Proceedings of the 1980 Winter Simulation Conference
T.I. Ören, C.M. Shub, P.F. Roth (eds.)

FUNDAMENTALS OF DIGITAL SIMULATION MODELING

J. W. Schmidt
Department of Industrial Engineering and Operations Research
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061

ABSTRACT: This paper and the tutorial session with which it is associated treat the fundamental concepts of digital simulation. The topics discussed include system modeling, simulation models and their advantages and disadvantages relative to mathematical models, the development of simulation and current applications, the role of simulation modeling in systems analysis and simulation languages. The paper and the tutorial are presented at a level which requires no previous exposure to digital simulation. However, familiarity with the fundamentals of probability, probability distributions and inferential statistics will facilitate the participant's understanding of the material presented.

1. INTRODUCTION

Since World War II organizational systems have grown in complexity to the degree that managers, individually and collectively, now find it difficult to control and in many cases to understand the organizations they have created. At the same time rampant inflation, pervasive government regulations and aggresive competition dictate increasing control of organizational behavior. The quantitative techniques loosely embodied in the discipline of operations research* are often employed in an attempt to better understand, predict and control system behavior.

The analysis of any system is generally the result of the need to better understand the behavior of the system. The analyst may wish to know how the system will function under a variety of conditions, whether the system should be modified to more efficiently achieve its intended function or simply to better understand the current operational characteristics of the system. Given sufficient time and financial resources each of these goals could be achieved through manipulation of the physical system itself. For example, one could implement successive modifications of the system in an attempt to achieve more efficient performance, or simply wait for certain conditions to arise and observe the behavior of the system under those conditions.

Manipulation of the physical system may be economically infeasible since it may seriously disrupt the overall operation of the organization of which it is a part. Simply waiting for those conditions under investigation to arise may be self-defeating in that failure to predict the effect of those conditions may lead to disastrous results with respect to the performance of the system. Thus, experimentation with the physical system is usually to be discouraged, although it is occasionally employed with satisfactory results.

2. MODELS

At the heart of most operations research studies is a system model which acts as an experimental substitute for the system itself. Models have long been used by scientists to explain physical phenomena. The acceleration of a particle under the force of gravity, the position of a planet in space and the flow of current across a conductor are simple examples. Engineers, social scientists and management scientists have extended the principles of modeling to the analysis of systems created and controlled

---

* The terms operations research, management science, systems analysis and systems engineering are often used synonymously.

by man.

In a broad sense a model may be thought of as any representation of reality which is not reality itself. For example, a painting of a ship at sea is a model of the visual characteristics of the vessel and its immediate environment. However, it captures neither the sounds of the sea nor the forces acting on the ship. A set of equations may be used to describe the flight path of a manned satallite in space but conveys no information concerning the visual characteristics of the satallite or the physical and emotional state of the crew. An architect's scale model of a medical complex is a three dimensional representation in miniature describing the relative location, shape and size of buildings, parking areas and walkways and may even include the details of the layout of each floor. However, as detailed as such a model may be it communicates little information relative to the cost of operation, staffing requirements or patient scheduling. The purpose of a profit and loss statement is to describe the financial position of a company; not to depict the building housing its corporate offices. An equation relating the variables of a quality control system to the total cost of quality control is not intended to provide a picture of a defective unit of product. Thus the objective of a model is to capture only certain aspects of reality. Even with this limited purpose in mind, the modeler can at best achieve an approximation to that part of reality with which he/she is concerned.

The models used by operations researchers are almost always mathematical in form and attempt to relate the variables and constants of the system to a measure or measures of system performance such as cost, profit or level of service. If the model approximates the behavior of the system with reasonable fidelity then the analyst may experiment with the values of the variables of the system and observe the resulting effect on the measures of system performance in an attempt to inferentially understand the behavior of the system under specified conditions or specify values of those variables under management control such that the system will perform in the optimum or at least an acceptable fashion.

2.1 Simulation Models

Although simulation models fall in the general class of mathematical models, a distinction is usually drawn between simulation models and mathematical models which consist solely of an equation or series of equations. A simulation model attempts to mimic the behavior of the system modeled by explicity capturing the time series of events which determine the behavior of the system, the interaction of those events and their impact upon the components of the system. Usually the model defines each event and its time of occurrence, appropriately altering the values of those variables which define the status of the system as events occur. Because the simulation model traces the behavior of the system over time, output drawn from the model at a sequence of points in time would be similar in form to that derived from observation of the physical system.

If a simulation model faithfully describes the behavior of the physical system, then the activity described by the model over a given time period may be treated as though it were drawn from direct observation of the system. In this sense a simulation model synthetically samples the activity of the system it describes, each time period simulated representing a possible sample of activity from the physical system.

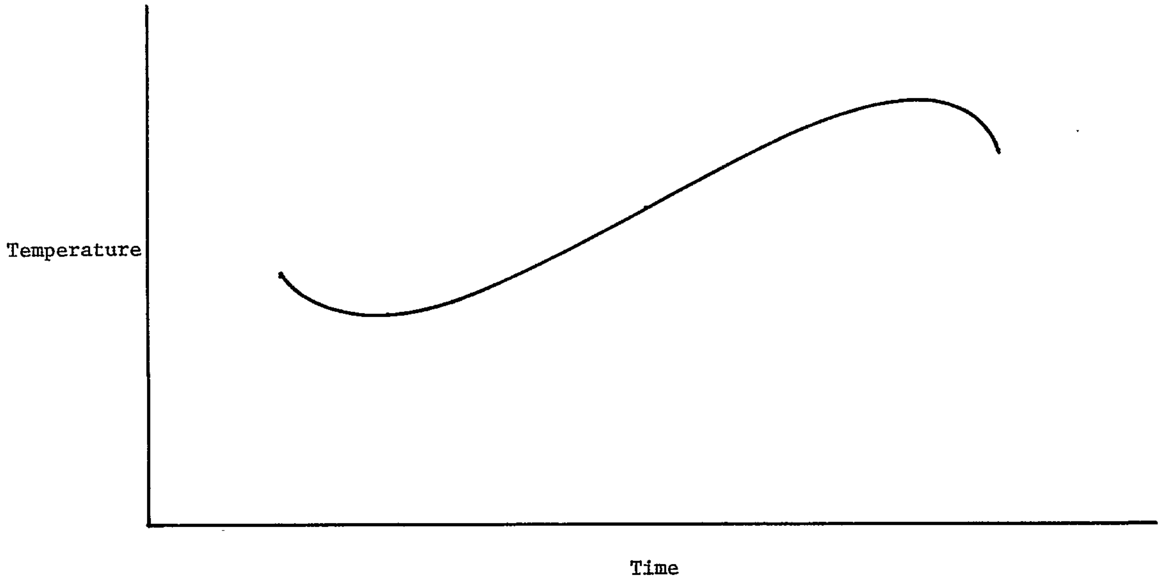
While simulation models may be either deterministic or stochastic, simulation modeling is usually applied to the analysis of stochastic systems where random variation in the time of occurrence of events and their effect on the state of the system is significant. Because a stochastic simulation model incorporates the random variability found in the physical system, two independent samples of system activity generated by the model for the same period of time are likely to yield different results in system behavior. However each sample of activity represents a possible outcome for that period of time.

Since the simulation process revolves around synthetic sampling, the output of a stochastic simulation model is comprised of values of one or more random variables. This characteristic is one of the principle distinctions between simulation models and more conventional mathematical models; the output of conventional mathematical models being a numerical constant or constants.

2.2 Digital Versus Analog Simulation

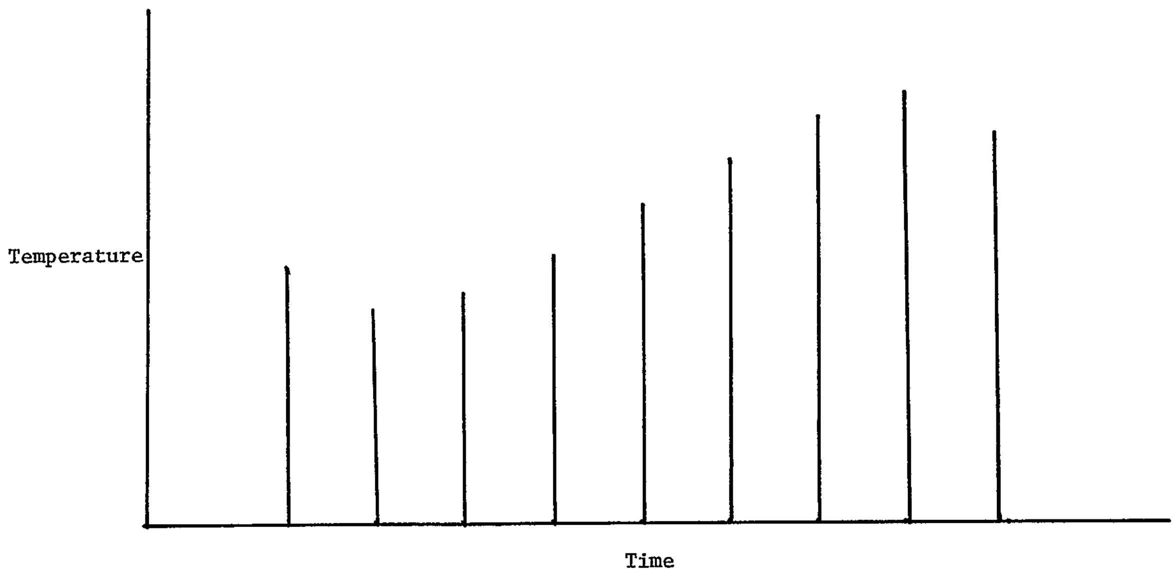
Although the primary focus of this paper is digital simulation, an understanding of the distinction between analog and digital simulation is useful. Analog simulation is employed where the system modeled is subject to continuous changes as a function of a continuous variable, usually time. Changes in the status of the system modeled through analog simulation are generally represented by proportionate changes in a continuous medium such as voltage or air pressure. Hence, the output of an analog simulator is a continuum of status changes in the system model. Sample output of an analog simulation is shown graphically in Figure 1.

A digital simulation model does not treat time in a continuous fashion, but rather models system changes at discrete points in time. In this sense a digital simulation model of a time dependent system



Output of an analog simulation model

Figure 1



Output of a digital simulation model

Figure 2

mimics the operation of an analog simulation. Hence, the output of a digital simulation model is discrete. A graphical representation of the output of a digital simulation model is shown in Figure 2.

2.3 Advantages and Disadvantages of Simulation

Since the most frequently employed alternative to a simulation model is a mathematical model consisting solely of one or a series of equations, the advantages and disadvantages of simulation will be discussed relative to this alternative. When the system under study can be modeled with equal validity using either modeling approach, a complete mathematical description is probably to be preferred. The output of a mathematical model is a numerical constant or constants or perhaps an analytical function while that of a simulation model is the value of one or more random variables. Hence interpretive value of the output of a simulation model is always clouded to some degree by the presence of random error. The random component in the output may lead to errors in conclusions concerning the effectiveness of a particular system and may be particularly troublesome when attempting to evaluate the effectiveness of one system versus another or several others. Since the output of a mathematical model is deterministic these issues do not arise.

The immediate objective of most conventional mathematical models and simulation models is estimation of the values of one or more measures of system performance. Since the simulation process requires synthetic sampling, replication of the sampling process is usually necessary to achieve acceptable precision in the estimator or estimators. When the system simulated is complex each replication may require significant execution time on a digital computer. Hence the number of replicates required for sufficiently precise estimation may be such that the resulting simulation experiment is quite expensive. While a mathematical analysis may also require execution on a digital computer, the time required for execution is usually not as great as that for a corresponding simulation model.

The principle disadvantages of simulation analysis are, then, the expense involved in model execution and the difficulty associated with the interpretation of output. However, as the complexity of the system to be modeled increases complete or even partial mathematical analysis may prove infeasible. In such cases simulation modeling often becomes "the only game in town" and hence the phrase "when all else fails, simulate". This suggests that the greatest advantage of simulation may be its versatility. While systems of only moderate complexity may not yield to mathematical analysis, simulation models can and have been developed for highly complex systems. In addition, even when a system can be modeled mathematically, the level of mathematical sophistication required may be beyond the background and training of the analyst while he/she possesses the requirements necessary for development of an adequate simulation model. Thus the principle advantages of simulation modeling relative to mathematical modeling are versatility and simplicity.

3. THE DEVELOPMENT OF SIMULATION

Some authors trace the origin of simulation to the early sampling experiments of W. S. Gosset, who published under the name Student, (1908). However, the foundations of modern simulation techniques are usually credited to the works of Von Neumann (1951) and Ulan (1951). Their work, conducted in the late 1940's, involved the analysis of nuclear-shielding problems through a technique termed "Monte Carlo Analysis" which subsequently became fundamental to simulation modeling. However, it was not until the early 1950's and the arrival of high-speed computing machinery that the horizons for application of simulation were broadened to include the practical analysis of engineering, business, and behaviorial systems. Since that time simulation has been applied in such diverse areas as:

- Analysis of Commerical Air Transportation Systems
- Analysis of Computing Facility Operations
- Military Operations Analysis
- Evaluation of Machine Replacement Policies
- Nuclear Fuel Cycle Analysis
- Management Gaming
- War Gaming
- Environmental Impact Analysis
- Forest Resource Management
- Corporate Planning
- Machine Requirements Analysis
- Evaluation of Health Care Delivery Systems
- Manpower Planning
- Job-shop Scheduling
- Instructional Modeling for Higher Education
- Transportation Planning
- Communication Network Analysis

Today simulation is one of the most powerful modeling techniques available for the analysis of a variety of complex systems. This conclusion is supported by Weston (1971) who reports that in a study of the 1000 largest firms in the U.S., 29% indicated that simulation was employed as a tool for analysis in corporate planning. The results of Weston's study are summarized in Table 1.

Table 1

Frequency of Application of Simulation

Tools Most Frequently Employed in Corporate Planning

Technique	Frequency	Percent
Simulation	60	29
Linear Programming	43	21
Network Analysis	28	14
Inventory Theory	24	12
Nonlinear Programming	16	8
Dynamic Programming	8	4
Integer Programming	7	3
Queueing Theory	7	3
Other	12	6
	205	100

4. SYSTEMS ANALYSIS THROUGH SIMULATION

In many respects the steps necessary for system analysis through simulation are the same as those taken when using any other approach to modeling and may be summarized as follows:

1. Identification of the problem
2. Specification of the objectives of the analysis
3. Analysis of the operating characteristics of the system and collection of data describing the behavior of the system
4. Formulation of the system model
5. Estimation of the values of the parameters of the model
6. Preliminary validation of the model
7. Development of required computer programs
8. Final model validation
9. Specification of the simulation experiments to be conducted
10. Analysis of results.

A procedural summary of the steps usually taken in analyzing a system are summarized in the flowchart in Figure 3. While the sequence of steps indicated may not be completely exhaustive nor in the proper chronological order for all problems, this outline may be used as a rough guide for the analysis of most systems problems.

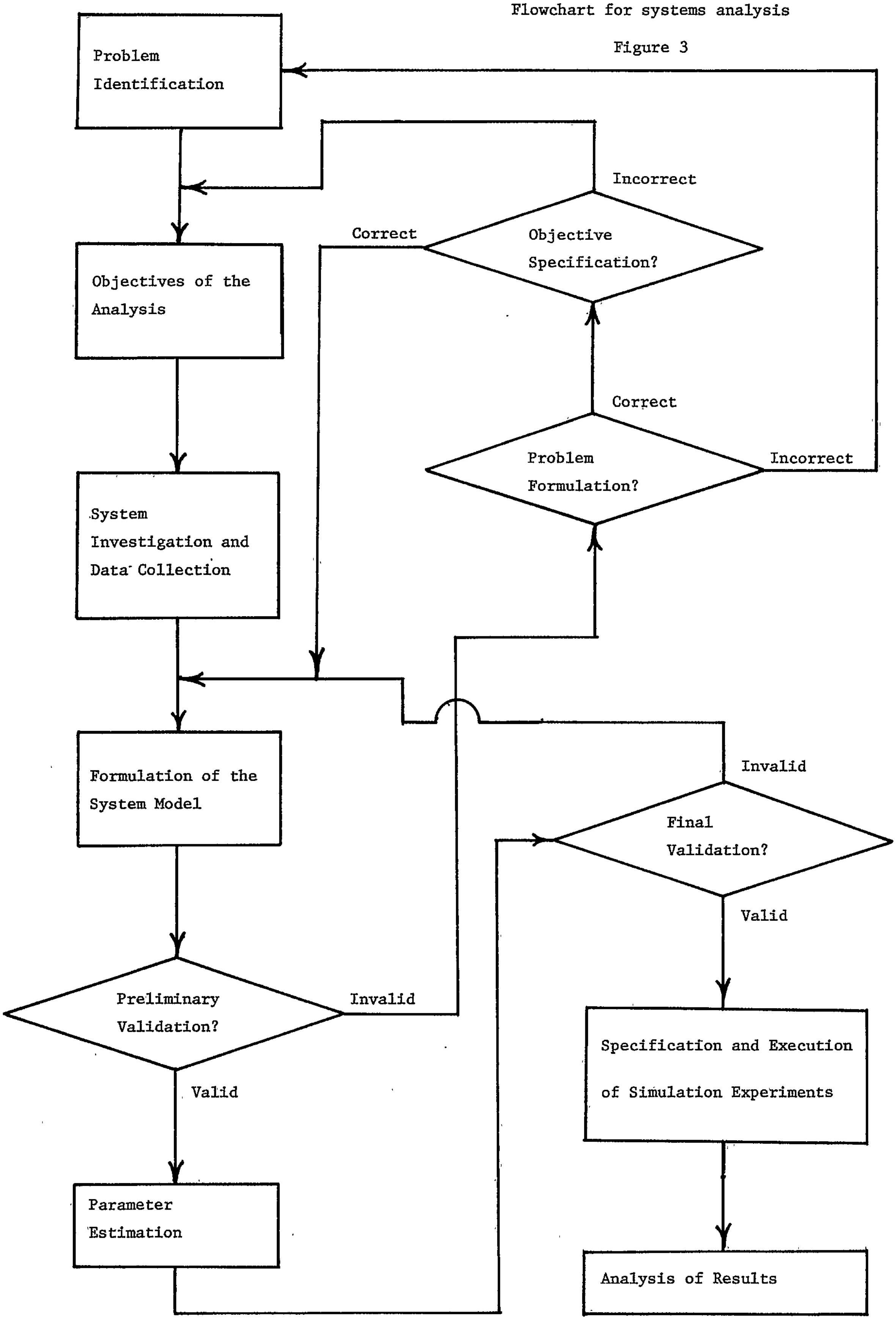
Although a complete discussion of systems analysis is beyond the scope of this paper, a brief treatment of the steps outlined above as they relate to simulation analysis may help to place the role of simulation modeling in perspective. To resolve any problem one must first understand the nature of the problem. Problem situations are recognized by the symptoms which they generate. While in some cases treatment of the symptoms may suitably arrest the problem, this solution may yield only short term benefits. Thus, the analyst is likely to be more interested in identification of the root causes of the problem and the formulation of means to resolve the problem itself rather than simply treating its symptoms.

Once the problem has been identified the analyst must outline a procedure which will hopefully lead to its resolution. To properly analyze the problem and develop procedures for its resolution, the objectives to be achieved as a result of the analysis must be carefully specified. In addition criteria should be defined whereby the degree of achievement of those objectives can be determined. For example, the objective of the study might be to reduce downtime at a given machine center. A solution which purports to achieve this objective carries little credibility unless the degree of achievement can be forecast and measured quantitatively. Hence, the problem solution should include an estimate of the reduction in downtime which will result from implementation of the recommendations resulting from the study or in general, an estimate of the benefits to be derived from the proposed solution.

The development of a valid model must be founded upon a complete understanding of the system itself. To this end the analyst should pursue a thorough investigation of the system to identify its purpose, operational characteristics, and to obtain data relevant to the behavior of the system under current and past operating conditions and to identify system changes contemplated for the future. Once a

Flowchart for systems analysis

Figure 3



thorough understanding of the system and its operation has been achieved, the analyst is in a position to proceed to formulation of the simulation model. The model developed at this point must be considered preliminary since it represents only the analyst's initial conceptualization of the system. A review of the assumptions underlying the model and its functioning with others intergrally involved with the system at this point is the first step in model validation. Rejection of the initial model is not at all unusual. However, after several attempts at model formulation, the analyst will hopefully arrive at a model which those familiar with the system accept as reasonable, at least in concept.

Final validation of the simulation model is a three stage process. The first step in this process, sometimes called verification, is to make sure that the simulation model is functioning in the manner intended by the analyst. Validation at this point consists largely of a logic check of computer programs developed for execution of the simulation model. The second stage of validation is much more difficult to deal with. Having ascertained that the model is functioning in its intended manner, the analyst must determine whether or not the intended functioning of the model conforms to reality. If the simulation model is used to represent the behavior of an operational system, the results of the simulation model can often be compared with those realized from the real world system where the exogeneous conditions governing both are the same. However, even when this comparison is favorable there is no guarantee that the simulation model will function in a manner representative of the real world system under conditions which have not yet been experienced. The problem of validation is compounded when the simulation model is intended to represent the functioning of a system which is not in existence at the present time but rather is planned for the future. In this case there is no system available which can be used to check the results of the model. In the final analysis then, complete validation of a simulation model is usually not possible. In most instances all the analyst can do is experiment with the simulation model under a variety of conditions, past, present and expected in the future, and compare the results with historical data, where available, and with what one might expect from the system under study should those conditions expected in the future arise.

The third and final stage of model validation, the acceptability of the model as a predictor of the future can be accomplished only after the model has been accepted for implementation at least tentatively or on a trial basis. In some instances the model is implemented on a limited basis and field tested under operating conditions. Its behavior is then observed and analyzed to determine whether or not it actually performs in an acceptable manner. The model may also be executed in parallel with the system without its implementation as an aid to decision making. In this mode of validation one is simply attempting to determine how well the model describes continuing system behavior without influencing that behavior.

In essence model development and model validation are opposite sides of the same coin. At every stage of model development the analyst asks whether or not it will work. Model validation attempts to answer that question. Model validation should continue even after a model is implemented. There is probably no such thing as a model which, without modification, will continue to describe system behavior indefinitely. Continuing periodic validation testing should be carried out in an attempt to identify degradation of the model and thus the need for modification. Ideally then, model development and validation should continue in a cyclic manner until the model is retired from use.

The final stage in the analysis of a system through simulation is the specification and execution of the simulation experiments to be carried out and the analysis of the results produced. Since simulation can be an expensive technique for systems analysis, the analyst should exercise care in specifying the experiments to be executed. The design of simulation experiments consists of specifying the conditions under which the simulation model will be executed and the number of simulation runs, replicates, to be executed under each condition. The set of conditions to be analyzed will be dictated by the objectives of the analysis. Ideally the analyst would hope to simulate the operation of the system studied under every condition which might be anticipated in the future. However time and budget limitations usually require a compromise in this regard.

To obtain a measure of the variability of an estimator the analyst would normally choose to replicate the simulation experiment under each set of conditions. The essential question to be answered here is how many replicates are necessary to achieve the required level of precision. In general, the precision of the estimate of a measure of system performance is improved by increasing the number of replicates of the simulation experiment. However, as the number of replicates is increased the cost of executing the experiment will increase proportionately. Thus again, the analyst is usually forced into a compromise situation.

5. SIMULATION LANGUAGES

Since most simulation experiments are carried out on a digital computer, the logic of the model must be translated into a medium which can be interpreted by the machine. Translation of the simulation model can be accomplished through a general purpose computer language or a special purpose language. General purpose languages such as FORTRAN, BASIC and PL/I provide the programmer with a tool for the analysis of a virtually limitless number of problems, of which simulation is only one. On the other hand, special purpose simulation languages are designed to address problems to be analyzed through simulation, although the variety of simulation problems which can be handled by these languages is quite broad. Included in the category of special purpose simulation languages are GPSS, SIMSCRIPT, GASP, DYNAMO and SIMULA, although there are many others.

Perhaps the principle advantage of general purpose languages lies in the fact that one of these languages is probably already known to the programmer. In addition these languages provide the analyst with a maximum of flexibility in the design of the analysis. However, because special purpose languages are oriented toward the specific application of simulation, the programming time required for translation of the model is generally less than that required in the case of general purpose languages since the time keeping mechanism and many of the subroutines normally required in any simulation model are built into the language. In addition, the structure of special purpose languages will often help the analyst to formulate the model. However, in using a special purpose language the analyst is restricted to a prescribed output format and may face increased computer run time.

REFERENCES

Chorafas, D.N., (1965), Systems Simulation, New York, Academic Press.

Churchman, C.W., Ackoff, R.L., and Arnoff, E.L., (1957), Introduction to Operations Research, New York: John Wiley & Sons, Inc.

Emshoff, J.R. and Sisson, R.L., (1970), Design and Use of Computer Simulation Models, New York: MacMillan Publishing Co., Inc.

Fishman, G.S., (1973), Concepts and Methods of Discrete Event Simulation, New York: John Wiley and Sons, Inc.

Gordon, G., (1978), Systems Simulation, Englewood Cliffs, N.J.: Prentice Hall, Inc.

Graybeal, W.J. and Pooch, U.W., (1980), Simulation: Principles and Methods, Cambridge, Mass.: Winthrop Publishers, Inc.

Hillier, F.S. and Lieberman, G.J., (1967), Introduction to Operations Research, San Francisco: Holden-Day, Inc.

Kleijnen, J.P.C., (1974), Statistical Techniques in Simulation, Parts I and II, New York: Marcel Dekker.

Martin, F.F., (1968), Computer Modeling and Simulation, New York: John Wiley and Sons, Inc.

McMillan, C. and Gonzalez, R.F., (1965), Systems Analysis: A Computer Approach to Decision Models, Homewood, Illinois: Richard D. Irwin, Inc.

Mihram, G.A., (1972), Simulation: Statistical Foundation and Methodology, New York: Academic Press.

Mize, J.H. and Cox, J.G., (1968), Essentials of Simulation, Englewood Cliffs, N.J.: Prentice-Hall, Inc.

Naylor, T.H., (1971a), Computer Simulation Experiments with Models of Economic Systems, New York: John Wiley and Sons, Inc.

Naylor, T.H., Balintfy, J.L., Burdick, D.S., and Chu, K., (1967b), Computer Simulation Techniques, New York: John Wiley and Sons, Inc.

Pritsker, A.A.B., (1977a), Modeling and Analysis Using Q-Gert Networks, New York: John Wiley and Sons, Inc.

Pritsker, A.A.B. and Kiviat, P.J., (1969b), Simulation with GASP II: A Fortran Based Simulation Language, Englewood Cliffs, N.J.: Prentice-Hall, Inc.

Reitman, J., (1971), Computer Simulation Applications, New York: John Wiley and Sons, Inc.

Schmidt, J.W. and Taylor, R.E., (1970), Simulation and Analysis of Industrial Systems, Homewood, Illinois: Richard R. Irwin Inc.

Shannon, R.E., (1975a), Systems Simulation: The Art and Science, Englewood Cliffs, N.J.: Prentice-Hall, Inc.

Shannon, R.E. and Biles, W.E., (1970b), "The Utility of Certain Curriculum Topics to Operations Research Practitioners", Operations Research, 18, 4.

Student, (1908), "On the Probable Error of a Mean", Biometrika, 6, 1.

Toucher, K.D., (1963), The Art of Simulation, London: The English Universities Press, LTD.

Ulam, S., (1951), "On the Monte Carlo Method", Proceedings of a Second Symposium on Large-Scale Digital Calculating Machinery, 207.

Von, Neuman, J., (1951), "Various Techniques Used in Connection with Random Digits, 'Monte Carlo Method'", National Bureau of Standards Applied Mathematics Series, 12.

Wagner, H.M., (1969), Principles of Operations Research with Applications to Managerial Decisions, Englewood Cliffs, N.J.: Prentice-Hall, Inc.

Weston, F.C., (1971), "O.R. Techniques Relevant to Corporate Planning Function Practices, An Investigative Look", Operations Research Bulletin, 19, 2.

Ziegler, B.P., (1976), Theory of Modeling and Simulation, New York: John Wiley and Sons, Inc.