CROGRO: AN INTERACTIVE FOREST GROWTH SIMULATOR

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ABSTRACT

CROGRO is an interactive computer model which simulates crown growth in trees as they compete with each other for available space and light. The model contains growth data for six species of trees. CROGRO was designed to be used as a teaching aid. It allows forestry students to observe in a few minutes a process which, in the real world, occurs over several decades. The system is programmed in APL and input/output is via a Tektronix 4015 graphics display terminal.

1. INTRODUCTION

CROGRO is an interactive dynamic computer model which simulates the growth of crowns in even-aged stands of trees. It is a competition model. The rate at which a tree can grow and the shape that its crown takes from year to year are determined by its species and the amount of competition it recieves from other trees. Growth at any time depends on current competitive conditions and on a tree's memory of the past as reflected by crown size - NOT on age alone. CROGRO models growth from the time seedlings are planted until the trees are harvested or until the stand matures and the trees eventually die.

The primary objective in building CROGRO was to create a student training simulator which would give future forest managers an understanding of the dynamics of stand development. A simulator is ideal for this purpose since it makes it possible to observe in a matter of minutes the complex processes of growth, competition, survival and death. These processes cannot be observed in the real world except over long periods of time. With a simulator the student can study stand performance, explore management alternatives, and carry out experiments without affecting the real world system.

A simulation modelling project to provide these facilities was undertaken as a joint project of the Faculty of Forestry and School of Computer Science at The University of New Brunswick. The model was developed to satisfy the following objectives and constraints:

- The model was to be able to grow several species of trees, having different growth characterictics and shade tolerances, with the emphasis on modelling crown shape. differentiation was to be demonstrated, and the model had to be able to simulate both opengrown and stand-grown trees. To limit the scope of the project only the most important processes governing the differential development of trees in the same stand were included. The responses to available light and stand density were included; the effects of such density-independent factors as weather, site quality, and disease were not.
- Since the simulation was to be used as a teaching aid graphic output was selected.
- The simulation was to be interactive to permit the user to selectively cut trees. It had to be "user friendly", prompting the user for in-

put in a logical, clear manner and to run fast enough to sustain user interest. Extensive error checking was required to insure that the simulation is always initialized with valid data. In addition, errors in input data were to be readily correctable and the simulation was to be easy to reinitialize and terminate.

- The simulation was to be implemented using existing computing facilities.
- The simulation was to be developed with approximately four man-months of effort.

Earlier simulation programs such as those described by Botkin (1972) and Mitchell (1969) have emphasized the modelling problems more than the user interface.

2. THE CROGRO PROGRAM

In order to understand the CROGRO model it is necessary to be familiar with the general processes of forest stand growth. During the first few years after planting the seedlings grow slowly in height and the tree crowns extend laterally close to the ground. This is followed by a period of rapid height growth in which the tree crowns extend vertically, and begin to actively compete for light by shading one another. Eventually adjacent crowns touch and then lift from the ground (the lower branches die when insufficient light reaches them to maintain their foliage). Those seedlings which have a slight competitive advantage grow faster than their neighbours and accentuate the advantage. As the crowns develop, competition causes differentiation into four crown classes: dominant, co-dominant, intermediate, and suppressed. The extent to which crowns differentiate depends upon the degree of competition for light, which is a function of the density of the stand, and of the shade toler-ance of the species, that is, its ability to survive at low levels of illumination.

As the height growth slows and approaches zero in older trees, the crowns continue to grow in width and volume. Both the intermediate and the co-dominant trees in the stand are gradually suppressed. After maturity is reached the crowns begin to shrink and with advancing age the stand starts to fall apart (Baker, 1950).

Shade tolerance is related to the evolutionary strategy for survival of a species. Shade intolerants grow rapidly in their initial development and are generally short-lived. Shade tolerant species grow more slowly and are generally long-lived. The key mechanism is the photosynthetic capacity of the foliage of a particular species with respect to low light intensities.

In mixed stands, the shade intolerant species dominate in the early years, but the more tolerant species in the stand survive and grow beneath this canopy. As holes are opened in the canopy by the death of trees in the overstory, the shade tolerant species are released from suppression, grow up through the canopy, and become dominant.

The CROGRO program models only even-aged stands of trees. It does not model the succession of species that would occur in a naturally regenerated forest.

It grows trees in a 2-dimensional world in which only height (vertical) and width (horizontal) growth are permitted. This is analogous to a profile strip or slice, up to 30 meters long, through a real stand. The user "plants a stand" on this strip to initialize the model by providing the number of seedlings to be grown as well as the species, height, and spacing for each seedling. The stand defined by the user is assumed to be bounded on both sides by stands which are identical to the user's stand. The user must provide the distance to this boundary when initializing the model. Thus the strip being grown can be viewed as being infinitely long with the stand repeated over and over again. It is therefore possible to plant a single tree and have it take the shape of an average stand-grown tree at any given spacing. Whenever the stand is displayed, the left-most tree of the right-bounding stand, and the right-most tree of the left-bounding stand, are also displayed.

In reply to prompts from the program, the user supplies initial values for the species, spacing, and height of each seedling. Tree species must be one of the six species coded into the program: Balsam Fir, Sugar Maple, White Pine, White Birch, Red Pine and Trembling Aspen. Two year old seedlings are assumed so the tree heights must be in the range 50 cm to 100 cm.

The user must also specify a display interval which is the number of years between displays. A display is always done at age 2 to permit the user to verify the initial conditions.

Each tree in the CROGRO program is described by 129 variables. The large number of variables needed to describe a tree in the program is a direct result of the requirement for a high degree of spatial resolution. The CROGRO program is based on representing the stand as shown in Figure 1. A tree consists of a bole (trunk), a leader (the growing top of the bole), and foliage to the left and to the right of the bole. The space in which the trees grow is divided into a number of horizontal layers. For the species currently modeled, sixty layers 0.5m thick have been found to be a suitable compromise between speed and graphic resolution.

A tree is described by:

- 1. its species
- 2. its height
- its location (from the left edge of the stand)
- the distance its foliage extends to the left of the bole in each layer.
- the distance its foliage extends to the right of the bole in each layer.
- the number of consecutive years the tree has been suppressed (growing very slowly in height).
- a shade tolerance factor which accounts for reduced tolerance in dying trees.

Each species is characterized by some twenty-four parameters (Sprague 1980). Within this representation the tree growth is computed repetitively until

all the trees have died or the user stops the simulation. A growth cycle can be any length of time. Two years have been found to be a good compromise between realism and computational requirements. During a growth cycle the stand is scanned layer by layer from the top and from left to right. When a leader is found it is grown an appropriate amount. If the growth takes it through a layer, some initial foliage is added in that layer.

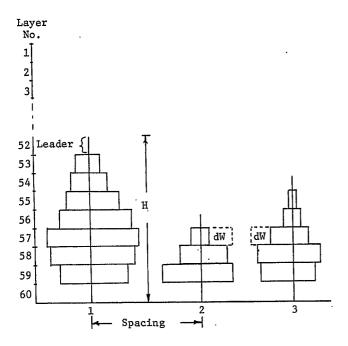


Figure 1: CROGRO forest representation

When foliage for a tree is encountered in a layer, the right side of the current tree and the left side of its right neighbour are simultaneously grown into the space between the two trees. If a layer contains foliage of only one tree, the tree becomes its own right neighbour. Neither foliage nor leader is grown into space already occupied by another tree.

The output routine draws a picture of the stand each display interval and then asks the user to select one of 4 options:

- P proceed (continue the current simulation run).
- C cut trees (kill the trees specified by the user).
- 3. R reset the display interval.
- 4. T terminate this run.

If the cut or reset options are chosen, the user is prompted for the required information. If the terminate option is chosen, the user may re-start with the same stand, specify a different stand, or terminate the simulation.

3. RESULTS

The script of a typical CROGRO simulation session is given in Figure 2. The resulting graphical output is shown in Figure 3. Five Balsam Fir were planted, all at the same initial height, with spacing varying between 0.6 m and 1.4 m. By age 10, the crowns have closed together and the trees have reached 3.5 m in height. At age 20 height differentiation due to competition has begun, and the crowns have all lifted from the ground, but by varying amounts depending upon the light intensity in the space between each pair of trees. By age 30, tree #3 has been overtopped by tree #4. At age 40 the stand is fully differentiated showing dominant (#5), co-dominant (#2 and #4), intermediate (#1) and suppressed (#3) trees. Trees #1 and #3 are slowly crowded out and die. The other trees take up the newly available space, mature by age 70, and by age 90 their crowns have begun to shrink due to senescence.

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CROCRO
CROWN GROWTH MODEL *** CROCKO *** IS READY TO GROW.
HOW MANY SEEDLINGS DO YOU WISH TO PLANT?
ENTER THE SPECIES OF EACH SEEDLING (F M W B R OR A):
F=BALSAM FIR
M=SUGAR MAPLE
W=WHITE PINE
B=WHITE BIRCH
        R=RED PINE
A=TREMBLING ASPEN
  -IF ALL TREES ARE OF THE SAME SPECIES ENTER ONLY 1 CODE.
ENTER SPECIES CODE(S):
ENTER THE HEIGHT OF EACH SEEDLING AT AGE 2 YRS ABOVE STUNP HEIGHT -HEIGHT MUST BE BETWEEN 50 AND 100 CM.
-IF ALL TREES ARE THE SAME HEIGHT ENTER ONLY 1 VALUE.
ENTER HEIGHT DATA (CM.)
DO YOU WANT THE SEEDLINGS TO BE EVENLY SPACED? REPLY \underline{Y} OR N.
ENTER THE SPACING DISTANCE BETWEEN TREES (M.)
BETHEEN 1 AND 2: 0.8
BETWEEN 2 AND 3: 1.2
BETWEEN 3 AND 4: 0.6
BETWEEN 4 AND 5: 1.4
HOW FAR (M.) IS THIS STAND AWAY FROM THE NEXT STAND?
        1.0
CHOOSE A DISPLAY FORMAT (1 2 OR 3):
1=CAPTIONED WINDOW (NORMAL)
       2=UNCAPTIONED WINDOW (FOR FILMING)
3=TIME SERIES (ERASE EVERY 5 DISPLAYS)
 ENTER THE DISPLAY FORMAT (DEFAULT=1)
 HOW OFTEN DO YOU WISH TO HAVE THE STAND DISPLAYED?
 MODEL INITIALIZED. DO YOU WISH TO PROCEED? REPLY Y OR N.
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Figure 2: Script of a typical CROGRO session

Figure 4 illustrates the results of a run in which one tree of each of the six different species incorporated in the program was planted. The trees were all started at the same height with an even

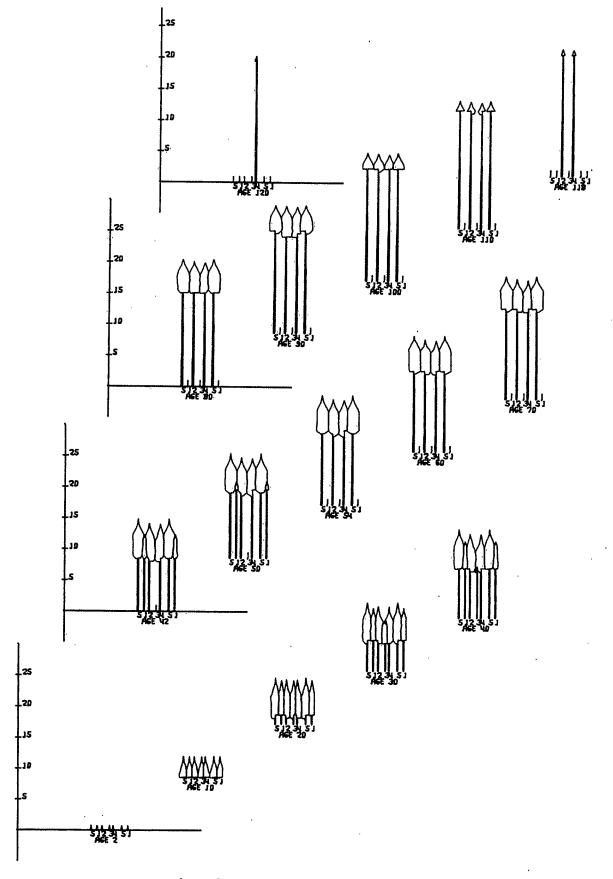


Figure 3: Balsam Fir growth - uneven spacing

spacing of 3 m. The Fir, Maple, and White Pine are the most shade tolerant species and their crowns are close to the ground whereas the intolerant Aspen has a very short crown. The Fir, Red Pine and Aspen are characterized by rapid early height growth in contrast to the Maple and White Pine which grow very slowly in height initially. Note that the hardwood crowns tend to spread more than those of the softwoods.

This figure also illustrates the second display mode of the simulator. The prompt at the bottom of the screen invites the user to cut any trees he/she wishes. The result of cutting trees 2 and 4 is shown in Figure 5. For comparison the same stand grown to age 50 but without cutting is shown in Figure 6.

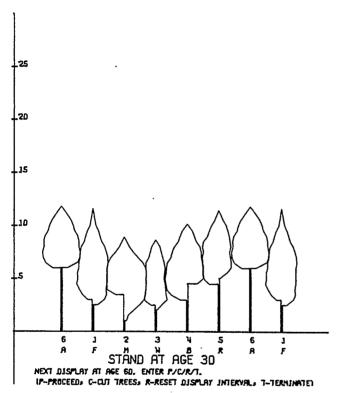


Figure 4: Example of species modelled - age 30

4. CROWN GROWTH MECHANISM

The approach used in modelling growth is to compute the growth rate which would occur in a dominant tree under optimum conditions. This is then modified by multiplying by a set of multiplier functions (less than or equal to l in value) which represent the various competitive effects acting on the tree. The growth rate functions are assumed to be functions of the current tree state (primarily height, crown area and health) and available light, but not of age. Data on growth rate is not readily available in this form.

The height growth rate, dH/dt, was found by taking height/age data for each species from yield tables (Plonski, 1974) with sites chosen to approximate New Brunswick growing conditions. A modified Weibull growth curve (see Figure 7) was then fitted to

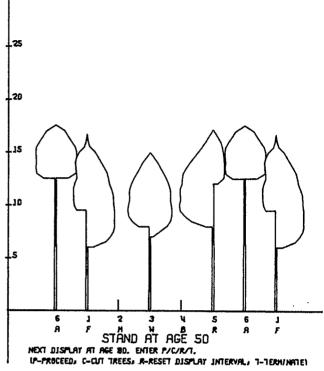


Figure 5: Effects of cutting

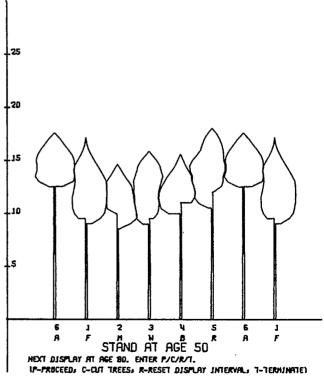


Figure 6: Example of species modelled - age 50

this data to produce a curve of the form given in Equation 1 (Yang, et al, 1978).

(1)
$$H(t) = HA * (1-e^{-HB*t})$$

where H is the height at age t; HA is the maximum height for the species; and HB and HC are constants which determine the shape of the curve.

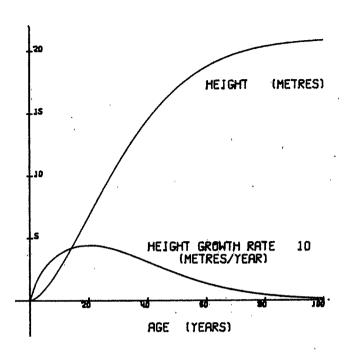


Figure 7: Height and height growth rate versus age - Balsam Fir

To remove the time dependency, the derivative of the inverse of this function was found and used to solve for the annual height growth rate, dH/dt as a function of H, the current height of the tree:

- (2a) $dH/dt = H1*(HA-H(t))*(-ln(1-H(t)/HA))^{H2}$
- (2b) $H1 = HC*HB^{1/HC}$
- (2c) H2 = (HC-1)/HC

Figure 8 shows the plot of dH/dt versus H for Balsam Fir.

Suppressed trees have a small crown which cannot produce enough assimilate via photosynthesis to permit full height growth. In open-grown trees nutrient is diverted into bole diameter growth and into sustaining the crown rather than into height growth. To simulate these processes, the model assumes that an optimal crown size for height growth exists and varies with tree height. The optimal crown size was chosen, on the basis of expert opinion, to be the crown area of a tree grown at 2 m spacing. For each species, a run of the model was done at this spacing and the crown area and tree height were recorded at each year. Crown area was then plotted as a function of height (Figure 9). A piecewise polynomial approximation to this function is used in the program.

Once the optimal crown area was determined, a height competition function was developed (Figure 10) which allows full height growth if the current crown area of the tree, TAREA, is within 20% of the optimal area for the current height; otherwise height growth is reduced.

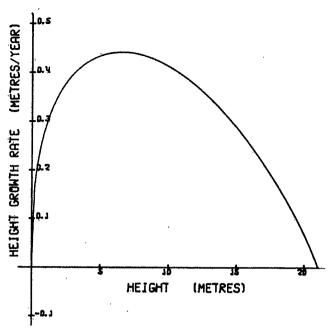


Figure 8: Height growth rate versus height - Balsam Fir

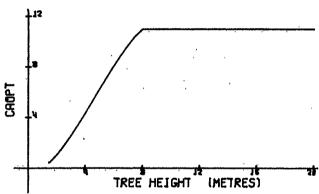


Figure 9: Optimal crown area as a function of height - Balsam Fir

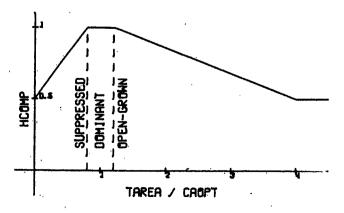


Figure 10: Height competition function - Balsam
Fir

Width growth is consderably more complex than height growth. Not only must the total area of the crown grow the proper amount, but the growth in each layer must be such that the crown takes on the shape characteristic of the species. As noted earlier, the model grows crowns in width by growing the right side of the current tree and the left side of its right-neighbour (in the current layer) into the space between the two trees, layer by layer. This approach was chosen for two reasons. First, it makes it possible to eliminate the bias that would be introduced if the left and right side of the current tree were grown as each tree is visited. Second, it makes it relatively simple to implement a spatially dependent vertical distribution of light intensity rather than have it averaged across the stand. This is very important in determinig the shape of individual trees.

To determine a tree's annual crown width growth increment in a particular layer, dW, which is the amount the tree crown would grow in width in the current layer in the absence of competition, is computed. A light competition factor is then calculated and applied. This factor specifies the ability of foliage to assimilate light as a function of light intensity and species. If the illumination is below the light extinction point for this species, the foliage is killed. If there is enough light to permit growth, two additional competition factors are applied: distance competition, which, mimics the growth reduction that occurs as two species of foliage get close to one another, even before the crowns touch, and health competition, which reduces width growth when annual height growth falls below the average annual height growth for the species. Health competition is based on the assumption that a tree with a crown which cannot produce full height growth is not in a healthy condition and will not be able to produce full width growth either.

If the sum of the two width growth increments is greater than the available space, the available space is apportioned according to their relative sizes. The foliage widths are then updated and the model proceeds to the next tree in the current layer, if any (Figure 1).

A Weibull growth curve was used to describe crown width growth over time and, using the same technique that was used for height growth, an equation was derived giving the width growth rate, dW/dt, as a function of crown width W:

(3)
$$dW/dt = W1*(WA-W(t))*(-ln(1-W(t)/WA))^{W2}$$

where W1 and W2 are constants. WA is the maximum crown radius in a given layer.

Crown shape is dependent on, among other things, the angle at which branches grow to the bole. The maximum crown radius, SWMAX, is a function of this angle, BA, and the maximum branch length, BLMAX, as shown in Figure 11. Thus above height HMIN:

(4) SWMAX = BLMAX * sin BA

where

(5) HMIN = SWMAX / tan BA

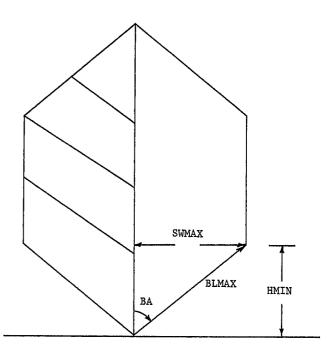


Figure 11: Tree geometry

At any height H below HMIN, the maximum crown radius is given by:

(6) SWMAX = H * tan BA

WA is then determined from Equation 7:

(7) WA = MIN(SWMAX, (H * tan BA))

where SWMAX is the maximum crown radius for the species, H is the height of the layer being grown above the ground, and BA is the branch angle described above.

Another factor which affects crown shape is whether the tree tends to be excurrent or deliquescent. Excurrent species, such as Fir and Spruce, have a well-defined leader but deliquescent species, such as Sugar Maple, have a bole which breaks up into branches without a definite leader. This is simulated in the model by making the width of a new layer of foliage a function of the height of the tree and SA, a species dependent angle of deliquescence.

Shade tolerance is the ability of a species to survive at low levels of illumination. As the light intensity diminishes, the ability of foliage to produce photosynthate is reduced slowly at first, followed by a sharp drop to zero at a species dependent light intensity known as the light compensation point (LCP). Below the LCP, net photosynthesis is zero and the foliage quickly dies (Wang, 1979). Ashby (1969) defines the light extinction point (LEP) as "the lowest percentage of full daylight in which a species is found under natural conditions". By combining the LCP and LEP concepts, the amount that a piece of foliage will grow over an entire growing season can be computed as a function of the average illumination.

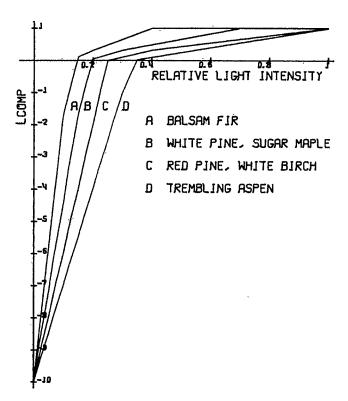


Figure 12: Light compétition function for crown width growth

The light competition function, LCOMP, is shown in Figure 12. It multiplies dW to adjust width growth. Note that at the LEP, LCOMP is zero and no width growth occurs. Below the LEP, LCOMP is negative so that when it is applied, dW also becomes negative and foliage dies.

The average light intensity at a given point in the stand depends on the amount and density of foliage above it. The function for light intensity in the growing space is based on Beer's Law (Horn 1971).

(8) IR =
$$e^{-FDEN}$$

The foliage density, FDEN, is a function of both the type and amount of foliage through which light has passed in order to reach the current layer. FDEN in the current growing space is the sum of the foliage densities in all layers directly above the current space. The foliage density, DEN, of any layer with a growing space bounded by trees L and R, distance D apart is given by:

(9) DEN=((WL*SFCON(SL))+(WR*SFCON(SR)))/(D*2)

where tree L is of species SL and has foliage of width WL in the growing space; tree R is of species SR and has foliage of width WR (Figure 1). SFCON is a species dependent constant which represents the ability of the foliage to block light.

As two pieces of crown approach each other, their rate of growth slows down. In the model, this is simulated with a distance competition function, DCOMP, which multiplies dW. DCOMP is a function of the distance to the neighbouring foliage in the growing space (Figure 13).

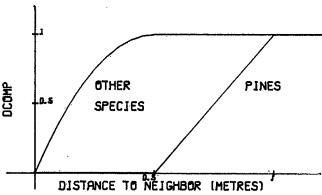


Figure 13: Distance competition function Health competition is used to reduce width growth in trees which are growing slowly due to old age or suppression. The average height growth rate, dHave/dt, for a tree is defined in the model as the maximum height a tree of a given species can attain divided by the age at which this height is reached. Height growth rate, dH/dt, in the model is a function of height and crown size, thus a small dH/dt value indicates a reduced growth capcity for the tree. The health competition function is

MIN(1, dH/dt/dHave/dt)

As a tree dies the crown shrinks from the bottom up. This happens slowly in some species and very quickly in others. In the model, a species dependent parameter, SOLD, specifies the number of years of slow growth a tree can withstand before it enters the irreversible death cycle. When a tree has been growing at less than half the average annual height growth rate for for more than SOLD years it is killed by gradually decreasing the shade tolerance of the remaining foliage at a species-dependent rate which is again a function of normal life-span and shade tolerance.

5. VALIDATION OF THE MODEL

CROGRO is a very difficult model to validate in the traditional sense. There is surprisingly little quantitative data available on how crown shapes change over time under different growing conditions, although there are many qualitative references. Any data that was found, such as Steill's paper on Red Pine (1966), was used to calibrate the model and, therefore, could not be used again for validation. The problem of matching growth rates in the model to site conditions in the real world makes quantitative comparisons with existing field data very difficult. The only way to validate the model in the classical sense is with sets of observations of individual crown development in stands over a period of 50 to 100 years. No such data is available and gathering it was beyond the scope of the CROGRO project.

Another approach to validation is to involve experts on the subject being modeled in both the development of the model and the analysis of its predictions. The object then is to produce a believable model which behaves at least qualitatively in a fashion that is consistent with what is known to occur in the real world. Holling (1978)

states that validation is in fact "nothing but hypothesis testing" and then goes on to say:

There is no sure way to decide what to predict and what level of detail to include in order to produce a believable model....a believable model should accurately predict the qualitative properties of the temporal and spatial patterns characteristic of the historical system.

This was the approach taken in validating CROGRO. The output of the model was exposed to forestry researchers and students and also compared with tree silhouettes taken from standard handbooks (Hosie 1972). Where a process in the model was shown to be in error, that is, yield unbelievable results, the model was changed. Other discrepencies, traced to program errors or to the improper calibration of parameters, were corrected.

6. PROGRAMMING CONSIDERATIONS

The limited amount of time available for this project dictated the maximum use of existing facilities, both hardware and software.

Initially it was proposed to build the software to run on a microcomputer. However, the available processors did not have floating point arithmetic hardware and had only a rudimentary graphics capability. Furthermore, nearly all the software would have to have been written from scratch. The university's central time-sharing system offered a choice of three languages and a sizeable software In addition, several Tektronix 4015 graphics display terminals attached to the central system were available for use during the development phase of the project. These terminals utilize a keyboard for input and a storage type cathode ray tube for output. They are capable of displaying high quality line drawings. It was felt that the availability of the central facility's hardware and software support would enable most of the project time to be devoted to analyzing and solving the problem as opposed to fiddling with hardware and support software. The central system was selected.

A suitable suite of APL functions to drive the 4015 terminal was available. This function was not available "off the shelf" for either FORTRAN or BASIC, the other languages available on the time-sharing system. APL was, therefore, chosen as the implementation language even though it is considerably slower than FORTRAN in execution.

The program consists of sixteen APL functions written to describe crown growth and thirteen functions, taken from the University of New Brunswick Computing Centre's APL public library, to perform the graphics display. Approximately 40,000 bytes of memory are required for the crown growth functions and 16,000 for the graphics functions. The data for each tree requires approximately 1024 bytes of memory. A stand of 8 trees can be simulated in a 64,000 byte APL workspace.

Running time is highly dependent on the number of trees, the length of the growing cycle, and the overall system load. It is directly proportional to the number of trees and inversely proportional to the length of the growing cycle. Ten trees is

about the maximum number most users are willing to wait for. It has been found that a growing cycle of two years produces trees nearly as realistic as those produced by a one year cycle and runs twice as fast. The realism deteriorates rapidly with growing cycles longer than two years.

The elapsed time to produce Figure 3 (the growing cycle was two years) was 8.5 minutes and required 62 seconds of CPU time on an IBM /370 3032 computer. These times are high, but acceptable from the user's viewpoint. In an instructional setting some of the running time is used for for discussion of the output obtained and consideration of alternative actions.

A project to rewrite the program in FORTRAN is under consideration. This should reduce the running times considerably since it will eliminate the interpreter overhead. Some less expensive output devices are also being considered.

Experience with a subsequent project to rewrite the program in BASIC for a microcomputer has shown the wisdom of our choice of a central time-sharing system. The microprocessor program runs slower, the output device cannot produce realistic looking trees, and much of the development time was indeed taken up in working around problems with the manufacturer's software or hardware.

The use of standard software components where possible has increased the flexibility and utility of the program. For example, only two changes were required to convert the program to produce reproduction quality drawings for this paper. The library driver for the Tektronix 4015 terminal was replaced with another library driver for an X-Y plotter; the plot size was altered to suit the paper layout.

Addition of new species requires modifications to some of the functions as well as changing some data elements. It requires a knowledge of APL, but is not a complex task. Once the species parameters have been determined, adding the species and testing it take about a day.

7. CONCLUSIONS

CROGRO successfully captures the most important dynamics of crown and stand development, but no claim is made that it is a definitive treatment of the subject. Rather, it demonstrates the feasibility of using the CROGRO model to tackle the problem. It has been well received by those students (and others) who have used it to date. The model has achieved all its objectives. As a teaching aid, it stimulates students to think about the processes at work in the forest.

CROGRO could be extended by adding features to the program or by modelling additional biological processes. For example, the user could be allowed to define the site conditions.

Modelling crown shape is in fact modelling branch growth. A more realistic, but more complex, model could be produced by doing so directly. This would also permit the modelling of the hollow crowns found on some species which would in turn produce better looking open grown trees by allowing their crowns to extend both wider and closer to the ground.

Mensurational data such as stand volume could be added to the model by implementing bole diameter simulation. Advanced growth could be introduced into the model by allowing the user to plant seedlings at any time during the simulation run.

The authors of The Limits to Growth (Meadows, et al., 1972) echo our feelings when they state:

The model we have constructed is, like every other model, imperfect, oversimplified and unfinished.

Nevertheless, CROGRO demonstrates that a very simple model, containing only those processes which are relevant to the objectives of the modelling exercise, can capture the dynamics of a system and produce good qualitative predictions.

REFERENCES

- Ashby, M. (1969), <u>Introduction</u> to <u>Plant Ecology</u>, Macmillan, London, 287 p.
- Baker, F.S. (1950), <u>Principles of Silviculture</u>, McGraw-Hill, New York, 414 p.
- Botkin, D.B., J.F.Janak, and J.R.Wallis (1972), Rationale, Limitations, and Assumptions of a Northeastern Forest Growth Simulator, IBM J. Res. Dev., Vol. 16, pp. 101-113.
- Holling, C.S. (ed.) (1978), Adaptive Environmental
 Assessment and Management, Wiley, New York, 377
- Horn, H.S. (1971), <u>The Adaptive Geometry of Trees</u>, Princeton University Press, Princeton, New Jersey, 144 p.
- Hosie, R.C. (1969), <u>Native Trees of Canada</u>, Canadian Forest Service, Dept. of the Environment, Ottawa, 380 p.
- Meadows, D.H., D.L.Meadows, J.Randers, W.W.Behrens III (1972), <u>The Limits to Growth</u>, Signet, New York, 207 p.
- Mitchell, K.J. (1969), Simulation of the Growth of
 Even-aged Stands of White Spruce, Bulletin
 No.75, Yale University, School of Forestry, 48
 p.
- Plonski, W.C. (1974), Normal Yield Tables (Metric) for Major Forest Species of Ontario, Ontario Ministry of Natural Resources, Toronto, 37p.
- Sprague, G.L. (1980), <u>GROGRO</u>: <u>The Dynamics of</u>
 <u>Forest Stand Development</u>, <u>a Crown Growth Model</u>,
 <u>MSc(CS) Report</u>, <u>University of New Brunswick</u>,
 <u>Fredericton</u>, New Brunswick, 92 p.
- Stiell, W.M. (1966), Red Pine Grown Development in Relation to Spacing, Pub. No. 1145, Canada Dept of Forestry.
- Wang, E.C. (1979), <u>Dynamic Modelling as an Aid to</u>

 <u>Understanding Cut-over Response</u>, MSc(F) Thesis,

 <u>University of New Brunswick</u>, Fredericton, New

 Brunswick.

Yang, R.C., A.Kozak, J.H.G.Smith (1978), The Potential of Weibull Type Functions as Flexible Growth Curves, <u>Can. J. For. Res.</u>, Vol. 8, pp. 424-431.