AN APPLICATION OF SIMULATION FOR STUDYING THE MULTI-NATIONAL CAPITAL BUDGETING PROBLEM

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ABSTRACT

Multi National Corporations are continually faced with the problem of selecting investment opportunities within specific budget constraints with the objective of maximizing the value of the firm at some future date. Not all prospective benefits are realized and therefore the value of the firm in the future is uncertain. The corporate decision maker is faced with the problem of selecting an appropriate measure of worth for evaluating the proposed investments and an appropriate decision procedure, or "policy", for selecting the "best" subset of opportunities.

Given the uncertain nature of exchange rates and the uncertain nature of future benefits, a comparison of the relative effectiveness of selected measures of worth and selected decision procedures in sequence of decisions would be impractical using analytical, closed form mathematics. Consequently, a computer based simulation model was developed to study the multi-national capital budgeting problem. The current results are presented below.

INTRODUCTION

During the economic development of a firm, its decision makers are faced, from time to time, with the problem of budgeting a limited supply of capital for reinvestment within the firm. We visualize a firm that makes decisions periodically. Each decision requires the selection of a subset from the set of investment opportunities presented for consideration at the decision time. The subset will be made up, principally, of investments within the firm, which we shall call productive investments, and residually of investments in the short term money market, which we shall call market investments. The investible capital will come from retained earnings and debt.

It is assumed that the objective of the capital budgeting decision is to select the subset of proposed investments that promises to maximize the firms future value subject to budget restrictions. The experienced decision maker knows that there are several methods of evaluating investment opportunities and selecting the 'best' subset. Henceforth, we will refer to an evaluation of an investment opportunity as a 'measure of worth' and to a procedure for selecting a subset as a 'decision prodedure'. Much attention has been given to the description of the qualities of specific measure of worth and of 'optimal' decision procedures but relatively little attention has been paid to their relative effectiveness in long sequences of decisions.

Large firms tend to be subdivided into divisions for operating purposes. When such is the case, the management of the firm must choose between centralized and decentralized capital budgeting procedures. The latter, in its extreme form, requires a priori allocation of the budget to the divisions, where each applies its own decision procedure. For the large multi-national firm the capital budgeting decision is further complicated by the introduction of currency exchange rates.

This paper reports specific results of a computer based model for simulating sequences of capital budgeting decisions in a hypothetical multi-division, multi-national firm. This simulation model has made possible a study of a variety of aspects of this very complex problem in a relatively short time. This approach was taken because the optimization models proposed for an analytical solution of this class of problems offers little promise of dealing with them in their full complexity.

THE DECISION SITUATION

The hypothetical firms studied in this research each had two-divisions located in different nations. At each decision time, the budget was known precisely and its only source was retained earnings; that is, borrowing was allowed only to make up short falls from prior decisions.

The productive investment opportunities were actually vectors of random cash flows, expressed in the inflated currency of the country in which the division was located. The decision maker saw, for each proposal, a randomly generated vector of cash flows which represented the divisional analysts 'best estimate' of the vector of expected cash flows. The bookkeeper received a randomly generated realization. It was only by chance that the decision maker and the bookkeeper ever received identical cash flow vectors.

It was assumed that the exchange rate tracked inflation precisely and the inflation rate was a random variable. The firm's decision maker had to estimate the relevant inflation rates for each division then he used those estimates to convert all cash flow vectors to equivalent constant purchasing power currency before making his decisions. The bookkeeper learned that actual value of the inflation rate and kept his records in constant purchasing power currency. Each decision maker had his favorite measure of worth and decision procedure and he used both consistently for the duration of the firms operation.

The growth rates of the proposals generated at each decision time constituted a random sample from a divisional investment function such as that illustrated in Figure 1. This function shows the fraction f of the budget horizontal axis that can be invested at or above growth rate g (vertical axis). The implication of this representation of the investment function is that productive investment opportunities grow with the capital growth of the firm. If investment opportunities grew less rapidly, the distance between the origin and f=1 would increase until a stable condition was reached. If they grew more rapidly that distance would decrease until a stable condition was reached. Productive opportunities that would appear at future decision times were unknown to the decision maker at each current decision time.

THE EXPERIMENTAL DESIGN

A principle objective of this study was the comparison of the relative effectiveness of different decision procedures in long sequences of decisions under statistically equivalent situations. Since the process consisted of a sequence of periodic choices of random variables, an outcome was a relaization of an incompletely described random process. The form of an outcome was a vector of future cash flows. By replicating the process, an estimate of the vector of average future cash flows was obtained. A comparison of the relative effectiveness of Decision Procedures A and B, would have involved a statistical comparison of two vectors. This problem was avoided by computing the average growth rate of the firm $\bar{g}_f$, which allowed simpler statistical comparisons.

In order to avoid bias in favor of one or another of the decision procedures, all were started with the same initial budget and all were exposed to the same sequence of sets of investment opportunities. The only exception being that the investment opportunities for each procedure were allowed to grow at the capital growth rate of that procedure. Otherwise the procedures realizing higher growth rates would have been penalized and those realizing lower growth rates would have been favored. Consequently, at each decision time the set of vectors available to each decision maker and the counterpart vectors available to the bookkeeper were equivalent, if not identical.

The detailed design is divided into six parts:

1) The generation of investment proposals
2) Random Perturbation
3) Correction for Inflation
4) The evaluation of the proposals
5) The application of the decision procedure
6) The bookkeeping of the data.

Proposal Generation

Random numbers were generated by random sampling from piece-wise linear cumulative distribution functions specified by the user. If a constant rather than a random number was desired, the user could simply concentrate all the probability on the value of that constant.

The number of productive investments to be generated at a decision time for each division was determined by sampling two distributions. Then the set of inflated currency growth rates (internal rates of return) for each division's productive investments were generated by random sampling from that division's inflated currency investment function.

For each growth rate a cash flow vector was generated in the following manner. The numbers of cash flows in the benefit and cost series were determined by sampling their respective distributions. The pattern -- arithmetic or geometric and the rate of increase or decrease -- of each component was determined by random sampling of two distributions. the benefit series was then scaled so that the resulting productive investment proposal yielded the proper growth rate.

The ratio of total first cost of productive investments to total budget was determined by random sampling as was the relative proportion of the first cost of each investment to the total first cost. These values provided final scaling factors for the set of investments.

Random Perturbation

To provide for uncertainty about cash flow forecasts, one random perturbation of the original vector was generated for the decision maker and another for the bookkeeper. These perturbations were done in such a way that the originally generated cash flow vector was the expected value of both perturbed cash flow vectors. In generating a perturbed vector, the three components were perturbed independently of each other -- (1) the first cost, (2) later costs, (3) the benefit series. Each of the three perturbations consisted of multiplication by a factor of the form $1 + e_1 + e_2$, where e_1 and e_2 represent random variables with mean zero whose realizations were obtained by sampling the appropriate distributions. Furthermore, the realization of e_1 was constant for each component for each decision whereas a realization of e_2 was generated independently for each vector component of each investment.

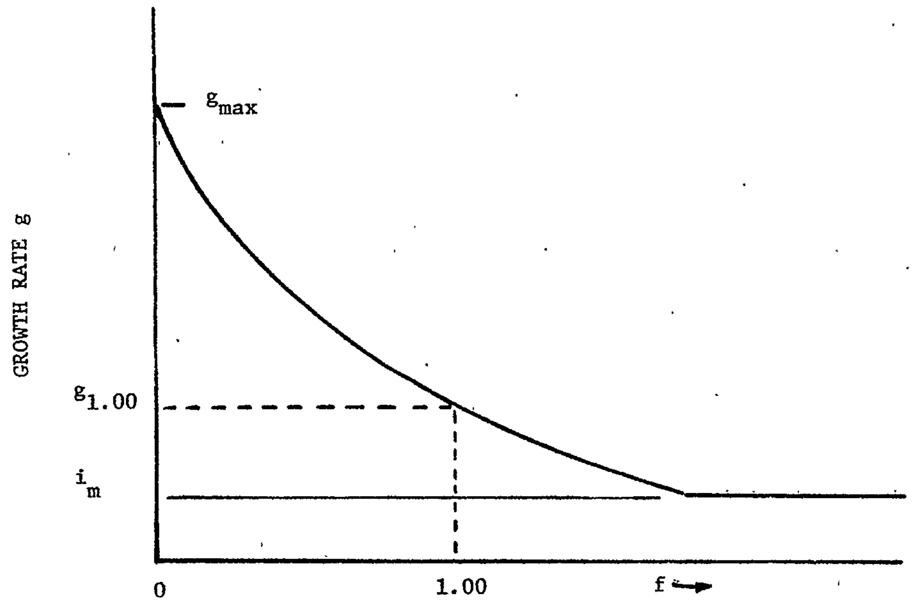


FIGURE 1. AN INVESTMENT OPPORTUNITY FUNCTION

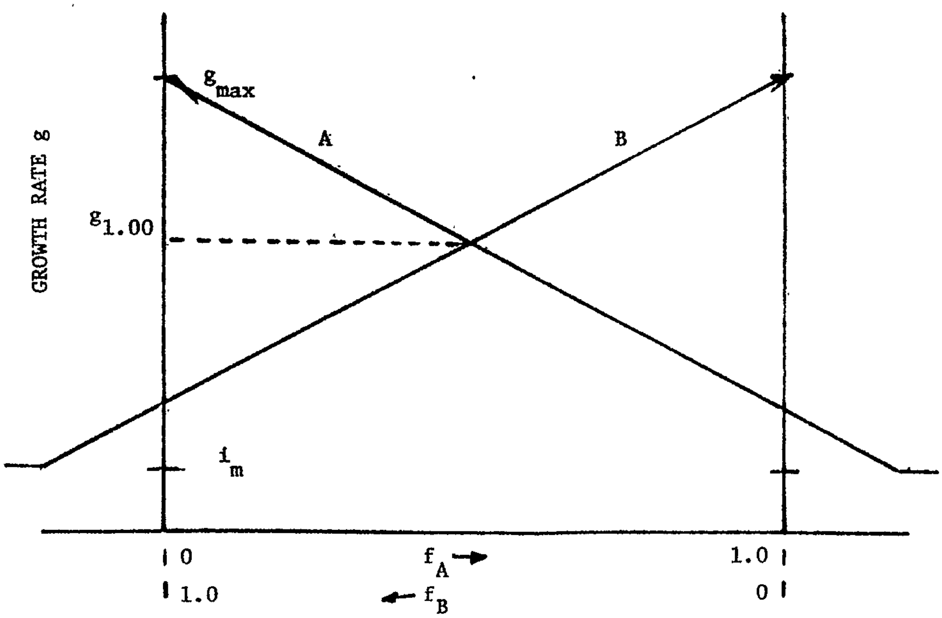


FIGURE 2. TWO DIVISION INVESTMENT OPPORTUNITY FUNCTION

Correction For Inflation

The firm's analyst received divisional productive investment vectors expressed in inflated currency of the respective divisions. Each division's vectors were converted to equivalent 'constant purchasing power' (CPP) cash flow vectors based on the decision maker's estimate of the inflation and his error in estimate. To accomplish this, each inflated cash flow at time t was divided by the factor

$$\prod_{k=1}^{t} (1+I_t+e_t)$$

where I_t represents the respective division's inflation rate for year t and e_t represents the decision maker's error in its estimate. The values of I_t and e_t were generated by sampling relevant distributions.

The bookkeeper made no errors in estimating inflation rates so his equivalent CPP cash flow vectors were computed by dividing a cash flow at time t by the respective division's factor of the form

$$\prod_{k=1}^{t} (1+I_t).$$

Evaluation of Proposals

Within each decision procedure, each proposal is evaluated by a prescribed measure of worth (MOW). No attempt was made to include all known measures of worth. Instead the ones selected were chosen on the basis of past studies in the areas of capital budgeting, engineering economy, and decision theory. Each of the four measures of worth was associated with a particular decision procedure. The first MOW was growth rate or "internal rate of return". For a complete definition see [1], [3]. The second MOW is the commonly called present worth function or for this paper Net Present Value (NPV). The third MOW is based on work by Oakford & Thuesen [1], [2] and is referred to in this paper as Adjusted Net Present Value (ANPV). ANPV is the NPV corrected for funds left in the market for one period. The fourth MOW is Pseudo-Utility (PU) which is the NPV less a positive constant times the standard deviation of the NPV. This MOW has been suggested by some authorities as a means of protecting against risk. Presumably it could be used to determine the "cost" on the performance of the firm to "insure" against ruin. As we shall see later this "insurance" cost could be prohibitive.

Decision Procedures

The decision procedures were also selected on the basis of past work and limited computer time. Not all known decision procedures were programmed but future users could include their favorite procedure and compare its effectiveness with that of those used here. Since it was intuitively evident that a consistent, rational decision procedure should be better than pure random choice, we included a decision procedure called Random. This procedure sampled randomly without replacement from the set of investment opportunities until the budget was exhausted. Each remaining proposal had an equally likely chance of being selected next. In all of the decision procedures, a small random fraction of the budget would be invested in the market due to the discrete nature of the sume of the first costs. The second decision procedure was Rank on Growth Rate (ROGR) which is known intuitively by most practical decision makers. Each proposal in the set is ranked according to its internal rate of return. Then starting with the highest growth rate, proposals are accepted until the budget is exhausted. This procedure was modified to a rank on growth rate with a cutoff at some hurdle rate, g_H; however this was found to be practically equivalent to the following procedure.

The third decision procedure was Rank on Net Present Value (RNPV). In this case each proposal was evaluated using the NPV measure of worth and then ranked high to low. Proposals were accepted until either the budget was exhausted or until the NPV was zero, whichever came first. Unused budget was placed in the market earning at rate i_m.

The fourth decision procedure was Rank on Adjusted Net Present Value (RANPV). As above, each proposal was evaluated using the ANPV measure of worth and then ranked high to low. Proposals were accepted until either the budget was exhausted or until the ANPV was zero, whichever came first. Unused budget was invested in the market. On the surface one might not expect substantial differences between RNPV and RANPV, but as we shall see these differences do occur and can be substantial.

The last decision procedure was Rank on Pseudo-Utility (RPU). The PU measure of worth was computed for each proposal and then the set was ranked from high to low on the values of PU. As before, proposals were selected until either the budget was exhausted or until the value of PU was zero, whichever came first.

Bookkeeping

At each decision time, each decision procedure was used on the set of proposals generated for that decision period. After each procedure was used to create the selected subset, the proposal data for that subset was sent to the bookkeeping function. In the case of certain cash flows, the selected subset for each decision procedure was sent intact. In the case of uncertain cash flows, the selected subset was a perturbed subset sent to the bookkeeper. In both cases, the bookkeeping function computed the firms' growth rate on the net cash flow vector after all decision periods were completed. Consequently, for each decision procedure operating consistently over 6 decision periods, one growth rate was computed. This process was repeated 9 times from which estimate of mean growth rate and standard deviation were computed. Deci-procedures could be compared on the basis of the average growth rate over the 9 replication under a variety of input parameters. The results are summarized below.

DETERMINISTIC SITUATION

Study of the results of the original model motivated an anlysis of a deterministic situation where each productive investment's constant purchasing power cash flow vector belongs to a class of semi-unit investments [1]. These have the desireable property, that the difference of two semi-unit cash flow vectors is a cash flow vector without a sign change. Consequently, the ROGR and RNPV decision procedures would always yield identical rankings. However, the two procedures would not necessarily yield identical decisions, as we shall see later. If, as we will assume in addition, the first cost is identical for all the semi-unit investments in a class, then RANPV would also yield an identical ranking.

Consider a decision maker who uses the ROGR decision procedure and has the CPP divisional investment functions shown in Figure 2. He would exhaust his budget by selecting all productive investments with growth rates greater than $g_{1.00}$, which is the growth rate at which the investment functions intersect. The subscript 1.00 indicates the fraction of the budget that is invested in productive investments. In general, we will use the symbol g_{fp} to represent the growth rate of the marginal productive investment when a fraction fp of the budget would be invested in productive investments. Of course, a fraction 1-fp would be invested in market investments at rate i_m. In the deterministic situation, we can readily compute the growth rate of the combination of proposals selected. A simple average of the component growth rates is usually a good approximation. The computed value would be the growth rate of the firm if the decision maker's investment function remained constant over time, i.e., if the firm's investment opportunities grew at its capital growth rate.

If, instead, the decision maker were to use either RNPV or RANPV and used a hurdle rate g_H less than or equal to $g_{1.00}$ as the discount rate in evaluating NPV then he would select the same set of proposals that was selected using ROGR. If he were to use RNPV and a hurdle rate g_H greater than $g_{1.00}$, he would reject those productive investments with growth rates less than g_H with the result that a fraction of his budget would be diverted to market investments at rate i_m. The distribution of growth rates resulting from that decision is illustrated in Figure 3. The growth rate of the selected combination can be computed in the deterministic case. In fact, the area subtended by the curve in Figure 3 would usually be a good approximation. It is obvious that the growth rate of the firm would be affected by the fraction of the budget that is diverted to the market.

We define $g'_{1.00}$ as the maximum value of g_H such that RANPV would select the set of proposals identical to that selected by ROGR. Furthermore $g'_{1.00}$ is greater than or equal to $g_{1.00}$. In the deterministic situation, it is not difficult to evaluate g'_{fP} when g_{fp} is known and vice versa.

The effect of the choice of the value of g_H on the growth rate of the firm can be computed with the aid of a hand-held calculator for each of the decision procedures. Figure 6.b, summarizes the results of the computations for the hypothetical single division investment function of Figure 6.a. If ROGR were used, the value of the growth rate of the firm, g_f, would be that of the upper horizontal line regardless of the value of g_H. If the Random procedure were used, the growth rate of the firm would be that of the horizontal dashed line. In this example $g_{1.00} = .10$ and $g'_{1.00} = .135$.

We were very surprised to find that when RNPV was used, g_f first increased as the value of g_H was increased beyond $g_{1.00}$. After reaching a maximum, g_f decreased until it reached $i_m = 0.05$ for $g_H \geq g_{0.0}$. Our intuition had led us to believe that diversion of any fraction of the budget from productive investments yielding more than $g_{1.00}$ to market investments yielding i_m, less than $g_{1.00}$, should result in a reduction in the growth rate of the firm. Such is the case often, but not always. Similarly, g_f first increased as the value of g_H exceeded $g'_{1.00}$ when RANPV was used, then decreased to $i_m = 0.05$ after reaching a maximum.

We will use the symbols m and m' respectively to represent the values of g_H at which the growth rate of the firm g_f would be maximized when RNPV and RANPV were used. We will refer to m as the marginal growth rate which is the growth rate for the marginal investment in this decision. More important, it would also be the growth rate for the marginal investment at each future decision in the deterministic situation if the decision maker's investment function remained constant over time.

The optimal allocation of the budget between divisions A and B would be that indicated by the points of intersection of the CPP divisional investment functions with the horizontal line passing through m as illustrated in Figure 4. If m were greater than $g_{1.00}$, a fraction of the budget would be diverted to market investments intentionally. If an allocation other than the above were made, for whatever reason, the growth rate of the firm would be less than the optimal value. If either too much or too little were allocated to both divisions, less than the optimal fraction would be diverted to market investments. If the correct fraction were allocated to market investments but the remainder was improperly divided between divisions, then marginally superior productive investments from one division would be replaced by marginally inferior productive investments from the other.

Three possible causes of non-optimal allocation are:

1. Basing the budget allocation decision on something other than economic considerations.
2. Errors in estimates of inflation rates.
3. Failure to evaluate the effect of intentionally diverting a fraction of the budget to market investments.

Figure 5 illustrates how errors in estimates of divisional inflation rates could result in an incorrect allocation of funds between divisions. In that figure, the solid lines (A and B) represent the CPP investment functions and the dashed line (A1 and B1) represent the decision maker's estimate of the solid lines. The indication is that the decision maker has underestimated the inflation rate for Division A and overestimated that for Division B. In this example, we assume that none of the budget should be intentionally diverted to the market, i.e., that the value of m corresponds to the point of intersection of the CPP investment functions.

Because of the errors in estimation of inflation rates, the allocation to division A would be f_{A1} rather than the optimal allocation f_{A0}. Consequently the growth rate of the firm would be reduced by an amount equal to the cross hatched area in the figure.

SUMMARY OF RESULTS

The data presented here is the result of running the above simulation with specific parameters. Many more results were obtained in the process of understanding the simulation and comparing its results with the hand computed deterministic result. However, these data seem to best represent the characteristics of the multi-national capital budgeting problem under various conditions. The data are presented in three groups - deterministic, random proposals with certain cash flows, and random proposals with uncertain cash flows - parameterized by two factors - hurdle rate g_H, and the relative inflation error η; The dependent variable was always the average growth rate of the firm, $\bar{g}_f$, over the entire decision sequence. However, two decision procedures and hence two firms are not affected by the decision maker's hurdle rate g_H - they are ROGR and Random. These firms are represented by single points on the graphs.

The first set of results are displayed in Figure 7. The solid lines are the deterministic group, the two dashed lines near the solid lines are the random-certain group, and the three dotted lines below are the random-uncertain group. All the curves are the average growth rate, $\bar{g}_f$, of the firm versus the decision maker's choice of the hurdle rate, g_H. All three curves are for the relative inflation factor $\eta = 1.00$ which means that the decision maker's estimate of the inflation rate is without any consistent error, however he may have a small random error with mean zero. We note that the first two groups confirm our analytical results of the previous section. Again RANPV consistently outperforms RNPV throughout a wide range of choices of the hurdle rate g_H. Furthermore as the magnitude of the perturbations is increased, the firm's optimal performance is decreased. ROGR outperforms RANPV by a small margin, within the limits of $\pm 1\sigma$ until g_H exceeds about 1.0; ROGR also outperforms RNPV when g_H exceeds about .3. Random selection is indicated by the point near $\bar{g}_f = .275$ for group 2 and near $\bar{g}_f = .298$ for group 3. By comparison we note that ROGR, and RANPV yield a substantially higher growth rate than Random for the deterministic and random-certain cash flow group. RANPV appears to do no worse than Random as the variability of the cash flows increase. However, RNPV and RPU yield surprising results. RNPV is quite sensitive to mis-estimating g_H and as the uncertainty increases, RNPV may actually yield a growth rate lower than Random.

Moreover, RNPV would appear to be less sensitive to large values of g_H under uncertain conditions than deterministic. RPU can be observed only under the condition of uncertainty. With low hurdle rates RPU yields a $\bar{g}_f$ slightly higher than Random but as g_H increases, RPU yields a $\bar{g}_f$ surprisingly lower than Random's $\bar{g}_f = .298$. Specifically, if RPU is used to insure against ruin, mis-estimating g_H or a large variability in the perturbed cash flows may cause the firm to achieve a much lower yield in $\bar{g}_f$ than using RANPV; the "insurance cost" may be prohibitive. We can also conclude that those who recommend <u>increasing</u> the discount rate to "adjust for risk", may make a significant error.[4] Increasing the discount rate is equivalent to increasing g_H. On the curves for RNPV, as the g_H is increased, yield in $\bar{g}_f$ is decreased as g_H exceeds the deterministic optimal. It should be observed that RPU can be moved arbitrarily close to RNPV by decreasing the multiplier of σ.

When the decision maker makes a consistent error in estimating the rate of inflation the relative inflation factor, η, will deviate from 1.00. In several of the runs we set η at 1.2 indicating a 100% underestimate of the 20% inflation rate. These results are depicted in Figure 8. Again the solid lines are the deterministic group, the dashed lines are the random proposals with certain cash flows and the dotted lines are the random proposals with uncertain cash flows. The general conclusion is that the critical g_H values are shifted, in this

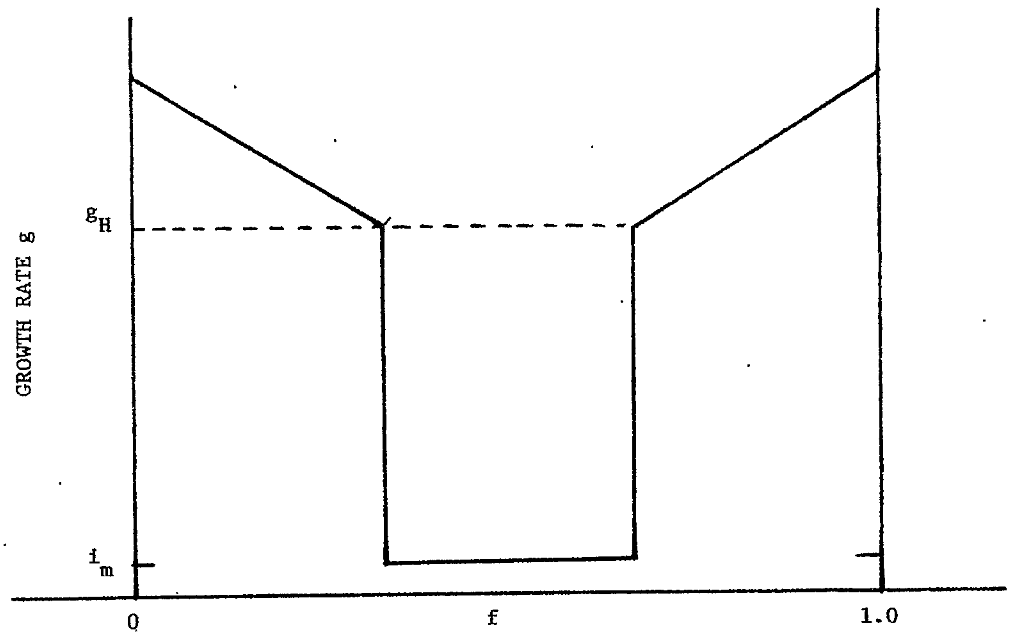


FIGURE 3. DISTRIBUTION OF RETURNS FROM THE INVESTMENT DECISION

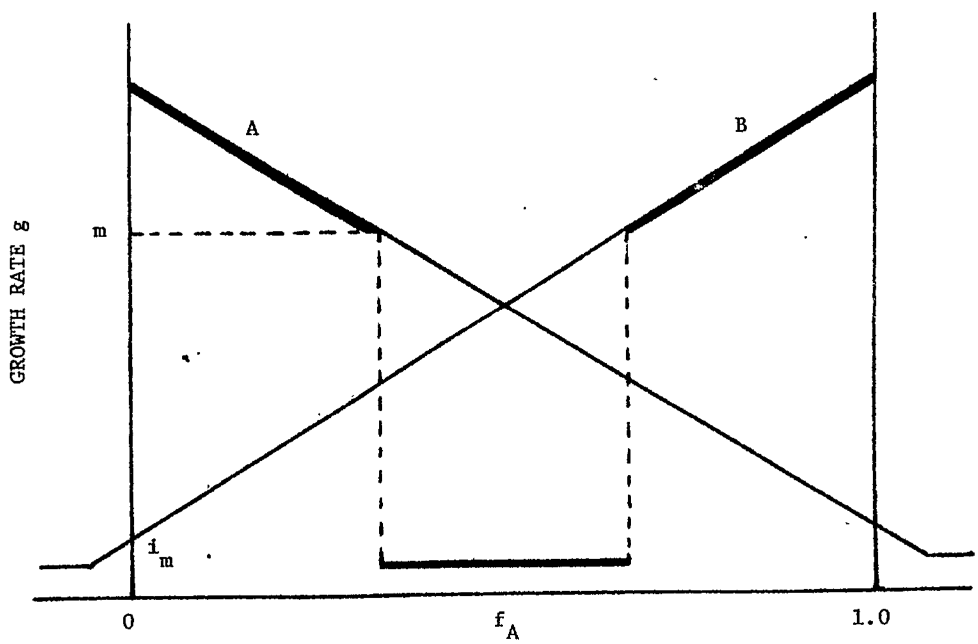


FIGURE 4. A FRACTION OF THE BUDGET IS INTENTIONALLY DIVERTED TO THE MARKET

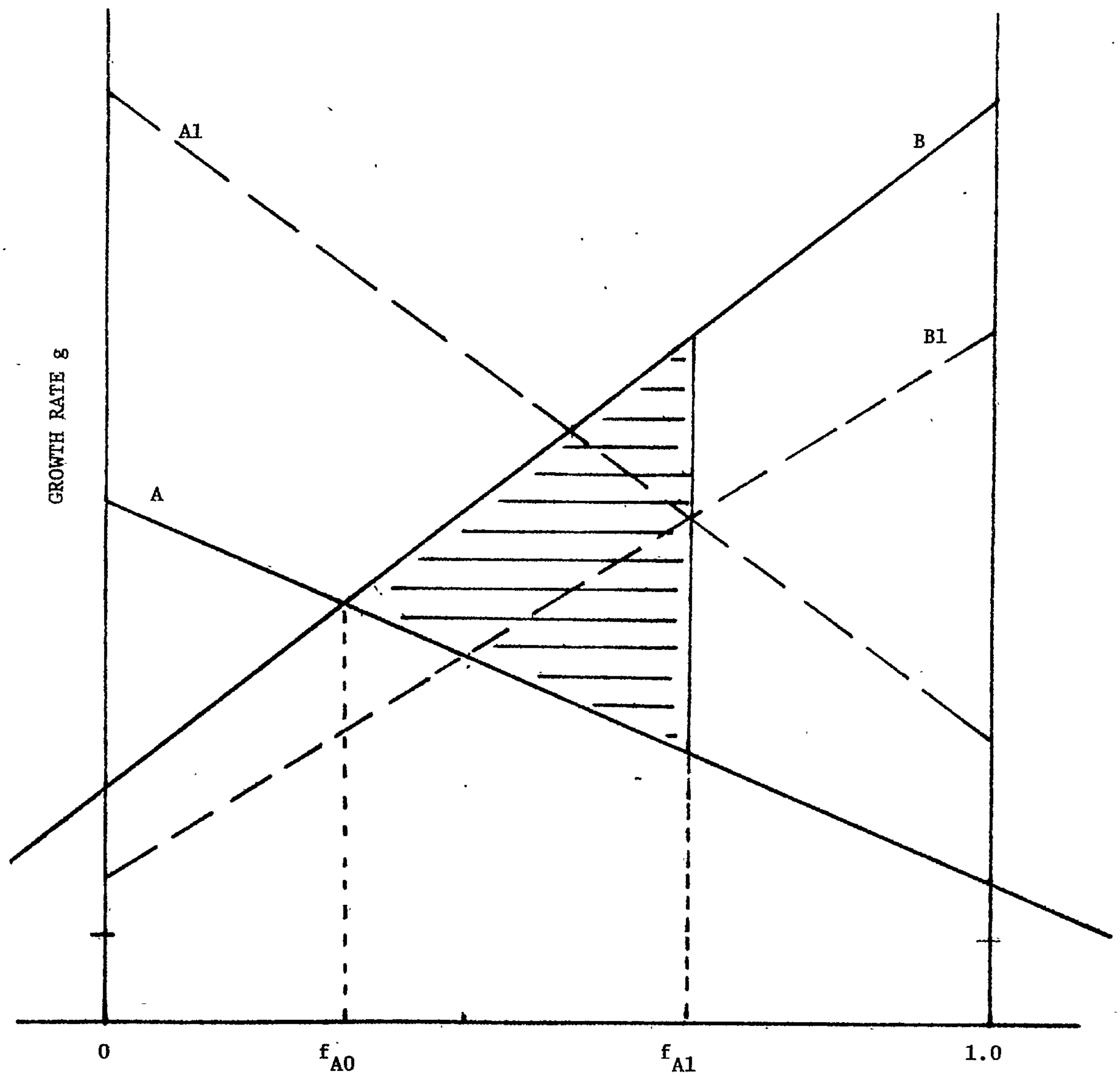


FIGURE 5. EFFECT OF MIS-ESTIMATING INFLATION RATES

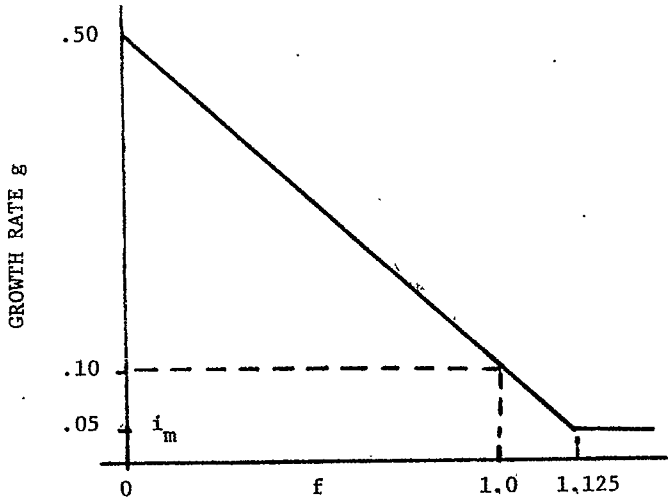


FIGURE 6a

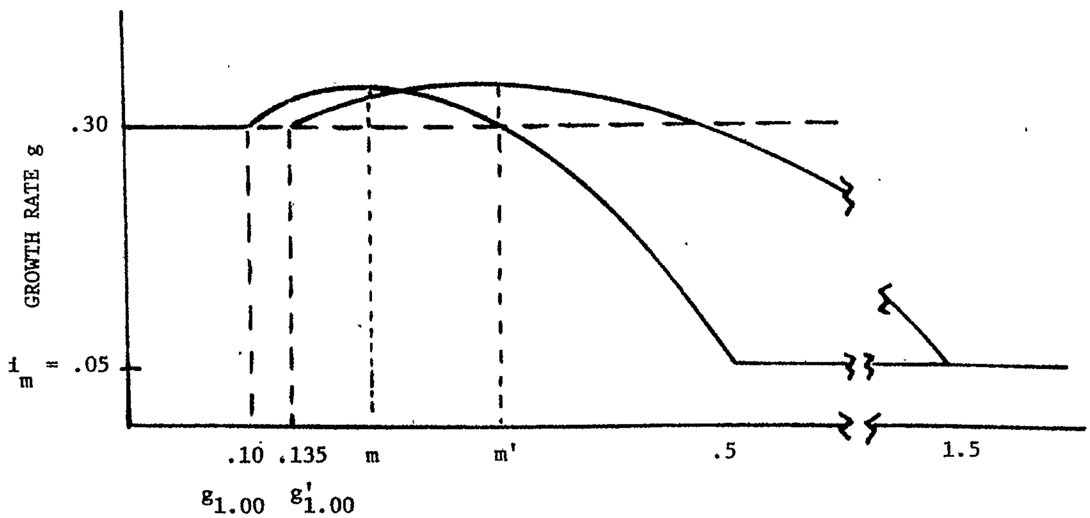


FIGURE 6b

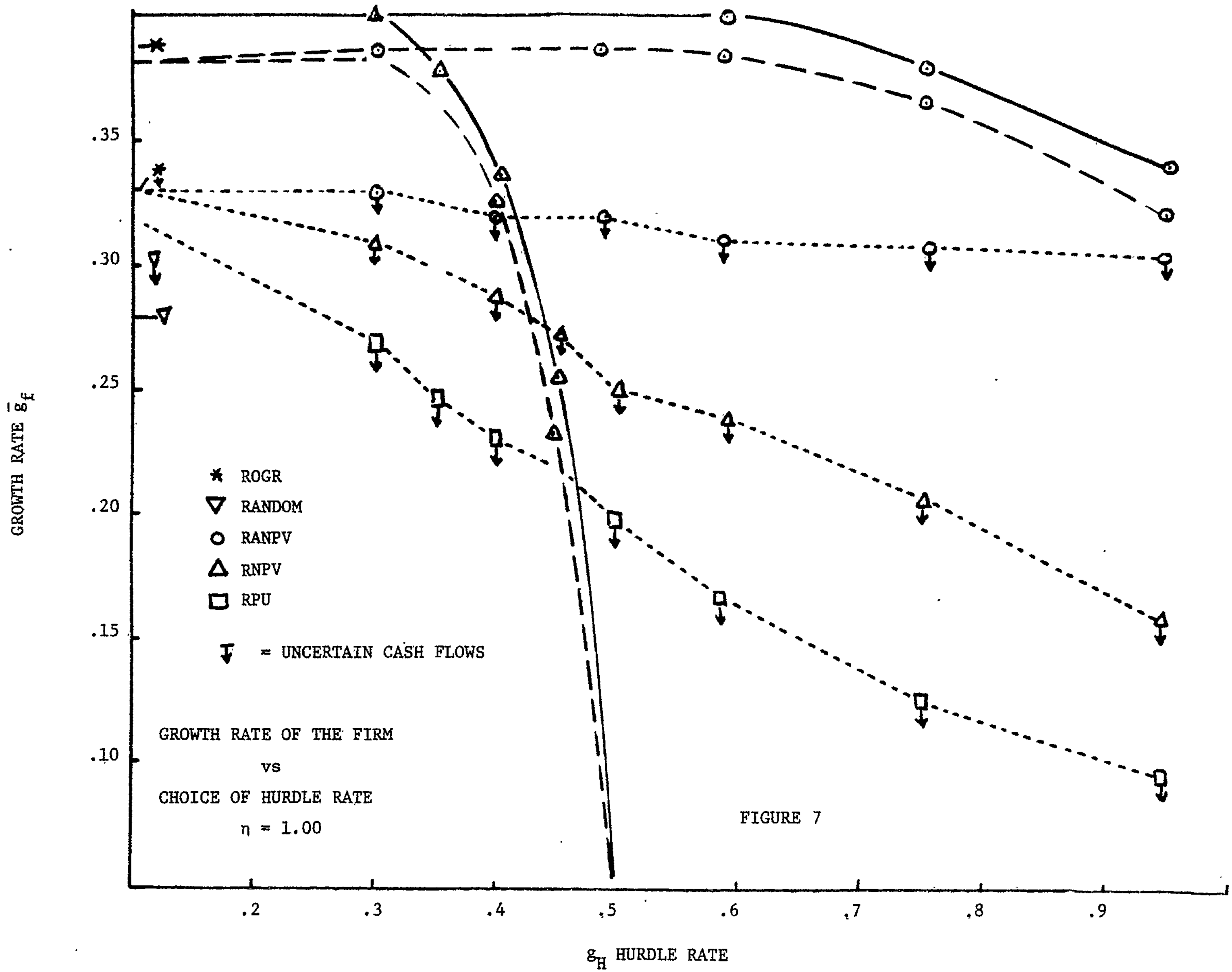
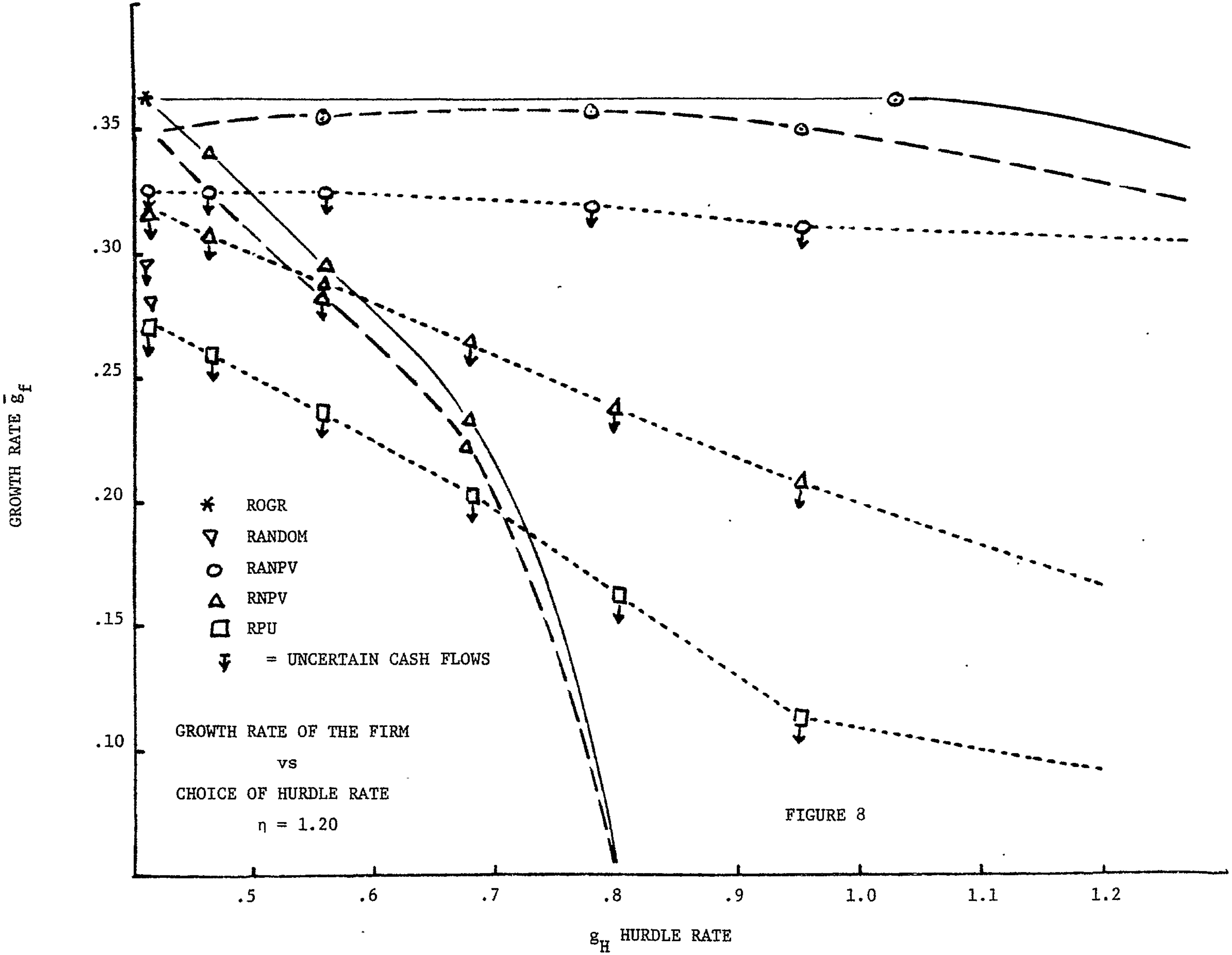

GROWTH RATE $\bar{g}_f$
.35
.30
.25
.20
.15
.10
.2
.3
.4
.5
.6
.7
.8
.9
g_H HURDLE RATE
ROGR
RANDOM
RANPV
RNPV
RPU
= UNCERTAIN CASH FLOWS
GROWTH RATE OF THE FIRM
vs
CHOICE OF HURDLE RATE
η = 1.00

FIGURE 7



GROWTH RATE $\bar{g}_f$
.35
.30
.25
.20
.15
.10
.5
.6
.7
.8
.9
1.0
1.1
1.2
g_H HURDLE RATE
ROGR
RANDOM
RANPV
RNPV
RPU
= UNCERTAIN CASH FLOWS
GROWTH RATE OF THE FIRM
vs
CHOICE OF HURDLE RATE
η = 1.20

FIGURE 8

case, to higher values than with the CPP curves in Figure 7. Furthermore the firm's yield in $\bar{g}_f$ is consistently lower when $\eta > 1.00$ than for $\eta = 1.00$ for all but Random. That is, mis-estimating the inflation rate cannot improve the yield and may cause significant reduction on the firm's yield. Looking at RPU we see that over the range of g_H shown, the RPU does not appear to be as good as Random choice in terms of $\bar{g}_f$.

In comparing one firm with $\eta = 1.00$ and another with $\eta = 1.20$, one might be tempted to conclude that for constant g_H, an improvement in $\bar{g}_f$ may be made by underestimating the inflation rate. For example RNPV at $\eta = 1.00$ and $g_H = .5$, $\bar{g}_f = .25$ while at $\eta = 1.20$ and $g_H = .5$, $\bar{g}_f = .30$. However, one should also note that RANPV remains almost constant at a higher $\bar{g}_f = .32$. That is poor decision procedure might be improved by poor estimates in inflation only because it makes it operate closer to the Random procedure.

Overall we are able to conclude that

1) Several decision procedures can be compared concurrently using the above simulation process
2) Statistically significant differences can be observed among these decision procedures
3) Hurdle rate and inflation rate estimates can significantly modify the firm's yield
4) Good decision procedures yield lower $\bar{g}_f$ under large variability in cash flows than with deterministic cash flows. Poor decision procedures are conversely affected.
5) Adjusting the discount rate upward to account for risk may not yield any improvement and may cause a reduction in a firm's $\bar{g}_f$.

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