A COMPUTER SIMULATION MODEL FOR PORTFOLIO STRATEGY FORMULATION

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ABSTRACT

A working computer simulation model for formulating investment strategy for a portfolio of capital assets is presented.

The use of modern capital market and portfolio theories in a flexible simulation network allows an investor to directly examine and compare the probable consequences of various static and dynamic investment and consumption policies and facilitates his decision-making process. Description of the model is illustrated by its application to the endowment portfolio of a university.

I. INTRODUCTION

This paper presents a computer-based model to assist investors in the formulation of a strategy for their long-term investment portfolios. The model synthesizes developments of modern portfolio and capital market theories and computer science so that investors may benefit from them without having to digest complicated details of mathematics and programming. After a brief review of some aspects of portfolio and capital market theories and computer simulation, we shall describe the model which was designed to assist the trustees of a university in the management of the university's endowment portfolio. The model is specific to the university endowment problem only to the extent that it takes into consideration the special constraints which might be imposed by the donors on spending the principal amounts of their gifts. Otherwise, the model is quite general and can be readily and usefully applied to the management of other portfolios.

PORTFOLIO STRATEGY

The owners or the trustees of a portfolio must define the objectives they expect their portfolio manager to achieve. Such a definition also provides a standard of evaluation for the portfolio manager's performance. Traditionally, such objectives have been defined in terms of maximization of returns (yield and gain), subject to certain constraints. Constraints might be placed on the minimum acceptable quality of securities in the portfolio, exclusion or inclusion of certain industries, proportions of the portfolio invested in fixed income securities, liquidity of the portfolio and the minimum acceptable yield (interest plus dividends). Modern portfolio theory suggests that risk of the entire portfolio, and not the risk of individual securities in the portfolio, should be the primary object of choice in the formulation of a portfolio strategy. The risk of individual securities is relevant only to the extent that it contributes to the risk of the entire portfolio. In a well-diversified portfolio, highly risky securities can be held without adding undue risk to the portfolio. The average return and the risk of securities are found to be positively associated. More risky securities offer higher returns on average. (7,9) The basic decision parameter for investors is the level of portfolio risk appropriate to their tastes. If they choose a high level of risk, they can expect that their portfolios, if efficiently managed, will bring them higher returns on average. The choice of a low-risk portfolio will have to be accompanied by lower expectations of returns.

Several studies (8,9,15) of the performance of mutual and other large funds have consistently shown that a portfolio cannot be expected to always "beat the market," however good the management of the portfolio might be. Therefore, investment objectives should be specified not in terms of beating the market but in terms of the level of riskiness of portfolio. Given a certain level of riskiness, efficient portfolio management can, on an average, be expected to produce a certain level of returns. Actual year to year returns, of course, will depend on the performance of the market portfolio in a given year. The model described in later sections of this paper is designed to assist the investors in understanding and specifying the key portfolio parameter to their portfolio managers--the level of risk of their portfolio.

PORTFOLIO RISK

Risk is generally understood to denote uncertainty of future events. A statistical measure of the risk of a portfolio may be made by estimating variability of returns on the portfolio. Standard deviation and mean absolute deviation are two such measures of variability. Total risk (or variability of returns) of a portfolio consists of two parts. One part, called the market risk, arises from price movements which are common to all securities in the market. The other part arises from price movements specific to the portfolio. The latter part is known as diversifiable risk because it can be eliminated entirely by diversification of the portfolio. For example, market portfolio is a fully diversified portfolio with no diversifiable risk. The market risk of a portfolio, however, cannot be eliminated by diversification. Market portfolio has a market risk of one.

Market risk of a portfolio is variously known to the professionals as volatility or sensitivity. If a market index change of ±5% is accompanied by a portfolio price change of ±5%, the market risk or volatility of the portfolio is one. If corresponding portfolio changes are ±10% and ±2.5%, then the volatility of the portfolio is 2 and 0.5, respectively. By preferring a high volatility portfolio, the investor opts for the possibility that the value of the portfolio may increase or decrease sharply, combined with higher average returns. By preferring a low volatility portfolio, the possibility of large positive or negative changes in the value of the investment is reduced but the average returns on the portfolio are also reduced accordingly. The most important decision for the portfolio strategist is to set the desirable level of market risk of his portfolio.

Investors often find it difficult to relate their risk preferences to a specific level of risk in their portfolio. This may be partly due to incomplete comprehension of its meaning and partly due to insufficient information on the consequences of choosing a certain level of risk. The result is that very considerable advances made in portfolio theory have remained unknown or underutilized by the investors. Relatively little research has been done on the ways of assisting the investors to choose the preferred risk levels.[1] The model described in the later parts of this paper is specifically designed to provide such assistance to the investors. It consists of producing and presenting the investors with probable consequences of portfolio options open to them. By direct examination of such consequences, we hope, the investors can make better strategic choices for their portfolios.

Probable consequences of portfolio decisions are generated by computer simulation. While a detailed description of the technique itself is beyond the scope of our paper (and can be found in several good references [12,14]), we shall describe it very briefly to maintain the continuity of this paper for those who may not be familiar with computer simulation.

COMPUTER SIMULATION

Computer simulation involves a hypothetical replication of real-world events. The first step in simulation consists of making a mathematical model of the events to be investigated and the effect of various policies on such events. Few future events, especially in the investment world, are known with certainty. It is assumed that even if a particular occurrence of an event cannot be predicted with certainty, the pattern and frequence of occurrences are known. This is referred to as probability distribution by the statisticians. For example, although we do not know whether a particular toss of a coin will yield heads or tails, we do know that if a fair coin is tossed many times it will yield heads about half the time. We also know that the result of a toss does not provide any help in predicting the result of the next or any future toss; in other words, results of the coin tosses are independent of each other.

In a simulation study of stock market behavior, we need a mathematical model of the behavior of and relationship between the stock prices. Though the stock prices are highly unpredictable, a statistician can find orderliness in the apparent disorder. Cootner (4) and Fama (6) present statistical models of stock-price behavior. These models suggest that for a given stock the frequency of occurrence of positive and negative relative price changes of various sizes follows a definite pattern and can be approximated by a "normal probability distribution." The same is true of the market index changes. If we assume that the stock market in the future will have the same basic behavior as it has had in the past, computer programs can be written to generate numbers which will appear similar to the price changes that the stock market is expected to produce. Note that a single price change produced by computer simulation may have no relevance to any real-life situation, but when such behavior is replicated many times, the relative frequencies of various price changes obtained will be close to reality.

By running a computer program for a given investment policy many times over, information on the likely consequences of the policy can be obtained. By comparing the consequences of various policies, an investor can choose a policy which is likely to yield consequences most preferable to the investor. The use of such an approach in management of portfolios has been suggested by Williamson (16), and Meckling and Jensen (11) for the university endowments and by Lorie and Hamilton (10) for the pension funds. Cohen and Elton (5) and Salazar and Sen (13) suggested the use of this approach for more general problems of portfolio management and capital budgeting respectively.

In the next section, the essential features of a model developed to assist investors to select an investment strategy are described. In Section III of the paper, information on likely consequences of two specific investment strategies is presented. Beyond this, information on consequences of any investment policy can be generated as a routine matter with the help of the computer program. An attempt has been made to present information in a format which is comprehensible to nonexperts in the theory of finance or statistics.

II. THE MODEL

The overall organization of the model is shown in Exhibit 1. Common stocks and bonds are the two types of capital assets available to the investor.[2] The portfolio can be divided between these two assets in any proportion. The model is discrete in the sense that the investment decisions are made, results are observed, and income is received only at specified intervals of time. This interval can be a month or any multiple of a month. The number of periods in a simulation run is limited only by the storage capacity available in the computer. For example, if the storage capacity permits ten periods, and a 12-month period length is chosen, the model can be used for portfolio simulation over ten years. Simulation over ten three-month periods will span an interval of two-and-a-half years. Thus, the model is flexible enough to suit the varying planning horizons of different investors.

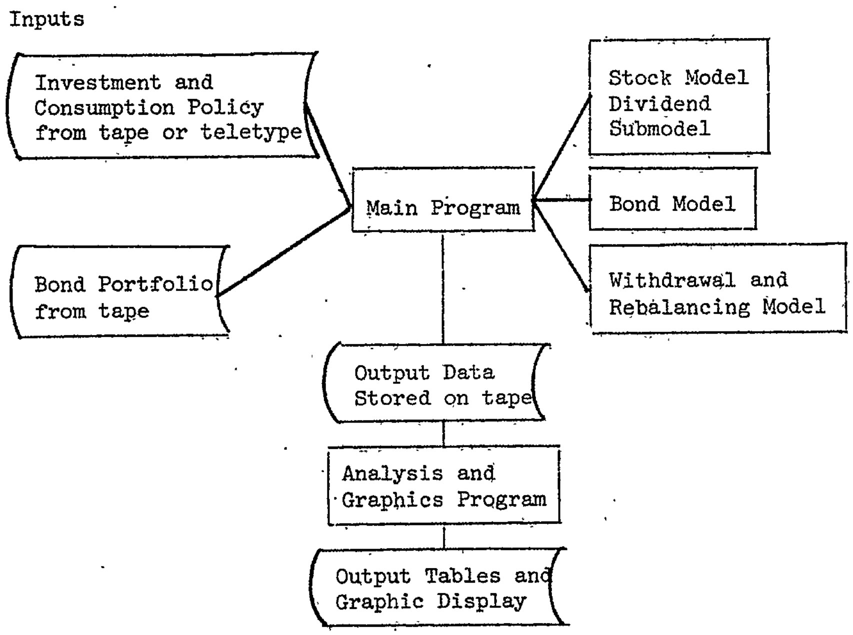
to the market. In addition to selecting a value of beta, the investor also specifies the number of different stocks in his portfolio to provide a measure of diversification.

Results from simulation of stock portfolio behavior are critically dependent on the assumed behavior of individual stock prices and the stock market. A brief discussion of the inherent assumptions of the model in this respect will be useful here.

Strong empirical evidence provided by Fama (6) and others (see Cootner [4]) supports the view that successive returns on common stocks are independent of each other. The probability distribution of the returns can be approximated by a bell-shaped curve--the normal distribution. Though deviations from normal probability distribution have been documented (see Fama [6] and Blattberg and Gonedes [2]), we believe that normal distribution provides reasonable approximation for purposes of this study. A normal probability distribution implies slightly fewer numbers of very small and very large price changes than are observed in the market data. Use of normal distribution considerably simplifies the implementation of the model on computers and is therefore justified in our opinion.

EXHIBIT 1

ORGANIZATION OF THE MODEL



COMMON STOCK PORTFOLIO

The investor selects the proportion of stocks in his portfolio in the range from zero to one hundred percent. The most important strategic decision about the stock part of the portfolio is its volatility (alternatively referred to as market risk or beta) with respect to the market. For large portfolios, feasible values of beta may be selected from the range 0.5 to 2.5. Values outside this range are harder, though not impossible, to obtain for large portfolios. As has been explained in an earlier section, a higher value of beta implies that on the average, the returns from the portfolio can be expected to be higher and the portfolio is more risky. If the investor expects the average market return in the future to be ten percent per annum, he can expect beta times ten percent return from his stock portfolio on average. Standard deviation of returns from his stock portfolio[3] also will be beta times the standard deviation of returns from the market portfolio. Thus, choice of a particular value of beta defines the relationship of the stock portfolio

Returns on portfolios, including the market portfolio, are also distributed normally. Successive market returns are independent of each other. Therefore, a series of pseudo-normal numbers with given average and standard deviation are used in simulation to represent the returns on the market portfolio. Average and standard deviation of market returns are selected by the investor to represent his estimate of the market performance in the future. Average and standard deviation of monthly returns on the New York Stock Exchange stocks during the forty-two year period from 1926 to 1968 are shown in Exhibit 2. These numbers may help the investor make his own estimates of the market behavior in the future.

EXHIBIT 2

HISTORIC AVERAGE AND STANDARD DEVIATION OF MONTHLY RETURNS ON NEW YORK STOCK EXCHANGE INDEX

	Period		
	2/26-6/68	2/26-12/45	1/46-6/68
Average Monthly Return	1.38%	1.62%	1.17%
Std. Dev. of Monthly Returns	8.53%	11.65%	4.13%

Returns on stock portfolios can be described by a simple relationship with the market returns:

Return on portfolio = beta × return on the market portfolio + a random error term.

The random error term is normally and independently distributed. On average, this term is zero and its standard deviation depends on the degree of diversification in the portfolio. The greater the diversification, the smaller the standard deviation. Therefore, for a well-diversified portfolio, this term is very small.

In summary, the user of this model must specify the proportion of his total investment to be invested in stocks, the beta or market risk of the stock portfolio, his estimate of average and standard deviation of the market returns in the future and the number of different stocks in the portfolio.

The managers of a university endowment are often concerned not only with the total return on stocks but also with the dividend income. While we do not agree with this concern,[4] the existence of legal constraints on spending capital gains from certain restricted gifts necessitated the incorporation of a dividend submodel in the stock part of the model. The information on decomposition of the total return between dividend and capital gains generated by this feature of the model may be useful to certain types of investors. This submodel does not in any way affect the generality of the model for the investors to whom the decomposition of total returns between dividends and gains is of no consequence.

FIXED INCOME PORTFOLIO

In its present form, the model assumes that all bonds are of the same quality and a common rate of yield is applied to the entire bond portfolio. However, the expansion of the model to include more than one class of bonds presents no difficulty and only minor programming modifications are necessary.

Bond yield is assumed to follow a random walk. In other words, it is assumed that yield at two successive points of time differs by a normally distributed independent random variable with mean zero and a specified standard deviation. Initial value of the bond yield and standard deviation of changes in bond yields must be specified by the user of the model. One possibility is to specify the current market yield for the appropriate class of bonds of, say, twenty years to maturity, as the initial value of yield. Standard deviation of annual yield changes on AA bonds is about 0.005. This might be used as a reasonable approximation.

In addition to the two parameters mentioned above, the user is required to provide information on his bond portfolio. If he already has a bond portfolio and wishes to simulate its behavior over time, the principal amounts, coupon rate, time to maturity for each security are specified. Given the current yield, the market value of each bond is recomputed in each period and the current value of the portfolio is reported along with interest income. Matured bonds are replaced in the portfolio by bonds of the same coupon rate and twenty years to maturity.

As has been mentioned earlier, if a single yield rate is considered inadequate to simulate all the bonds in the portfolio, the model can be slightly altered to include different risk classes of bonds.

WITHDRAWALS AND REBALANCING

This part of the model is designed to accomodate different expenditure patterns of the investors. Since the ultimate purpose of investment is consumption, investors may like to know what consumption levels are commensurate with a specified investment strategy. Such information may be valuable in selecting the investment strategy. Computer simulation enjoys a major advantage for simulation of consumption behavior because it is possible to simulate and evaluate various kinds of dynamic consumption policies in the model. For example, instead of planning for a specified dollar of consumption, an investor may like to know the consequences of consuming a given fraction of the current market value of his portfolio during each year. The model is fully capable of simulating this and more complex consumption policies.

Another feature of this part of the model is designed to meet specific needs of university endowments with respect to restrictions on spending capital gains on certain portions of their portfolio. Though the legal necessity as well as the economic justification for such restrictions are highly questionable (see [1], [3], [11]), we have designed the model to keep separate accounts of the discretionary and restricted parts of the portfolios. Investors who believe in justification of these restrictions may adopt different investment strategies for the two parts of the portfolio. The present model will be helpful in making such decisions. This feature of the model can be entirely bypassed by other investors simply by specifying that the proportion of their money in the restricted part of the portfolio is zero.

The reinvestment of cash income and periodic rebalancing of portfolios is another feature of this part of the model. Investors may choose to reinvest interest and dividend income in stocks and bonds in any proportion. Since relative market values of the stock and bond portfolios will change from time to time, the transfer of funds from one type of assets to another may be desirable. The model is capable of simulating portfolio rebalancing policies.

TRANSACTIONS COSTS

In its present form, the model assumes that transactions costs are zero. These costs are highly dependent on the volume of speculative trading by portfolio managers and can be substantially reduced by adopting a buy-and-hold strategy. However, if an investor feels that these costs are a necessary part of his portfolio simulation, these too can be incorporated in the model.

III. SAMPLE RESULTS

In this section we present the results from a sample simulation of the performance of a large portfolio under specified investment policies. Space limitations prevent us from giving a comparative analysis of portfolio performance under different investment policies. The following discussion, however, should provide considerable understanding of the capabilities of the model. The computer program was written in Fortran IV and was implemented on IBM System 360-67 TSS at Carnegie-Mellon University.

The total market value of the initial investment is 99 million dollars. Of this, $50.2 million is to be invested in a stock portfolio with 48 different stocks and a market risk of 0.8. The average and standard deviation of expected market returns are 9% and 18.9% respectively. The remaining $48.8 million is to be invested in 171 different bonds with a current average yield to maturity of 6% per annum. Standard deviation of yearly changes in yield to maturity is 0.005.

Out of a total of $99 million, $55.7 are permanent funds in the sense that only interest and dividend income from these funds can be consumed. The balance is a discretionary fund which carries no such restrictions. The investor has decided to invest 16.6% of the permanent fund in stocks and 83.4% in bonds. Similarly, 94.6% of the discretionary funds has to be invested in stocks and the balance in bonds. The investor plans to withdraw for consumption $5.6 million at the end of the first year and will increase his consumption by $0.1 million after every three years.

Since the dividend submodel assumes a dividend payout rate which is dependent on total return and payout rate for the stock market during the previous three years, this initializing information also is provided by the investor. During the previous three years, stock market returns and dividend payout rates were 10%, 15%, 13% and 4%, 5%, 5.5%, respectively.

Suppose the investment strategist wishes to examine the results of the above mentioned policy and assumptions over the next ten years on a yearly basis. Stocks and bonds are the two asset classes. He believes that one hundred iterations of the model will give him sufficient information about the likely results of this investment strategy and consumption pattern. All the input information described here is summarized in Exhibit 3. This information and the data on individual bonds (principal, coupon rate and years to maturity) are supplied to the computer program. The output of the program consists of values of fifteen different output variables in each period and each iteration of the simulation. This output is stored on a tape or disc storage device. A second program is then used to provide detailed analysis of any aspect of portfolio performance that the investor may like to have. The fifteen variables on which information is provided are listed in Exhibit 4. We shall now give a few examples of performance analysis and the format of the output presented to the investor.

EXHIBIT 3

INPUT INFORMATION FOR PORTFOLIO SIMULATION

Number of Periods (NP)	10
Length of Each Period (PERLEN)	12 months
Number of Iterations (NI)	100
Number of Asset Classes (NASS)	2
Initial Value of Total Portfolio (VALUE)	$99.0 million
Initial Value of Stock Portfolio (VIN(1))	$50.2 million
Number of Stocks in the Portfolio (NSTOK)	48
Beta of Stock Portfolio (BETA)	0.8
% Average Return on Stock Market (DRETM(1))	9.0
Standard Deviation of Return on Stock Market (DSTD(1))	0.189
% Average Bond Yield to Maturity (DRETM(2))	6.0
Standard Deviation of Bond Yield Changes (DSTD(2))	0.005
Number of Bonds in the Portfolio (NBOND)	171
Initial Value of Permanent Fund (VPERM)	$55.7 million
Initial % of Permanent Fund in Stocks (SHARE 3)	16.6

Initializing Values:

	Year 0	Year 1	Year 2
Dividend Yield on Stock Portfolio	0.04	0.05	0.055
Total Return on Stock Portfolio	0.10	0.15	0.13

Planned Withdrawals: (in millions of dollars)

Year	1	2	3	4	5	6	7	8	9	10
Amount	5.6	5.6	5.6	5.7	5.7	5.7	5.8	5.8	5.8	5.9

Principal amount, coupon rate and years to maturity of bond portfolio are also supplied to the program.

EXHIBIT 4

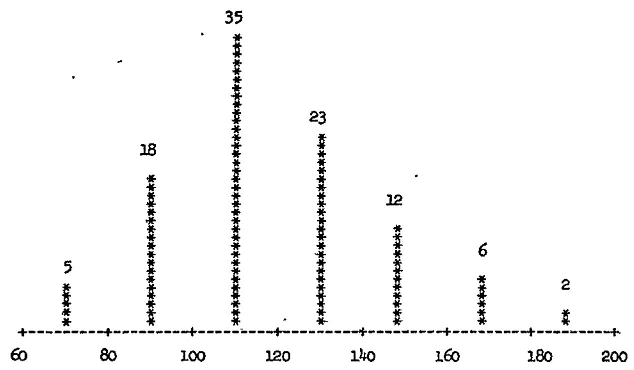
LIST OF OUTPUT VARIABLES

1. Market Value of the Total Portfolio
2. Market Value of the Stock Portfolio
3. Market Value of the Bond Portfolio
4. Market Value of the Permanent Fund
5. Market Value of the Discretionary Fund
6. Dividend Income
7. Interest Income
8. Permanent Fund Income
9. Discretionary Fund Income
10. Withdrawals (for dynamic withdrawals policy)
11. Return on Stock Portfolio
12. Value of Stocks in Permanent Fund/Value of All Stocks (%)
13. Value of Bonds in Permanent Fund/Value of All Bonds (%)
14. Value of Stocks in Permanent Fund/Value of Permanent Fund (%)
15. Value of Stocks in Discretionary Fund/Value of Discretionary Fund (%)

Probably the most important information to the investor is the likely market values of the portfolio at different points in time in the future. In this simulation (over a ten-year period with one hundred iterations) one hundred values of the portfolio are obtained for each of the ten years. The investor cannot easily comprehend the meaning and significance of so many numbers. These numbers can be aggregated and summarized to be more informative. Exhibit 5 is a histogram of portfolio values five years after the initial investment. The height of each bar

EXHIBIT 5

HISTOGRAM OF THE MARKET VALUE OF THE TOTAL PORTFOLIO FIVE YEARS AFTER BEGINNING



Portfolio Value (in millions of dollars)

Average Value = 120 million
Minimum Value = 65.4 million
Maximum Value = 191 million
Most Likely Range = 100 to 120 million

in the exhibit indicates the relative chances of the portfolio value lying in the given range. The most likely range, given the investment and consumption strategy mentioned earlier, is $100 million to $120 million. The expected value of the portfolio five years from the beginning is $120 million. Similar histograms can be prepared by the computer program for portfolio values at the end of each of the ten years.

Another method of summarizing the information from simulation is shown in Exhibit 6. Fractile ranges of the portfolio values at the end of each of the ten years are plotted against time. Letter "M" indicates the median value, i.e., the actual portfolio value is equally likely to be greater and smaller than this value. There is a fifty percent chance that the portfolio value will lie between digits "5" and ninety percent chance that the actual portfolio value will lie between digits "9." In other words, for any given year, there is only five percent chance that the value of the portfolio will be above the top "9" and five percent chance that it will be below the bottom "9." Exhibit 7 tabulates the numerical value of the points plotted in Exhibit 6. In addition, the table gives the minimum and maximum realized values of the portfolio at the end of each year realized in the simulation. Average annualized rate of growth of the portfolio calculated on the basis of median and mean values is also given.

The histograms, plots and tables shown in Exhibits 4, 5, and 6 can be prepared for each of the fifteen variables listed in Exhibit 4.

INTERPRETATION OF RESULTS

Exhibits 5, 6, and 7 and similar exhibits for other variables listed in exhibit 3 can be used by the investor to evaluate his investment strategy and consumption pattern specified in Exhibit 3. Exhibit 6 shows the degree of uncertainty involved if this strategy is followed. Increasing spread between fractiles indicates the increasing uncertainty about the value of the portfolio far out in the future. More detailed information on the distribution of portfolio value in a given year can be provided by histograms. Histograms and plots convey substantial information about the degree of risk involved in a given strategy. This will be made clearer by comparing exhibits 6 and 8. The latter exhibit plots the market value for the same portfolio when the investment strategy and consumption pattern is unchanged in all respects except that the market risk of the stock portfolio is increased from 0.8 to 1.5. As mentioned earlier in the description of the model, increased risk will bring higher returns on average, combined with greater variability. This is borne out by comparing results from the simulations. For example, the median value of the higher risk portfolio after ten years is $148 million compared to $144 million for the lower risk portfolio. At the same time, the market value of the higher risk portfolio will lie between $95.6 million and $285 million with fifty percent chance, i.e., in a range of $189.4 million. For the lower risk portfolio this range is only $71 million (from $112 million to $183 million). Other comparisons between the consequences of the two policies after

EXHIBIT 6

PLOT OF THE MARKET VALUE OF THE TOTAL PORTFOLIO

Market Risk of Stock Portfolio (Beta) = 0.8

1 Total Value (Disc. + Perm. Funds) in Dollars Millions

Frame 1

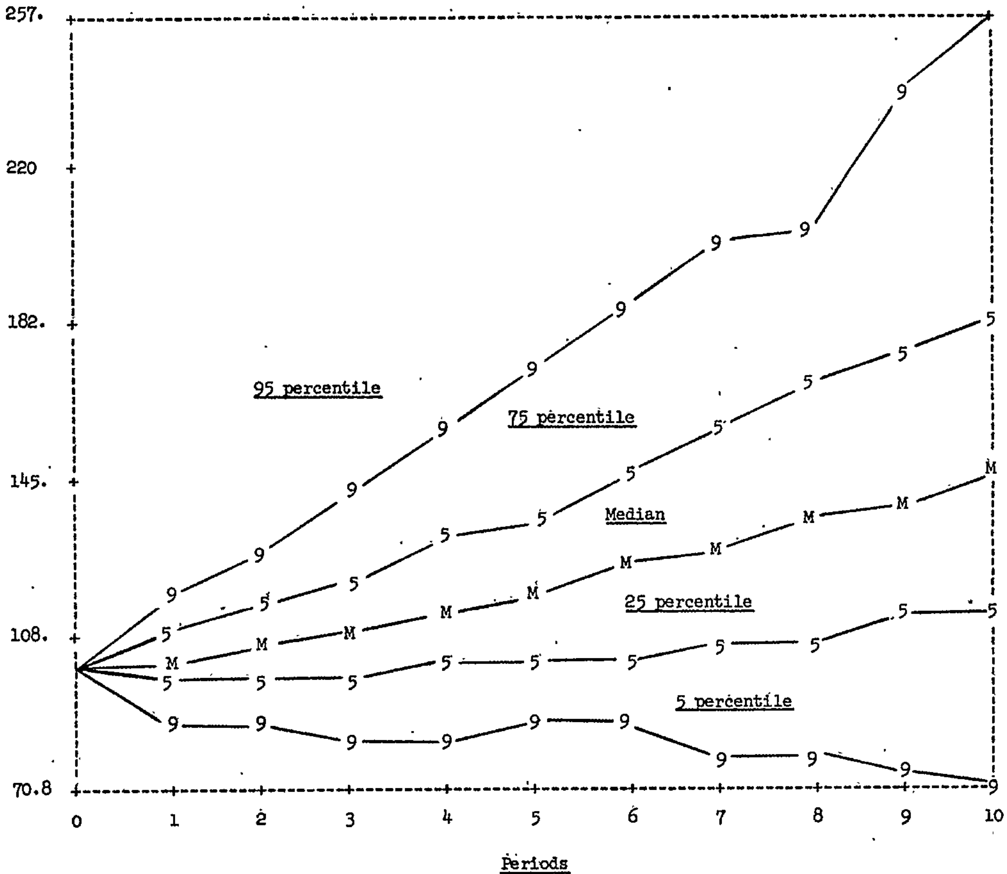


EXHIBIT 7

TABLE OF THE MARKET VALUE OF THE TOTAL PORTFOLIO
MARKET RISK OF STOCK PORTFOLIO (BETA) = 0.8

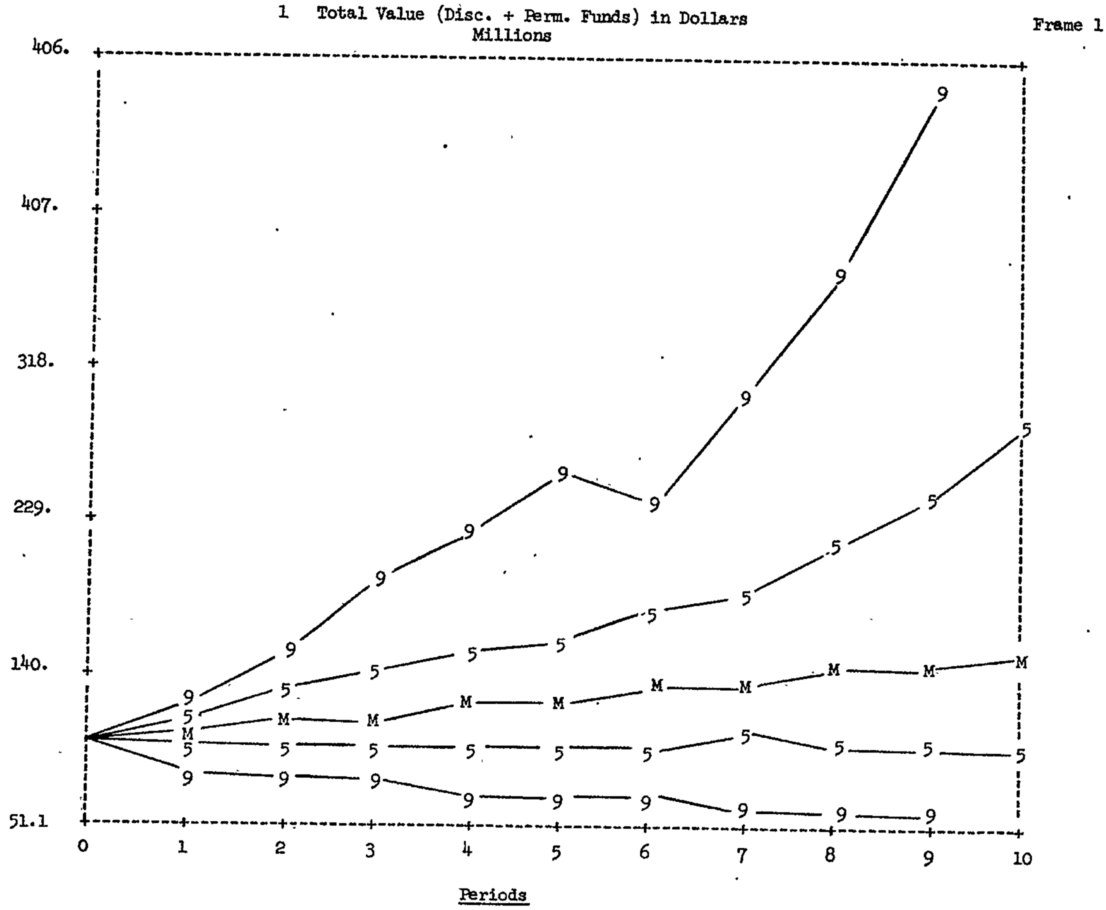
1. Total Value (Disc. + Perm. Funds) in Dollars

Initial Value = 99.0 millions Ending Median Value = 144. Millions Ending Mean Value = 152. Millions
Equivalent Annual Growth Rate over Entire Period = 3.83 % Compounded Annually, Based on Median,
4.39% Based on Mean

End of Year	1	2	3	4	5	6	7	8	9	10
Means	102.	107.	109.	115.	120.	126.	133.	138.	145	152.
Std. Devn.	9.08	12.4	16.7	21.3	26.0	30.1	37.9	42.7	48.5	57.1
Maximum	120.	132.	153.	165.	191.	212.	259.	257.	285.	371.
95 percentile	116.	128.	141.	156.	171.	186.	202.	207.	237.	257.
75 percentile	108.	117.	121.	130.	133.	144.	155.	166.	177.	183.
50 percentile	102.	106.	108.	112.	116.	122.	127.	135.	139.	144.
25 percentile	95.8	97.6	97.5	99.6	102.	102.	105.	104.	111.	112.
5 percentile	84.5	87.2	81.2	81.6	83.9	83.9	79.0	78.5	74.5	71.6
Minimum	80.4	79.8	73.7	70.8	65.4	65.7	68.6	67.8	61.2	62.4

EXHIBIT 8

PLOT OF THE MARKET VALUE OF THE TOTAL PORTFOLIO
Market Risk of the Stock Portfolio (Beta) = 1.5



ten years are shown in Exhibit 9. Such comparisons of portfolio performance at different intervals and with different investment strategy are likely to prove very useful to the investor in selecting his

EXHIBIT 9

COMPARISON BETWEEN PORTFOLIO PERFORMANCES WITH BETA = 0.8 AND BETA = 1.5
Values at the end of ten years in millions of dollars

	Beta = 0.8			Beta = 1.5		
	25 Percentile	Median	75 Percentile	25 Percentile	Median	75 Percentile
Total Value of Portfolio	112	144	183	95.6	148	285
Geometric Average Annual Growth Rate Based on Median	--	3.83%	--	--	6.07%	--
Value of Stock Portfolio	61.4	90.5	129	41.4	96.8	231
Value of Discretionary Fund	49.7	74.7	109	34.1	80.2	193

policy. Comparisons similar to those shown above can be made for other variables such as cash income and values of stock and bond portfolios. For dynamic consumption policies (when dollar value of consumption is not fixed in advance and is dependent on portfolio performance), comparisons for consumption levels also can be made. Since consumption remains the ultimate purpose of investment, it is unlikely that investment strategy decisions can be made by investors without fully comprehending the consequences of such policies for their consumption levels. This model, we believe, provides a convenient and easily understood vehicle for making these decisions.

IV. CONCLUDING REMARKS

The model presented in this paper provides a means for (1) examining and comparing probable consequences of proposed investment and consumption policies, (2) selecting a proportion of various kinds of assets in the portfolio on the basis of the effect of such choice on performance measures which are deemed important by the manager and (3) selecting a level of risk for stocks and bonds in the portfolio, by presenting the likely consequences of various policies in realistic and comprehensible form to the investor. The simulation approach provides a framework in which relatively sophisticated and modern concepts of capital market theory can be applied to practical problems of an average investor to yield useful results. We might also mention here that this model is not a guide to the selection of specific stocks and bonds for a portfolio. Such selection comprises the tactical decisions of portfolio management within the framework of portfolio strategy. Stock selection decisions lie within the purview of the portfolio manager, while the strategy decisions must be made by the owner himself. This model is directed to the latter type of decisions. Space limitations prevent us from including description of another use of this model: evaluation of the performance of the portfolio manager by the owner of the portfolio.

This model was developed to assist trustees of a university in laying down the portfolio strategy for its endowment portfolio. The model has since been implemented, but it is yet too early to evaluate the results from its use. Some special features of the model, such as the distinction between permanent and discretionary funds, were necessitated by this application. But other investors using the model can simply bypass such features if they find them unnecessary. For example, if consumption levels are set to zero for all periods, the model will simulate the pure investment strategy without consumption. Therefore, the model is much more general than it may first appear to be. The approach of the model, however, is still more general as the model can be expanded and further specialized or generalized to serve a variety of investment situations. Mathematical features of the model can be refined further depending on the time and effort available to the users of the model. This presentation of a working model for portfolio strategic decision is only one step in bringing the theory and practice of investment closer together.

ACKNOWLEDGEMENTS

Others who worked on designing this model include Kalman J. Cohen, Richard Roll and Robert Boldin, and my debt to them is gratefully acknowledged.

FOOTNOTES

1. Williamson [16] and Meckling and Jensen [11] present simulation models for setting risk of stock portfolios. Our model includes modeling of bonds in addition to stocks in the portfolio.
2. It is easy to expand the model to include other types of assets such as real estate, once the behavior of returns from such assets is defined.
3. Assuming that it is a well diversified portfolio.
4. See Ford Foundation [1], Cary and Bright [3], and Meckling and Jensen [11] studies for detailed analysis of the problem.

REFERENCES

1. Advisory Committee on Endowment Mangement. Managing Educational Endowments. Ford Foundation, New York, 1969.
2. Blattberg, Robert C., and Nicholas J. Gonedes. "A Comparison of Stable and Student Distributions as Statistical Models for Stock Prices," forthcoming in the Journal of Business.
3. Cary, William L., and Craig B. Bright. The Law and the Lore of Endowment Funds: Report to the Ford Foundation. Ford Foundation, New York, 1969.
4. Cootner, Paul H., ed. The Random Character of Stock Market Prices. MIT Press, Cambridge, 1964.
5. Cohen, Kalman J., and Edwin J. Elton. "Inter-temporal Portfolio Analysis Based upon Simulation of Joint Returns," in Management Science, Vol. 14, (September, 1967).
6. Fama, Eugene. "The Behavior of Stock Market Prices," in Journal of Business, Vol. 47, (January, 1965).
7. Fama, Eugene, and James D. MacBeth. "Risk, Return and Equilibrium: Empirical Tests," in Journal of Political Economy, Vol. 81, (May/June, 1973).
8. Jensen, Michael C. "The Performance of Mutual Funds in the Period 1945-64," in Journal of Finance, Vol. 23, (May, 1968).
9. Jensen, Michael C. "Risk, Capital Asset Pricing, and Evaluation of Investment Portfolios," in Journal of Business, Vol. 42, (April, 1969).
10. Lorie, James H., and Mary T. Hamilton. "New Focus for Investment Counselling," in Financial Analysts Journal, Vol. 29, (July/August, 1973).
11. Meckling, William H., and Michael C. Jensen. "University Endowments and Spending Policies," mimeographed report, University of Rochester, 1970.
12. Meier, Robert C., William T. Newell, and Harold L. Pazer. Simulation in Business and Economics. Prentice-Hall, Englewood Cliffs, N.J., 1969.
13. Salazar, Rudolfo C., and Subrata K. Sen. "A Simulation Model of Capital Budgeting under Uncertainty," in Management Science, Vol. 15, (December, 1968).
14. Schmidt, Joseph W., and R. E. Taylor. Simulation and Analysis of Industrial Systems. R. D. Irwin, Homewood, Ill., 1970.
15. Sharpe, William F. "Mutual Fund Performance," in Journal of Business, Vol. 39, (January, 1966).
16. Williamson, J. Peter. "Performance Measurement and Investment Objectives for Educational Endowment Funds," The Common Fund, 1972. Copies available from Money Market Reports, Box 618, Naugatuck, Conn. 06770.