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AN ENGAGEMENT EFFECTIVENESS MODEL FOR SURFACE SHIPS

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ABSTRACT

A stochastic model is presented that can be used to evaluate the effectiveness of surface ships as they operate in their various military roles. The development is undertaken from the defensive point of view; both a surface-to-air missile system and a gun system are considered. Input data are obtained from a complementary model in the form of platform reaction time distributions. Once the scenario is specified overall performance measures are derived from ship survivability and cumulative kill probability calculations. A sample engagement is outlined.

1. INTRODUCTION

The effectiveness and survivability of the surface fleet are continuing concerns of military planners. In recent years the surface-effect-ship (SES) has been given serious consideration as a naval combatant and has undergone extensive analysis (Saunders *et al* 1971),(Saunders *et al* 1975). This is the second of two papers which reports on the analytic techniques used in these studies. The first paper (Bard 1981) described the model that was developed to evaluate the reaction time of the platform. We will now describe the model that was used to measure the outcome of engagements between friendly and hostile forces.

Surface warfare can be analyzed from either an offensive or defensive point of view. We will view the engagement from the perspective of the defensive platform be it a surface-effect-ship or an enemy destroyer. Both combatants are credited with two possible methods of countering a multiple cruise missile attack - the assumed threat. Depending on the state of the engagement and the established operational parameters, either a surface-to-air missile (SAM) system or a gun system may be employed. Two complementary models have been developed and subsequently combined to account for each phase of the engagement. Since the SAMs will be first to engage the anti-ship cruise missiles (ASCM), the effectiveness attending the missile defense system will be independent of any later activity. The effectiveness of the gun system, however, is critically linked to both the presence of a first line defense and the number of ASCMs that actually (in probabilistic terms) penetrate this "barrier".

In general, the effectiveness of this defense is a function of the reaction time of the platform's surveillance, command and control, and fire control systems, as well as the performance characteristics attributable to the weapons suite. The overall Measures of Effectiveness for anti-air warfare and surface warfare are platform survivability and platform kill, respectively. Of corollary interest is the expected number of missiles or bullets required to achieve the desired end-game.

In the final stages of analysis, the demonstrated capabilities of both systems (if more than one is available) are combined to yield the survivability of the platform vis-a-vis conventional and nuclear tipped warheads. From an offensive perspective, the outcome is a measure of the ASCMs' performance, and can be extrapolated to reflect an engagement against any number of ships.

In Sections 2 and 3 the missile defense model and the gun system model are described. This is followed by a discussion of the effectiveness of the combined weapon suite. Finally, a sample engagement is outlined and results are presented.

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2. MISSILE DEFENSE MODEL

This sub-model provides the structure for the engagement by simulating the actual dynamics and time-dependent sequences of the duel. However, since the kinematics governing the battle have been reduced to one dimension, the effects of maneuvering targets and fluctuating flight profiles must be introduced via appropriate degradations in the respective performance parameters.

The general philosophy guiding the defense is a shoot-look-shoot firing doctrine; i.e., serially fire a salvo of SAMs at each member of the raid, wait for intercept, assess engagement results, and if unsuccessful, fire a second salvo after computing flight time and cycle time delays. When the operational envelope of the SAMs is exceeded, the gun system is called upon to counter any remaining ASCMs. Model inputs, influential parameters, and the computational and combinatorial theory are discussed below.

2.1 Inputs

The engagement model is initialized by two sets of input data. The first is statistical in nature and is derived from the reaction time model. The cumulative reaction time probability curve indicates the likelihood that the defense will commence when the penetrator is at a specified range (time). A sample curve is shown in Fig. 1. The second set of parameters delimits the performance characteristics of the missiles and weapon suites; establishes the number of ASCMs, SAMs, platforms, and launchers per platform; and delineates time delays. Of particular importance are:

OFIRE - spacing between ASCMs

TFC - time to reload launcher tubes and solve the fire control equation

ASSESS - time required to evaluate the outcome of an engagement

PKSAM - effectiveness of the SAM (can be a function of range)

PKCM - kill probability attributed to the ASCM, i.e., P_k given a hit assuming penetration; equals one if nuclear tipped.

Once these operational parameters have been defined, SAM effectiveness can be calculated.

2.2 Mathematics

The outcome of a SAM-ASCM engagement will depend on two factors:

(1) The time (range) at which the SAM can be fired; and

(2) Its effectiveness.

As stated above, the first factor is inextricably tied to the reaction time of the platform and, coupled with the operational envelope of the SAM, will in effect determine the maximum number of defensive missiles that can be fired. Against

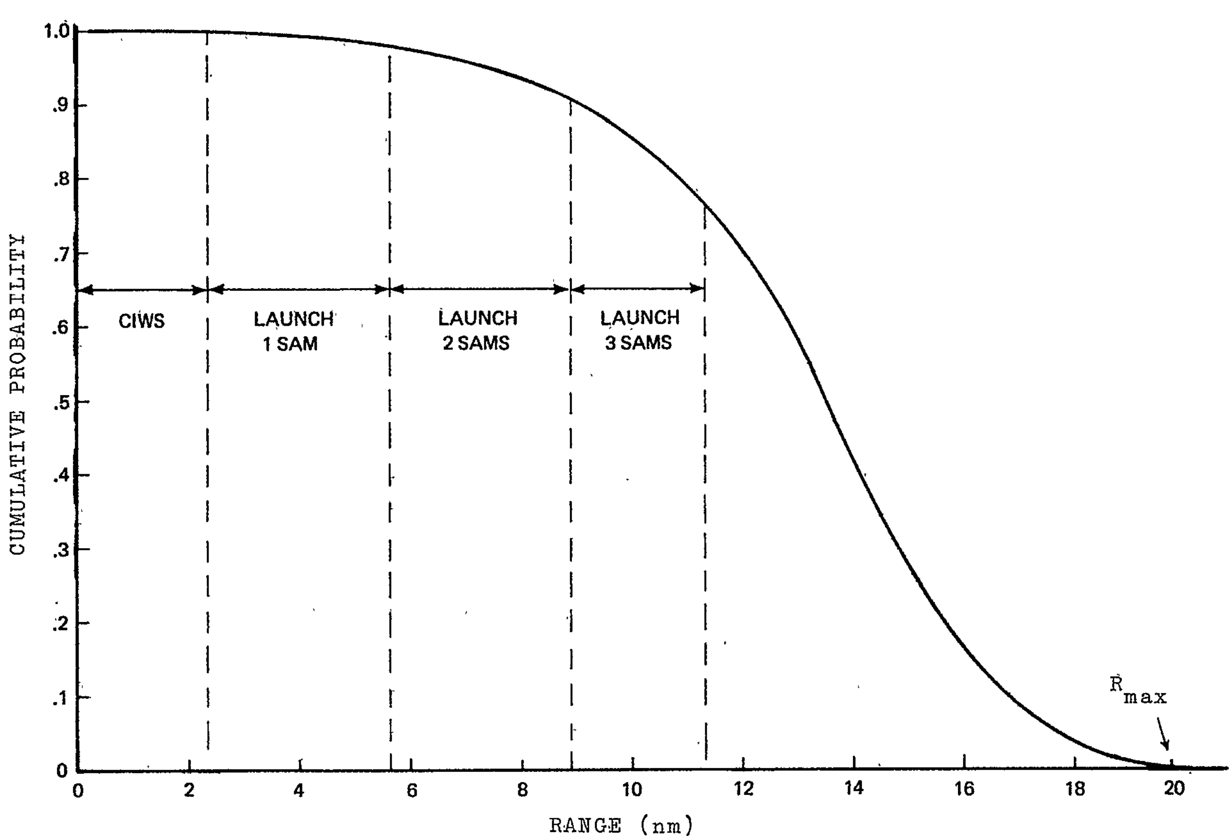

Figure 1 -- Reaction Time Distribution and Launch Envelopes for SES Engagement against SS-N-7 Missiles

the first ASCM, the expected probability of kill attributed to the first SAM is given by:

$$P^1_{KSAM} = F_{K_L} + \int_{R_{max}}^{R_{min}} p_{K/L}(R)\, f_L(R)\, dR \tag{1}$$

where:

$p_{K/L}$ = Effectiveness of SAM given launch; i.e., instantaneous kill probability

f_L = Reaction time density function

R_{min} = Minimum launch range of SAM

R_{max} = Maximum launch range of SAM; if greater than range at which ASCM becomes detectable, it is equal to the latter

F_{K_L} = $p_{K/L}(R_{max})\ F_L(R_{max})$ if R_{max} is less than $R_{detectable}$; otherwise zero

F_L = Cumulative reaction time probability

The term F_{K_L} must be included in (1) to account for the fact that a state of readiness might prevail prior to the time the ASCM entered the SAM's launch envelope. Upon entrance the SAMs are fired. Note that if $p_{K/L}$ is indeed range dependent, R_{max} can be adjusted to coincide with maximum effectiveness.

The launching of subsequent SAMs, if necessary, is keyed to where (and with what probability) the first was fired. In order to compute the kill probability for the ith SAM the limits and arguments of integration in (1) must be modified as follows:

$$P^i_{KSAM} = F_{K_L} + \int_{R_i}^{R_{min}} p_{K/L}(R)\, f_L(R+\Delta R)\, dR \tag{2}$$

where: R_i = New target location after ith engagement

ΔR = Aggregate range traversed by ASCM during previous engagements ($R_i+\Delta R = R_{max}$)

Of course when R_i becomes less than R_{min} the SAM envelope is exceeded and the gun system takes over the defense.

Recognizing that the serial firing of SAMs (shoot-look-shoot) are independent events, the cumulative P_{KSAM_1} associated with the missile defense system, against the first ASCM can be calculated from:

$$P_{KSAM_1} = \left(\cdots\left(\left(P^1_{KSAM}(1-P^2_{KSAM})+P^2_{KSAM}\right)(1-P^3_{KSAM})+P^3_{KSAM}\right)\cdots(1-P^i_{KSAM})+P^i_{KSAM}\right) \tag{3}$$

If more than one ASCM is on the horizon, (1),(2), and (3) can be employed to compute their respective chances of penetration. The results from this phase of the analysis will be joined with those of the next phase in order to derive the engagement measures of effectiveness.

3. GUN SYSTEM MODEL

A close-in weapons system (CIWS) is expected to provide a "last ditch" defense against the ASCMs that have not been already killed or neutralized. Unlike the firing doctrine stipulated for the surface-to-air missile, the gun system is required to be sequentially dedicated; i.e., it must effect a kill against the first attacker before it can be slewed to counter the second, etc. Needless to say, the limited range capabilities attending gun systems permit the engagement of at most, three to four targets. However, in actuality, the likelihood of ASCM attrition imparted by a first line defense is usually high enough to preclude more than two or three penetrations.

With these precepts as guidelines the equations governing a multiple cruise missile attack against a platform defended by a gun system will be derived, prior attrition notwithstanding. Once results have been obtained for the independent case, these P_K's will be combined with the probabilities of penetration previously computed in the above section to determine the true effectiveness of the gun system.

3.1 Inputs

Here again the subprogram inputs can be separated into reaction time probabilities, and mission and operational parameters. Key descriptors associated with the latter are operational envelope, rate of fire, number of mounts per platform, muzzle velocity, fire control delays and single projectile effectiveness. Projectile effectiveness is a range (time) dependent function since it is well known that bullet dispersion at the target resulting from gun jump, noise in the fire control system, and wind, decreases with time. In the event that gun performance data is unavailable, circular coverage functions, tabularized in (Jarnagin 1966) can be invoked to fill the void. These functions are based on a two dimensional Gaussian dispersion and require only knowledge of the target cross sectional area, the lethal area of the bullet, and the standard deviation of the gun.

3.2 Theoretical Formulation

The mathematics surrounding the operation of a sequentially dedicated, rapid-fire gun system are coincidentally similar to those governing pulse radar detection (see Skolnick 1962); however, a different formulation will be given. If $F_U(R)$ is defined as the cumulative probability that an approaching target or target complex remains unkilled until its range has dropped to a value R or less, then the relative or percentage decrease $dF_U(R)/F_U(R)$ in the cumulative probability of remaining unkilled in the time interval dt when the (first) target is at range R is given by:

$$\frac{dF_U}{F_U} = -\rho\ p_{K/U}(R)\ F_L(R)\ dt = \frac{\rho}{V} p_{K/U}(R)\ F_L(R)\ dR \tag{4}$$

where:

ρ = Rounds per second

V = Relative target velocity

$p_{K/U}$ = Probability of kill assuming no kill has occurred beyond R; i.e., instantaneous kill probability

F_L = Cumulative reaction time probability

$dR = -Vdt$

Integrating (4) from the maximum open-fire range, R_{max} to R we get:

$$\ln F_U(R_{max}) - \ln F_U(R) = \frac{\rho}{V}\int_{R_{max}}^{R} p_{K/U}(R)\ F_L(R)\ dR \tag{5}$$

Noting that the probability of being unkilled at the outset of the engagement, $F_U(R_{max})$, is one; and, that by definition $P_K(R) = 1 - F_U(R)$, where P_K is the cumulative probability of kill, the above yields:

$$P_{K_1} = 1 - \exp\left(-\frac{\rho}{V}\int_{R_{max}}^{R_{min}} p_{K/U}(R)\ F_L(R)\ dR\right) \tag{6}$$

The formulation for successive targets is basically the same; however, the following modifications must be made. For the ith target:

$$P_{K_i} = 1 - \exp\left(-\frac{\rho}{V}\int_{R_i}^{R_{min}} p_{K/U}(R)\ P_{i-1}(R+\Delta R)\ dR\right) \tag{7}$$

where:

R_i = Range at which the ith target is initially engaged

P_{i-1} = Cumulative probability that the i-1 targets have been killed and/or the platform survived the i-1 attacks

ΔR = Total range traversed by the i-1 targets during the respective engagements.

Figure 2 illustrates the effectiveness of a close-in weapon system versus four Mach 1.2 cruise missiles (no SAM defense assumed).

<u>Conditional Gun Kill Probability</u>

Consideration will now be given to the probability that one or more attackers actually penetrate the first line SAM defense. Equations (6) and (7) are unconditional in the sense that the *a priori* presence of i targets is assumed. In order to account for prior attrition, we will compute the gun system probability of kill conditioned on the first target (i.e., first target engaged by the missile defense) penetrating, the second, etc., i.e., the probability that one or more ASCMs remain unkilled. Set theory provides the framework for these computations. Accordingly, we will note that given n mutually exclusive events $A_1, A_2, \ldots, A_n$, whose sum equals the certain event (probability space) S, with B an arbitrary event included in S, it can be shown that

$$P(B) = P(B/A_1)\ P(A_1) + \cdots + P(B/A_n)\ P(A_n)$$

where by definition

$$P(A_iA_j) = 0; \qquad i \neq j = 1,2,\ldots,n\ .$$

In the situation under discussion the event B_j will be defined conditionally as jth target kill given jth target penetration and P_{KGUN_j} will be correspondingly defined as $P(B_j)$. The probability of jth target penetration (PP_j) is simply $1-P_{KSAM_j}$, where P_{KSAM_j} follows from (1), (2), and (3). The event A_i denotes i-1 other penetrations prior to the ith, where $i = 1,2,\ldots,j$; and $P(B_j/A_i) = P_{K_i}$.

For the first target, S is composed of one element, A_1 where A_1 is the event that zero other targets penetrate exclusive of the first. Clearly, $P(A_1) = 1$. Hence:

$$P(B_1) = P(B_1/A_1)\ P(A_1)$$

or

$$P_{KGUN_1} = P_{K_1}\ .$$

For the second target two possibilities arise (A_1 and A_2): 1) the first target has already been killed by SAMs, and 2) the first target as well as the second must be countered by the gun. Therefore, realizing that

$$P(A_1) = 1 - PP_1 = P_{KSAM_1}$$

$$P(A_2) = PP_1$$

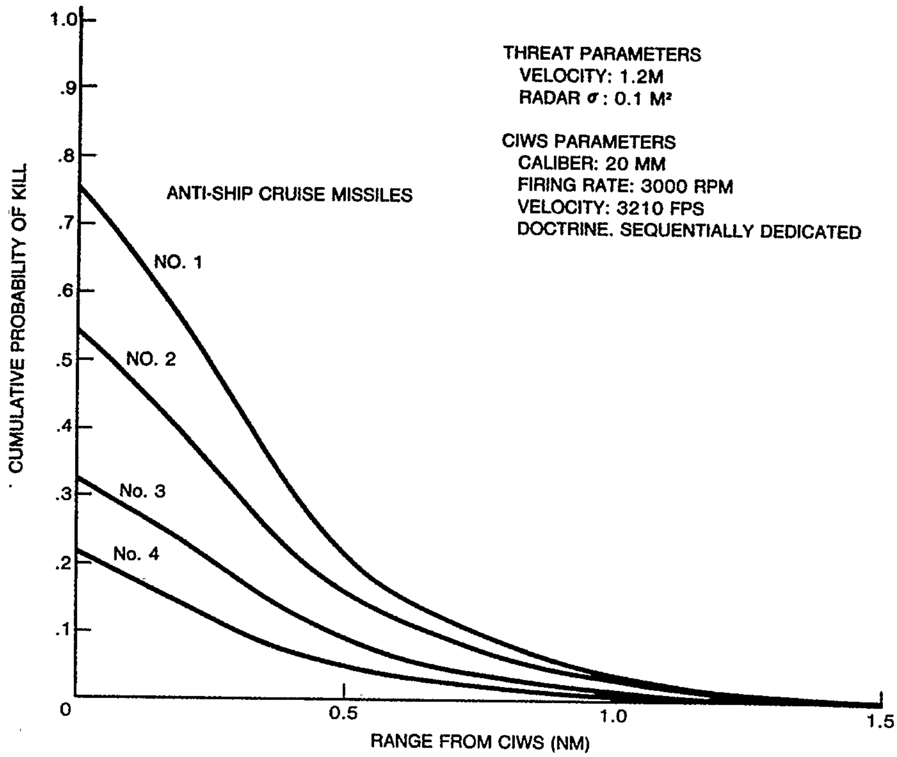

Figure 2 -- CIWS Effectiveness Against Four Simultaneous Attackers

it follows that

$$P(B_2) = P(B_2/A_1)\,P(A_1) + P(B_2/A_2)\,P(A_2)$$

or

$$P_{KGUN_2} = P_{K_1}\,P_{KSAM_1} + P_{K_2}\,PP_1 .$$

In general, for the nth target:

$$\left.\begin{aligned}
P(A_1) &= P_{KSAM_1} \cdots P_{KSAM_{n-1}} \\
P(A_2) &= PP_1\,P_{KSAM_2} \cdots P_{KSAM_{n-1}} \\
&\quad + P_{KSAM_1}\,PP_2 \cdots P_{KSAM_{n-1}} \\
&\quad + \cdots + P_{KSAM_1} \cdots P_{KSAM_{n-2}}\,PP_{n-1} \\
P(A_3) &= PP_1\,PP_2\,P_{KSAM_3} \cdots P_{KSAM_{n-1}} \\
&\quad + \cdots + P_{KSAM_1} \cdots P_{KSAM_{n-3}}PP_{n-2}PP_{n-1} \\
&\;\;\vdots \\
P(A_n) &= PP_1\,PP_2 \cdots PP_{n-1}
\end{aligned}\right\} \quad (8)$$

where it is important to note that ASCM penetrations are independent events, and thus,

if PEN_i = ith penetration

and $PP_i = P(PEN_i)$ (as defined above)

then $P(PEN_i\ PEN_j) = PP_i\ PP_j \qquad i \neq j = 1,\ldots,n$

3.3 Expected Number of Rounds

The expected number of rounds fired during an engagement is of notable importance, particularly when the thermal or firing capacity of the gun are approached. Two factors influence this quantity: 1) the rate of fire, ρ; and 2) the corresponding cumulative effectiveness. It follows that the expected number of rounds can be given by:

$$ER = \sum_{i=1}^{n} \left(RDS_{i-1} + \int_{R_i}^{R_{min}} \rho F_{U_i}(R)\, dR \right) PPN_i$$

and

$$RDS_i = RDS_{i-1} + \int_{R_i}^{R_{min}} \rho F_{U_i}(R)\, dR$$

where n = number of attackers

$RDS_0 = 0$

PPN_i = probability that exactly i attackers penetrate

4. TOTAL WEAPON SUITE EFFECTIVENESS

At this point in the analysis we have two distinct kill probabilities that must be properly combined:

P_{KSAM_i}, which is a measure of the SAM system defense, and P_{KGUN_i}, which is a measure of the gun system's performance conditioned on the event that the ith target has penetrated. Recognizing that the two events -- kill with SAMs and kill with gun -- must be mutually exclusive and that the probability of the joint event -- (PEN) and uncondional gun kill (K_{GUN}) -- is given by

$$P(K_{GUN_i}\ PEN_i) = P(K_{GUN_i}/PEN_i)\ P(PEN_i)$$

$$= P_{KGUN_i}\ PP_i$$

then the composite effectiveness of the weapon suite against the ith target becomes:

$$P_{WS_i} = P_{KSAM_i} + P_{KGUN_i}\ PP_i$$

i.e., the probability that the SAM was successful in countering the ith ASCM, plus the probability that the gun system was successful if the former was not.

4.1 Survivability

The platform's principal measure of effectiveness is its ability to counter and/or survive a cruise missile attack. If the hostile projectiles are conventionally armed, survivability will be assured by the occurrence of one of two interrelated events: either the attacker is killed or it missed its mark. In the case of nuclear warheads it is presumed that the attacker must be killed prior to surface impact. Generally, the kill probability attributed to the warhead will reflect the nature of the desired kill (e.g., seaworthiness, defensive firepower, etc.) or the number of hits required to effect that kill.

Consequently, a raid consisting of n units has an expected kill probability that can be calculated from the following equation:

$$P_K = \sum_{j=1}^{n} PPN_j p_{K_j} .$$

From this we infer that the ship's survivability is given by:

$$P(Surv) = 1 - P_K$$

where PPN_j = probability that j missiles penetrate the ship's defenses

p_{K_j} = probability of kill attributed to j penetrators

PPN_j is a function of the weapon suite's effectiveness (P_{WS_i}) and is computed by a procedure similar to that detailed in (8).

4.2 Sample Engagement

For purposes of illustration, let us assume that a SES is operating defensively in the presence of 4 Soviet SS-N-7 ASCMs. The SES is equipped with Mach 1.7 SAMs which have a radius of 6.2 nautical miles (nm). The SS-N-7 has a velocity of Mach 1.2 and a launch range of 20 nm (see Preston 1972). The kinematics of the engagement indicate that the first SAM can be launched only when the SS-N-7 is within 11.2 nm of the SES. The CIWS will be brought into action when it is no longer possible to deploy the SAMs. If enough time is available for three SAMs to be fired, the gun system's operational envelope must be reduced from 1.32 nm to approximately 0.5 nm. This reduction will have little effect, however, because the CIWS has marginal capability at ranges greater than 1000 feet.

A typical reaction time cumulative distribution (see Bard 1981) for the SES is shown in Fig. 1. The ranges (and probabilities) beyond which one, two, and three SAMs can be launched are also marked. According to this curve, 0.95 probability exists that sufficient time will be available to fire three defensive missles at each ASCM. These results are futher discussed in (Saunders *et al* 1975). Other scenarios involving a variety of networks and platforms are also examined.

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