

A SIMULATION LANGUAGE TO FACILITATE THE UNDERSTANDING OF RISK  
 WITH AN ILLUSTRATION FROM CAPITAL BUDGETING

R. V. Oakford

Dept. of Ind. Engr. & Engr. Mgmt, Stanford University, Stanford, Ca.

Arturo Salazar

School of Business, San Francisco State University, San Francisco, Ca.

R. S. Skertchly

Dept. of Ind. Engr. & Engr. Mgmt, Stanford University, Stanford, Ca.

ABSTRACT

Most risk analysis models use a measure of dispersion as a surrogate of risk. This paper presents an alternative method of risk analysis which uses a formalized dictionary definition of risk: the probability of occurrence of an undesirable event. The context of the Capital Budgeting Decision Process is used to discuss fundamental weaknesses of classical risk analysis, the limitations of probability theory to treat risk (properly) as probability of occurrence and the use of Monte Carlo Analysis in risk evaluation. In particular, the objective of maximizing future wealth of the firm, subject to an acceptable risk of ruin is discussed in detail. A description of the Capital Budgeting Decision Situation is included, since, judging from the existing literature, its true nature is not well understood. The development of a simulation language to facilitate risk evaluation is proposed; it should impose no more structure on the user than would the editorial requirements of a good journal. The use of the current version of the proposed language is illustrated with a hypothetical numeric example and a brief discussion of the results obtained is presented.

1.0 Introduction

Defining a risk as the probability of occurrence of an undesirable event is consistent with a commonly understood dictionary definition-- "the chance of injury, damage or loss." Risk "analysis" models in capital budgeting have avoided this definition and have adopted surrogate measures of risk. One popular surrogate is variance, which is a measure of dispersion but not of risk per se. To some extent this approach avoids the inability of mathematical probability theory to evaluate risk in many practical situations. Decision analysis recommends that Monte Carlo analysis be used to estimate the probability distribution of the decision criterion but it uses expected utility as a surrogate measure of risk.

We think that Monte Carlo analysis would be a practical tool for directly evaluating risk in capital budgeting decisions provided a method were developed that would enable the decision maker (henceforth DM) to describe the decision situation in terms of random variables and functional relations among them. The limitations of mathematical probability theory and the potential of an "unstructured" simulation language for describing a Monte Carlo analysis of risk are illustrated with a hypothetical capital budgeting example.

In this article we start with a formal adaptation of the common dictionary definition of risk to the language of probability theory. The limitations of mathematical probability theory and popular risk surrogates for dealing with risk in capital budgeting are discussed. A realistic capital budgeting situation and the factors affecting the risk of ruin are explicitly described, since their descriptions do not appear elsewhere in the literature. A hypothetical numerical example is described formally; the Monte Carlo analysis of the risk of ruin is described in the current version of our "unstructured" simulation language and the results of the analysis are presented. We conclude with a discussion of the requirements of the language and our plans for the future.

## 2.0 Risk Evaluation in Capital Budgeting

Webster's Dictionary defines risk, in the context we use it, as "the chance of injury, damage or loss." We adopt this definition by expressing it in formal terms of probability theory: A risk is the probability of occurrence of an undesirable event.

In this article, we will be particularly concerned with risks associated with the capital budgeting process in the practical world, which will be described later. Probability of occurrence, a seemingly obvious measure of risk, has been avoided by most capital budgeting theoreticians<sup>1</sup> in favor of surrogates that do not, except indirectly in a few special cases, measure risks. We maintain that this is the direct result of several factors; the insistence that measures be computable in analytic closed forms, the limitations of mathematical probability theory in providing such computational procedures for evaluating risk, and a possible confusion between the roles of risk and uncertainty.

### 2.1 Limitations of Probability Theory in Evaluation of Risks.

Mathematical probability theory is concerned, among other things, with the development of analytic formulas (computational procedures) for evaluating the probability of occurrence of random events when the governing probability distributions are known. For some probability distributions, e.g., the univariate normal with known mean and variance, evaluation of a probability may require only a little computation followed by a reference to an appropriate statistical table. For other probability distributions, e.g., many multivariate distributions, statistical tables are not available and evaluation of the desired probability by analytic methods of mathematical probability theory may be impracticable or even impossible. Such is the case for many risks perceived by the capital rationing DM.

Computation of the means and variances of the net cash flows would be a relatively simple matter if each were a sum of independent random variables. If, in addition, the random variables summed were either normally distributed or were identically distributed and the central limit theorem of probability theory could be invoked, then the probability distributions of the net cash flows would either be normal or approximately normal. If the central limit theorem could not be invoked, Chebyshev's inequality might be used to obtain a bound (probably very loose) on the probability that a net cash flow realization would lie outside a specified range of values. Risks measured by such probability could thus be evaluated (or bounded). When the central limit theorem is inapplicable and Chebyshev's inequality (because of its loose bounds) fails to indicate acceptance, as often would be the case, analytic methods of risk analysis would be impracticable, but simulation might be quite practical.

### 2.2 Objectives and Undesirable Events in Capital Budgeting.

A firm would commonly have many objectives. An attempt to enumerate all those that are directly related to the capital budgeting process would be outside the scope of this article, however the following are illustrative objectives: maintain a comfortable cash position, pay dividends regularly, optimize the future internal wealth of the firm, optimize the future share price of the firm's common stock, and avoid ruin.

Some of these objectives could be expressed as reducing certain risks (e.g., those of ruin or non-payment of dividends) to an acceptable level. For most, if not all, of these objectives there exists a critical value of a related random variable such that a lesser outcome would define an undesirable event whose risk is to be made acceptable. Thus, there could be a large number of risks that a DM would like evaluated and there will normally be trade-offs between conflicting objectives and risks. The problem of balancing such trade-offs is complex and interesting, but it is outside the scope of this paper; our primary concern is with the precise definition and evaluation of risks whose control constitutes an essential objective of a firm.

Ruin would occur if the firm's cash position became such that it could not meet its financial obligations in a manner satisfactory to its creditors and they took over. If ruin occurred, none of the other firm's objectives could be realized. Therefore, it seems reasonable to regard ruin as the most serious of all outcomes. To facilitate the presentation, we postulate that the firm's objective is to maximize its future internal wealth, subject to an acceptable risk of ruin; we are particularly concerned with its evaluation and will assume that the firm has identified a capital rationing decision procedure effective in maximizing future internal wealth<sup>2</sup>

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1 Hertz, for example, states: "The methods described in this article assume that uncertainty- that is, the spread of distribution of potential returns around the "expected value," or average of all outcomes- is a useful measure of risk." [4]

2 For a discussion of the relative effectiveness of various decision procedures see [2,11,13].

### 2.3 Variance as a Surrogate Measure of Risk.

Mathematically oriented capital budgeting theoreticians have tended to avoid, as stated earlier, probability of occurrence as a measure of risk and to adopt, instead, a surrogate measure of risk, which is in reality a measure of dispersion. The variance of the (rate of) return,  $R$ , of an investment is a very popular surrogate.<sup>3</sup> One popular "mean-variance" model has the form:

$$E(U) = \bar{R} - ks_r$$

where  $\bar{R}$  is the mean and  $s_r$  is the standard deviation of the random variable  $R$ ;  $k \geq 0$  is a real number specified by the DM; and  $E(U)$  is frequently referred to as (the DM's) expected utility (for the random variable  $R$ ). This model contains several fundamental weaknesses:

- a) standard deviation (variance) is a measure of dispersion, not risk per se. In general, the relationship between a risk associated with  $R$  and the variance of  $R$  can be expressed if, and only if, the probability distribution of  $R$  is known.
- b) In practical capital budgeting situations, the variance of  $R$  may be extremely difficult to evaluate analytically unless  $R$  is a sum of independent random variables.
- c) The most serious risk, that of ruin, can be expressed in terms of a critical value of  $R$  only in very special cases. The relationship between the risk of ruin and the variance of  $R$  may be difficult or impossible to establish (e.g. if a factor independent of  $R$  affects the risk of ruin).
- d) It treats each decision as though it were independent of all other decisions.

The methodology of decision analysis [5] recommends a three phase procedure for analysing risk in capital budgeting: a) information gathering, b) calculation and c) decision. In the calculation phase, the information about uncertainty is used to obtain a probability distribution of the decision criterion, usually net present value. Decision Analysis recommends a Monte Carlo simulation procedure to overcome the limitations of mathematical probability theory in combining the input probability distributions. In the decision phase the analysis is based on stochastic dominance when applicable; otherwise a utility function is estimated and used to compute an expected utility, the risk preference decision criterion.

From the viewpoint of risk evaluation, the basic weakness of this approach is that its focus on expected utility does not address risk per se. Unless first degree stochastic dominance exists, maximization of expected utility does not necessarily minimize (any) risk. Information would be available to evaluate a risk defined on NPV, but the most important risk, that of ruin, is ignored since it cannot be defined by a critical value of NPV.

### 3.0 Potential of Monte Carlo Analysis.

The potential of Monte Carlo analysis has been recognized since the advent of stored program electronic computers. In 1964 Hertz [3] demonstrated its power for dealing with the stochastic nature of an engineering economy decision, but he did not address the full complexity of risk analysis in capital budgeting. Kryzanowski, Lusztig, and Schwab [6] and Thuesen [14] contributed further evidence of the power of Monte Carlo simulation in the analysis of uncertainty in engineering economy decisions. Decision analysis recommends Monte Carlo simulation for obtaining estimates of the probability distribution of the decision criterion.

Oakford and Thuesen [12] studied the relative effectiveness of several capital rationing procedures in an environment of incomplete information and certain cash flows by simulating sequences of decisions in hypothetical firms. Parra and Oakford [13] extended that work to study the relative effectiveness of (a) sequential and batch decision processes and (b) logically exact and approximate selection algorithms. Oakford, Salazar, and DiGiulio [10] studied the long term effectiveness of expected net present value maximization in an environment of incomplete information and uncertain cash flows. Their conclusions include the following statements:

- a) consistent use of Rank On Net Present Value will, in probability, maximize the expected capital growth rate of a firm if the discount rate is properly selected and the expected cash flows are accurately estimated, however
- b) failure to meet either of these conditions can have a seriously adverse effect on the growth rate of the firm, potentially reducing it to that achieved by a random selection procedure.

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<sup>3</sup> We recognize that many models have been developed that attempt to avoid measuring dispersion in the desirable direction as unattractive. Our fundamental objections to this type of model applies equally. For a comparison of different quantitative risk surrogates see [1].

DiGiulio and Oakford [2] studied the effect of errors in estimating differential inflation rates by a multinational firm in a similar environment. Lohmann [7] studied the effect of debt policy on the capital growth rate and risk of ruin of a firm.

An impediment to the use of simulation in the evaluation of risk has been the time consuming and tedious work of writing and testing computer programs to perform the simulation. Special purpose languages, such as SIMSCRIPT and GPSS, designed to facilitate the creation of computer simulation programs exist, but their relatively fixed (queing systems oriented) structures limit their usefulness for studying risk.

We are developing a General Purpose Risk Analysis Simulation Language, henceforth referred to as RAL, that will permit the user to describe both analytical and empirical probability distributions rather easily and to specify almost arbitrary functional relationships among the random variables in a rather "natural" way. The user's "natural" descriptions will be translated into an APL program that will perform the actual simulation and present results in a readily usable form.

#### 4.0 The Decision situation

With few exceptions, capital budgeting models present the unrealistic situation in which the DM has complete information and the decision is independent of all others. Consequently, we must describe the realistic decision situation we visualized in some detail. The current decision is one in a long sequence of periodic, e.g., annual, capital rationing decisions that started long ago and is expected to continue far into the future, a situation representative of an established corporation that co-ordinates its capital rationing decisions with its annual budget review. At each decision time the DM is presented with a set of productive investment opportunities (henceforth referred to as PIOs) summarized by the expected net cash flows that would result from accepting the opportunities. To simplify the presentation, it is assumed that the relatively simple computations required to convert before tax, inflated cash flows to after tax, inflation corrected cash flows have already been performed. At each decision time the DM expects to be presented with a new and different set of PIOs.

In our capital budgeting model, described in [10], we view the PIOs presented and the cash flows realized from those selected as being generated by a complex stable random process. Consequently the capital budgeting process of the firm can be viewed as a stochastic process with an absorbing barrier, ruin, in which the outcome of each decision is a random event (a vector of cash flows) from a complex multivariate probability distribution. If ruin is avoided, and the sequence of decisions is long, then the DM will, in probability, achieve his objective. An attempt to evaluate the elements of the transition probability matrix needed to analyze a typical capital budgeting decision as a stochastic process would encounter the same kinds of limitations that mathematical probability theory would encounter in evaluating risks.

The DM would like an estimate of the probability of ruin associated with the combination of current investment opportunities that promises to maximize the firm's expected future wealth. Formally, the DM would like an estimate for each time  $t = 0, 1, 2, \dots, H$  (where  $H$  is the horizon time for the current decision) of the probability,  $p[t]$ , that the firm's cash position,  $CP[t]$ , will be less than a specified solvency level,  $SL[t]$ . Depending upon its short term credit line and its attitude toward the use of short term credit to finance current investments,  $SL[t]$  may be positive, zero or negative.

The firm's cash position,  $CP[t]$ , at time  $t$  (prior to that capital rationing decision) is determined by the net cash released then as a result of prior investment and borrowing decisions. If  $CP[t]$ , augmented by the unused short term line of credit were less than the solvency level,  $SL[t]$ , the firm would be deemed bankrupt and its sequence of capital rationing decisions would be terminated. Thus a firm might be deemed bankrupt before the current decision as a consequence of prior capital rationing decisions. Another possibility is that the firm would be solvent prior to the current investment decision and become insolvent after it because of errors in estimation of the first costs of the current investments.

An estimate, made at the current time, of the firm's cash position just prior to a future capital rationing decision could be divided into three component estimates of the cash that will be released at that time as a result of:

1. decisions prior to the current one,
2. the current decision, and
3. decisions subsequent to the current one.

An estimate of the first component would involve updating prior forecasts of future cash flows; that of the second component would be based on estimates of future cash flows from investments and borrowings selected currently; and that of the third component would involve estimates of future cash releases from currently unknown investments and borrowings that will appear and be selected in the future.

5.0 A Hypothetical Numerical Example

The limitations of mathematical probability theory and the potential of Monte Carlo analysis for compensating for those limitations are illustrated in the following numerical example. This example, although artificially simple, illustrates estimation of the three components of future cash position and also the limited ability of mathematical probability theory to evaluate risks associated with capital rationing decisions. The random variables and the relationships among them, in this example, were arbitrarily chosen for illustrative purposes.

5.1 The Effect of The Current Decision.

A capital rationing decision (in the visualized environment) involves the selection of a sub-set of investments (and borrowings) from the set of available opportunities. This sub-set can be visualized as a single (composite) investment. The money released at the current time from past decisions is represented by the symbol  $M[0]$ , which is assigned the value of 1.0 in our numerical example. The firm's debt policy requires that a fraction,  $f_d = 0.50$ , of the current budget,  $B$ , will be financed by long term debt, LTD. Formally,

$$B = M[0] + LTD = M[0]/(1-f_d) = 2.00 \tag{1}$$

To assure liquidity, the firm has a policy of investing a fraction,  $f_m = 0.10$ , of its budget in highly liquid market investments that yield  $i = 0.03$ . Consequently it plans to invest (disburse) an amount

$$\hat{P} = -B*(1-f_m) = -1.80 \tag{2}$$

in productive investments that are expected to yield a rate of return  $g_p = 0.25$ ; but, because of uncertainty about  $\hat{P}$ , the actual disbursement will be the random variable

$$P = \hat{P}*RV1 \tag{3}$$

where  $RV1$  represents a random variable with the triangular distribution<sup>4</sup> shown in Figure 1. The firm perceives the most likely payoff from the productive investments would be two equal receipts of an amount

$$R = -P*g_p/(1 - 1/(1+g_p)^2) \tag{4}$$

A two year life was chosen to facilitate presentation of the three components of future cash position.

The firm's uncertainty about the payoff,  $R[1]$ , at time 1 is dependent on the realization of the random variable  $RV1$  as described by the formula

$$R[1] = R * \begin{cases} RV2a & \text{if } 0.75 \leq RV1 < 0.9 \\ RV2b & \text{if } 0.90 \leq RV1 < 1.2 \\ RV2c & \text{if } 1.20 \leq RV1 \leq 1.5 \end{cases} \tag{5}$$

where the random variables  $RV2a$ ,  $RV2b$ , and  $RV2c$  have the triangular distributions shown in Figure 1. The long tail of  $RV2a$  reflects the potential for improved yield when the cost of productive investments is appreciably less than the most likely and vice-versa for  $RV2c$ . In other words, a negative correlation between  $P$  and  $R[1]$  is introduced for extreme values of  $P$ .

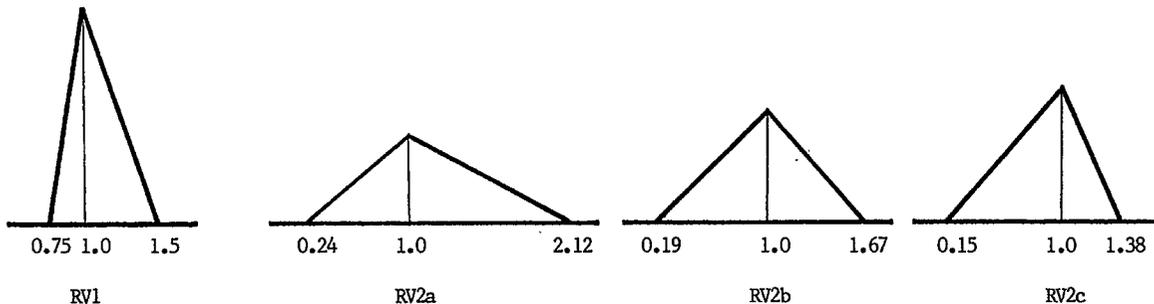


Figure 1.

4 A triangular distribution has the following desirable features: (1) it is easily described by three numbers, (2) easily visualized, and (3) permits both symmetry and asymmetry.

The value of the payoff,  $R[2]$ , at time 2 would be

$$R[2] = R[1]*RV2b \quad (6)$$

The long term debt incurred at the current decision would be

$$LTD = B*f_d \quad (7)$$

which must be repaid in two equal installments with value

$$d = LTD*\underline{i}_d / (1 - 1/(1+\underline{i}_d)^2) \quad (8)$$

where  $\underline{i}_d = 0.06$ . There is no uncertainty about the amount of long term debt incurred or the consequent payments.

If the cost of productive investments were to exceed the budget, the firm would incur short term debt in an amount

$$STD = \max(0, -(B+P)) \quad (9)$$

that must be repaid in one year together with interest at rate  $\underline{i}_d$ . If, instead, the budget were to exceed the cost of productive investments, the firm would invest in market investments the amount

$$MKT = \max(0, B+P) \quad (10)$$

for one year at interest rate  $\underline{i}$ .

The net amount released next year and the year after as a consequence of the current investments and borrowings would be

$$S[1] = R[1] - d - STD*(1+\underline{i}_d) + MKT*(1+\underline{i}) \quad (11)$$

$$S[2] = R[2] - d \quad (12)$$

and the net present value of the current decision would be the random variable

$$NPV = -M[0] + S[1]/(1+g_m) + S[2]/(1+g_m)^2 \quad (13)$$

In evaluating risks associated with this decision, the firm might desire probability distributions of any or all of the random variables--  $P$ ,  $STD$ ,  $MKT$ ,  $S[1]$ ,  $S[2]$ , or  $NPV$ . Of these, the distribution of  $P$  is given and those for  $STD$  and  $MKT$  could be derived mathematically, but, even in this simple example, those for  $S[1]$ ,  $S[2]$ , and  $NPV$  could not.

## 5.2 The Effect of Past Decisions on Future Cash Flows

The future net cash flows currently anticipated from a past decision are a subset of the net cash flows anticipated at the time of that decision. It seems likely, however, that the descriptions of the relevant random variables or functional relationships might be modified to reflect information acquired since the decision was made. The relevant variables could be combined in RAL with those for the current decision.

To illustrate, we now consider the distributions of  $M[1]$ , the cash that will be released next year, and  $M[2]$ , that released the following year, as a result of decisions made prior to the current one. To simplify the discussion, we will assume that the current decision of our example is representative, statistically, of the two prior decisions. This assumption permits us to derive the distribution of  $M[1]$  by first recognizing that all the components of  $M[0]$  are currently known realizations of the random variables appearing on the right hand side of the formula

$$M[0] = R_{-1}[1] + R_{-2}[2] + MKT_{-1}*(1+\underline{i}) - d_{-1} - d_{-2} - STD_{-1}*(1+\underline{i}_d) \quad (14)$$

where the subscripts  $-1$  and  $-2$  refer to decisions made one and two years ago. Consequently,

$$M[1] = R_{-1}[2] - d_{-1} = R_{-1}[1]*RV2b - d_{-1} \quad (15)$$

and  $M[2] = 0$ .

Consequently, the formula for the firm's cash position one period hence would be

$$\begin{aligned} CP[1] &= M[1] + S[1] \\ &= R_{-1}[1]*RV2b - d_{-1} + R[1] - d - STD*(1+\underline{i}_d) + MKT*(1+\underline{i}) \end{aligned} \quad (16)$$

The distribution of CP[1] could not be determined by mathematical analysis but its cumulative relative frequency, an empirical estimate of the cumulative distribution function henceforth called CRF, shown in Figure 2, was obtained by introducing M[1] and repeating the earlier simulation.

5.3 The Effect of Future Decisions

RAL can be used to simulate all three components simultaneously. To illustrate, we return to the example and consider the value of the firm's cash position two periods hence, CP[2], which will be affected by the next decision as well as the current and past one. CP[2] will be the sum of M[2], S[2], and S+1[1] resulting from the past, current, and proximate decisions. In this example, as in a typical real situation, the DM will not know, at the current time, the actual investment and borrowing opportunities that will be available for consideration at the proximate decision time. We can, however, use RAL to simulate a 'typical' investment of the realization of CP[1] and use its realization as an estimated realization of S+1[1]. This result could be used to obtain a CRF for CP[2], as shown in Figure 2. The DM would then have a basis for assessing the risk of ruin associated with the current decision.

We ask how a DM should interpret a CRF for the Net Future Value

$$NFV(g_m) = NPV(g_m) * (1 + g_m)^2 \tag{17}$$

of the current decision? Considered by itself,  $NFV(g_m)$  would represent an estimate of the difference between the value to the firm of the current decision and that of the hypothetical alternative of investing its entire budget at rate  $g_m$ . Are there any practical interpretations of  $g_m$  that would be meaningful? If  $g_m$  represented the average growth rate of the firm, where would the firm find the implied alternative investment? If, instead,  $g_m$  represented the growth rate of future marginal investments, as it should in an incremental analysis, the value of  $NFV(g_m)$  (or  $NPV(g_m)$ ) per se would have no meaningful practical interpretation, but the difference between the values of  $NFV(g_m)$  for alternative capital rationing decisions would -- it would represent the prospective monetary difference between the alternative decisions [8,9,11].

5.4 Analysis of Results.

The CRF of MKT-STD, together with a value of the firm's line of short term credit, provides a basis for estimating the probability that the firm will become insolvent at  $t = 0$ , after the current investment decision has been implemented. That probability would be approximately 380/1000 (see Figure 2a.) if the firm's line of short term credit were zero, and would decrease to about 1/1000 if it were one and a half times M[0]. (The simulation allowed unlimited short term credit.)

Similarly, the CRFs of CP[1] and CP[2] provide information for estimating the risk of ruin at times 1 and 2, prior to the actual decisions then. Recall that the estimate of CP[2] required a simulated decision at time 1. These risks would be about 15/1000 and 34/1000, if the firm's line of short term credit were zero. A line of short term credit equal to M[0] would reduce those risks to less than 1/1000. The probabilities that S[1], S[2], and NFV would be negative were about 43/1000, 119/1000, and 395/1000, but none of these measure the risk of ruin, even indirectly, so the CRFs for those variables are not displayed.

The results of the simulation indicate further that the firm's capital growth rate would be approximately 34% provided that the current investment were typical of future ones and ruin were avoided.

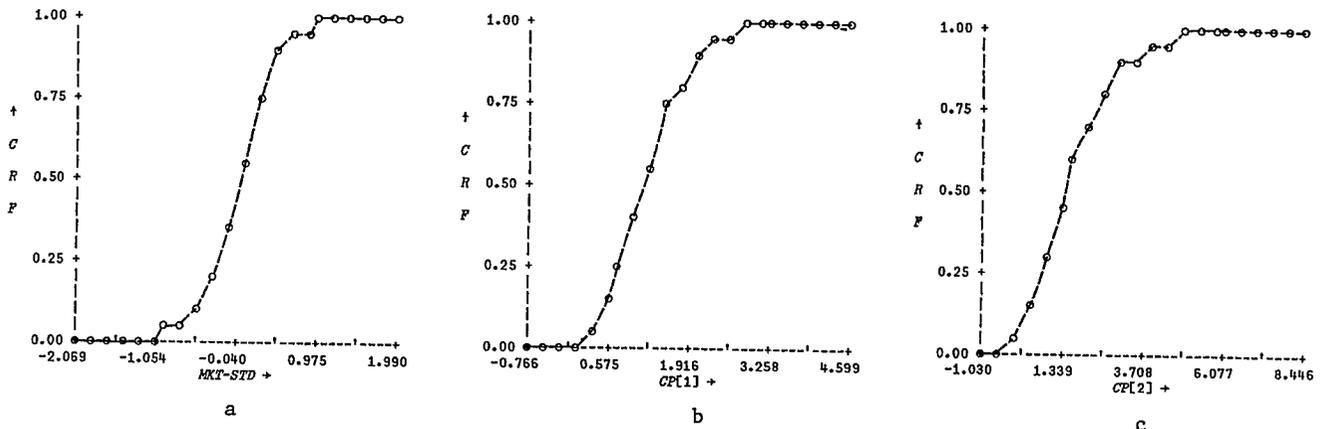


Figure 2. CRFs of MKT-STD, CP[1], and CP[2]

## 6.0 Requirements of a "Risk Simulation Language"

One of the objectives in a capital budgeting decision is to limit the risk of ruin to an acceptable level. To (properly) assess this risk, the DM would need probability distributions of the net cash flows from (or to) the firm at times  $t = 0, 1, 2, \dots, H$ . These probability distributions are determined by random variables to which component cash flows are functionally related. For example, market acceptance of a product is a causal random variable that influences the firm's sales volume, which, in turn, will affect sales revenue and cost of sales. In its turn, market acceptance may depend upon other causal random effects, such as the state of the economy in general or in a particular region or industry. Depreciation allowances furnish a more explicit example when estimated first cost is recognized as a random variable, because the functional relationship between first cost and tax savings from depreciation is established by accounting practice.

Thus we have independent and dependent random variables. The latter are functionally related to an independent variable either directly or else indirectly through another dependent variable. To study our capital rationing problem properly, we need to estimate the probability distributions of causal random variables and formulate the functional relationships among random variables that ultimately define the distributions of the net cash flows at times  $t = 0, 1, \dots, H$ .

If evaluation of risk is to be facilitated, we believe that the "simulation language" should permit the user to describe random variables and functional relationships among them in a language that is intended to facilitate communication with another person rather than with a computer. For example, the language would accommodate the formal descriptions appearing in the numerical example. The "simulation language" should impose no more structure on the description than would be imposed by editorial requirements of a journal in which the results of an experiments might be reported.

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V CAPBUDSIM
[1] A
[2] A --- PAST DECISION
[3] DECPROC 1*1.25
[4] SPD+R[;2]-D
[5] A
[6] A --- CURRENT DECISION
[7] DECPROC 1
[8] S[;1]*R[;1]+(MKT*1+I)-(D+(STD*1+ID))
[9] S[;2]*R[;2]-D
[10] NPV+GM CNPV(-1),S
[11] CP[;1]*SPD+S[;1]
[12] A
[13] A --- FUTURE DECISION
[14] DECPROC CP[;1]
[15] CP[;2]*S[;2]+R[;1]+(MKT*1+I)-(D+(STD*1+ID))
V

V DECPROC MO
[1] B+MO*(1-FD)
[2] P*-B*(1-FM)
[3] RV1+1000 SAMPLE TRIANG 0.7 1 1.5
[4] P*P*RV1
[5] R*-P*GP*(1-1*(1+GP)*2)
[6] RV2*TRIANG(0.24 1 2.12) OR(0.19 1 1.67) OR(0.15 1 1.38)
[7] R*R*SAMPLE(RV1, 0.9 1.2 1.5) SELECT RV2
[8] R*R AND R*1000 SAMPLE TRIANG 0.19 1 1.67
[9] LTD*B*FD
[10] D*LTD*ID*(1-1*(1+ID)*2)
[11] STD*MAX 0 AND(-B+P)
[12] MKT*MAX 0 AND(B+P)
[13] A
[14] A --- DECISION PROCEDURE
[15] A
V

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Figure 3.

## 6.1 Current Status

Figure 3 displays the RAL program that evaluated the CRFs of  $S[1]$ ,  $S[2]$ ,  $NFV$ ,  $CP[1]$ ,  $CP[2]$  and  $MKT-STD$ , of which the last three are displayed in Figure 2. The main program,  $CAPBUDSIM$  has three sections which compute the effects of past, current and future decisions on future cash flows. In each section a twelve line procedure  $DECPROC$  is called.

The first line in  $DECPROC$  is just formula 1 for evaluating  $B$  from the formal description of the numerical example. The second line evaluates  $P$  (formula 2 in the formal description). Line three describes the triangular distribution of the random variable  $RV1$  and causes 1,000 realizations to be generated. The fourth line is just formula 3 that evaluates realizations of  $P$ , while line five is formula 4 that evaluates  $R$ . The sixth line describes the distributions of the three conditional random variables  $RV2a$ ,  $RV2b$ ,  $RV2c$  and line 7 describes the rule for choosing among them, and evaluates realizations of  $R[1]$  (formula 5). Finally lines nine through twelve are simply formulas 7 to 10 for evaluating  $LTD$ ,  $D$ ,  $STD$ , and  $MKT$ .

The third line in  $CAPBUDSIM$  causes  $DECPROC$  to evaluate 1,000 realizations of the previous decision with  $M_{-1}[0]=1/1.25$ , and assigns the value of the cash flow  $S[2]$  for that decision to  $SPD$ . The seventh line causes  $DECPROC$  to evaluate realizations of the current decision with  $M[0]=1$ . Lines 8 and 9 evaluate the realizations of  $S[1]$  and  $S[2]$  as specified by formulas 11 and 12. The tenth line evaluates  $NFV$ . Line 11 completes the evaluation of the realizations of  $CP[1]$  as specified by formula 16. Line 14 causes  $DECPROC$  to evaluate realizations of the proximate decision with  $M_{+1}[0]=CP[1]$ . Line 15 completes the evaluation of  $CP[2]$  by adding the realizations of  $S_{+1}[1]$  and  $S[2]$ .

## 6.2 Plans for the Future

This example indicates that one who is conversant with APL should have little difficulty in programming a solution to almost any risk evaluation problem. Our objective, though, is to provide for a large body of potential users by eliminating their need to be conversant with APL. To accomplish this, we plan to develop software that will translate descriptions of probability distributions of random variables and functional relationships among them to an equivalent APL program that will use Monte Carlo simulation to generate CRFs (and other useful statistics) of specified random variables and evaluate estimates of probabilities of specified undesirable events.

Provision must be made for a complete set of discrete and continuous theoretical distributions, for almost arbitrary discrete or piecewise continuous empirical distributions, e.g., histograms, and for decision trees. In addition, provision must be made for describing almost arbitrary conditional relationships among random variables.

## 7.0 Conclusion.

In this article, we have illustrated the inability of mathematical probability theory to provide formulas for (properly) evaluating risks in most practical capital budgeting situations. We have also shown that most so-called risk analysis procedures appearing in the capital budgeting literature confuse risk with uncertainty and provide no information that would enable the DM to evaluate the ultimate risk, that of ruin. Since most of these models represent unrealistic situations of complete information, we describe a (realistic) situation of incomplete information and uncertain cash flows that would be representative of those firms that coordinate capital budgeting decisions with annual budget reviews. We have used the current version of our General Purpose Risk Analysis Language, RAL, in a hypothetical numeric example to compute estimates of probability distributions that a DM would need to (properly) assess the risk of ruin associated with a particular capital budgeting decision. The example illustrates the power of APL to perform a Monte Carlo analysis for a wide range of risk evaluation problems. Our objective, in the development of RAL, is to provide a risk evaluation tool for the wide body of (potential) users who would not be conversant with APL. The restrictions imposed on the user in his description of the relevant random variables and the functional relationships among them should be no greater than those of a good journal in which he might publish his results.

REFERENCES

- [1] Cooley, P. L., R. L. Roenfeldt, and N. K. Modani, "Interdependence of Market Returns," Journal of Business, Vol. 50, No. 3, 1977, pp. 356-363.
- [2] DiGiulio, H.A. and R.V. Oakford, "An Application of Simulation for Studying the Multinational Capital Budgeting Problem," Proceedings of the Winter Simulation Conference 1978, Miami Beach Fla., Dec.1978, pp. 931-942.
- [3] Hertz, David B., "Risk Analysis in Capital Investment," Harvard Business Review, January-February, 1964.
- [4] Hertz, David B., "Investment Policies That Pay Off," Harvard Business Review, Jan.-Feb. 1968.
- [5] Howard, Ronald A., Readings in Decision Analysis, Decision Analysis Group, Stanford Research Institute, Menlo Park, 2nd Ed., 1977.
- [6] Kryzanowski, L., P. Lusztig, and B. Schwab, "Monte Carlo Simulation and Capital Expenditure Decisions: A Case Study," The Engineering Economist, Vol.18, No. 1, Fall 1972, pp. 31-48.
- [7] Lohmann, Jack R., "Understanding Risk in Capital Budgeting Decisions," Presented in the 87th Annual Conference of the American Society for Engineering Education, Baton Rouge, Louisiana, June 25-28, 1979.
- [8] Oakford, R.V., Capital Budgeting, Ronald Press Co., New York, 1970.
- [9] Oakford, R.V., Sultan A. Bhimjee, and Arturo Salazar, "The Maximum Prospective Utility Criterion, Utility Maximization in Capital Rationing Under Incomplete Information," Presented at the TMS/ORSA Conference on Capital Budgeting, Phoenix, Arizona, Feb. 1, 1978.
- [10] Oakford, R.V., Arturo Salazar, and H.A. DiGiulio, "The Long Term Effectiveness of Expected Net Present Value Maximization," Proceedings of the 1979 Fall Industrial Engineering Conference, Houston, Texas, Nov. 1979.
- [11] Oakford, R.V. and G.J. Thuesen, "The Maximum Prospective Value Criterion," The Engineering Economist, Vol. 13, No. 3, Spring 1968, pp. 141-164.
- [12] \_\_\_\_\_, "The Effectiveness of the Maximum Prospective Value Criterion for Capital Budgeting Decisions," Proceedings of the 19th Annual Institute Conference and Convention, AIIE, May, 1968, pp. 271-278.
- [13] Parra-Vasquez, A.S. and R.V. Oakford, "Simulation as a Technique for Comparing Decision Procedures," The Engineering Economist, Vol. 21, No. 4, Summer 1976, pp. 221-236.
- [14] Thuesen, Gerald J., "Nuclear Versus Fossil Power Plants: Evolution of Economic Evaluation Techniques," The Engineering Economist, Vol. 21, No. 1, Fall 1975, pp. 21-38.