And if w is that cost for the lower limit, the average cost per unit time will be:

$(\varepsilon w)(\phi/P)$

15. If F is the average cost for adjustment and repair when a point falls outside the upper limit, the average cost per unit time will be:

$(\varepsilon F)(\eta/P)$

And if f is that cost to put the process in a state of better quality than the controlled level for a point outside the lower limit, the corresponding cost will be:

$(\varepsilon f)(\phi/P)$

16. When the process mean shifts from p" to p"+δσ" due to the occurrence of the assignable cause, the proportion of defective items will be increased. V_0 is that average income per unit time from operation of the process at the standard level p" or below it, and V_1 is that income from the operation at the new level p"+δσ".

17. In the long run, the probability of a sample point falling outside the upper limit is assumed to be equal that probability for a point that falls under the lower limit.

18. The average income per unit time from the occurrence moment of an assignable cause until noticing a sample point falling outside the upper limit and shutting down the process is:

$V_1(ah+en+l)(\eta/P)/((1/\lambda)+B)$

And the income per unit time until noticing a point falling under the lower limit is:

$V_0(ah+en+l)(\phi/P)/((1/\lambda)+B)$

19. The cost per unit time for keeping the control chart is assumed to be given by the simple linear function:

$(b/h)+(cn/h)$

where b is the cost of sampling and charting per sample which is independent of the sample size, and c is the cost of measuring an item and other control chart operations directly related to the sample size.

CONSTRUCTION OF THE MODEL

The average net income per unit time for a long period of operation is:

$I = [\beta V_0 + V_0(ah+en+1)(\phi/P)/((1/\lambda)+B) + V_1(ah+en+1)(\eta/P)/((1/\lambda)+B)] - [(\beta\alpha_1 T/h) + (\beta\alpha_2 t/h) + \varepsilon W(\eta/P) + \varepsilon w(\phi/P) + \varepsilon F(\eta/P) + \varepsilon f(\phi/P) + (b+cn)/h]$

$= V_0 - V_0 U/((1/\lambda)+B) - Mu(\eta/P)/((1/\lambda)+B) - (\beta\alpha_1 T/h) - (\beta\alpha_2 t/h) - \varepsilon(W+F)(\eta/P) - (w+f)(\phi/P) - (b+cn)/h$

where,

$M = V_0 - V_1$

$U = (D+G)(\eta/P) + (d+g)(\phi/P)$

$u = ah+en+1$

I, may be expressed as composed of only two terms:

V_0, the average income per unit time from operation of the process at the standard level, and S to be called the loss cost function, i.e.

$I = V_0 - S$, where

$S = \{[(V_0 \lambda U) - M\lambda u(\eta/P) + (\alpha_1 T/h) + (\alpha_2 t/h) + \lambda(W+F)(\eta/P) + (w+f)(\phi/P)] / (1+B)\} + (b+cn)/h$

The average net income I will be a maximum when S is a minimum for certain values of n, h and k, since V_0 is independent of these variables. Hence, the optimum solution will be obtained by minimizing S.

SOLUTION OF THE MODEL

The computer is used to solve the model. The program has been designed using Fortran IV language. Different practical values at suitable intervals are assigned to n, h and k, seeking for the minimum S. These values are:

n	10-100	interval 5
h	1-25	interval 1
k	2-4	interval 0.2

The use of the computer is demonstrated by solving 25 examples given in Table 1, which also contains the results. In the 19 independent parameters considered for each example, the cost is in dollars and the time is in hours.

The 25 examples are constructed such that each parameter is assigned four values in order to investigate the influence of any parameter on the optimum design.

Throughout all the 25 examples, the following proportionalities are assumed:

$V_0 : V_1$	2
e:l	1:10
D:d:G:g	2:1:2:1
T:t:W:w:F:f	4:2:2:1:6:3
b:c	5

CONCLUSIONS

The general conclusions which may be drawn from the presented study are:

1. S always increases with the increase of any of the 19 independent parameters except with p", e and l where it decreases.

2. The optimum values of n decrease with the parameters p", δ, b and c, while they are constant with e and l. For the other parameters n shows increasing values.

3. The optimum values of h show constant figure at the upper practical limit with all the 19 independent parameters.

4. The optimum values of k decrease with δ, λ, D, d, G, g, b and c. Value of k is constant with p", e and l at the upper limit (4σ), and has a value of 3.8σ with the parameters T,t,W,w,F and f. Values of k increase with increasing of V_0 and V_1 values.

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Ex. no.	Parameters involved in the examples																			Optimum conditions			
	P"	δ	λ	V_0	V_1	T	t	W	w	F	f	D	d	G	g	e	l	b	c	n	h	k	S
1	.01	0.2	.01	200	100	50	25	25	12.5	75	37.5	1.0	0.50	1.0	0.50	.01	0.1	1	0.2	75	25	4.0	2.78
2	.02																			95	25	4.0	3.00
3	.05																			40	25	4.0	1.65
4	.10																			20	25	4.0	1.32
5		0.5																		85	25	4.0	2.83
6		1.0																		65	25	4.0	2.99
7		2.0																		20	25	3.8	3.36
8			.02																	80	25	3.8	3.66
9			.05																	90	25	3.6	4.67
10			.10																	90	25	3.6	5.07
11				20	10															50	25	3.8	1.45
12				40	20															50	25	3.8	1.61
13				100	50															75	25	4.0	2.05
14						10	5	5	2.5	15	7.5									25	25	3.8	2.14
15						20	10	10	5	30	15									25	25	3.8	2.36
16						100	50	50	25	150	75									80	25	3.8	3.45
17												0.5	0.25	0.5	0.25					75	25	4.0	2.09
18												2.0	1.0	2.0	1.0					80	25	3.8	4.15
19												5.0	2.5	5.0	2.5					100	25	3.4	7.94
20																.02	0.2			75	25	4.0	2.77
21																.05	0.5			75	25	4.0	2.74
22																.10	1.0			75	25	4.0	2.69
23																		2	0.4	25	25	3.8	3.23
24																		5	1.0	10	25	3.2	3.60
25																		10	2.0	10	25	3.2	4.20

p.s. Blank spaces refer to corresponding figures stated in example 1 at the top of the table.

Table 1. Examples of the optimum design of percent defective control charts.

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