

# IRRIGATION SYSTEM SELECTION FOR MAXIMUM CROP PROFIT

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## ABSTRACT

The selection of irrigation systems and management of irrigation water to maximize profit from crop production implies an optimization process of some sort. Such an optimization of irrigation system design is a complicated task and most often it is done by intuition. It involves the physical system, crop growth patterns, crop response to water and fertility and on-farm management practices.

This paper presents a methodology for relating the selection of irrigation system design parameters to the crop production profitability. Irrigation system uniformity along with the scheduling of applied water has an important effect on crop yield and thus net farm income. A computerized technique is used which relates system uniformity and scheduling practices to crop growth and production. For a given irrigation system design, furrow, sprinkle, or trickle, and a particular scheduling scheme, the seasonal water application which will maximize the net profit from a specified crop can be estimated.

The computerized mathematical model requires data relating system costs and design parameters to water application uniformity, crop-water production functions (from field data or crop growth models), production costs, and system capital costs. This systematized design process allows the designer to evaluate relative trade-offs between water, system and crop production costs, and returns with relative ease. Without this capability many irrigation systems, as a result of minimal effort expended in their design, are less than adequate for the field conditions in which they must operate. The inclusion of more relevant variables into a workable design process should prove helpful in increasing the designers' awareness of the interaction of crop production with irrigation systems. Field based crop production and economic data are used to illustrate the technique.

## INTRODUCTION

The selection of irrigation systems and management of irrigation water to maximize profit from crop production implies an optimization process of some sort. Such an optimization of irrigation system design is an involved task. It includes the physical system, crop growth patterns, crop response to water and fertility and on-farm management practices. The many interactions existing in agricultural production processes create a very complex system. Keller, Peterson and Peterson (5), proposed that this complex system be disaggregated into less complex, more manageable components, thus permitting identification of the factors affecting each. They outlined a strategy for optimizing research on agricultural systems involving water management which is applicable also to design processes.

The work reported herein is a part of that continuing effort to identify non-site specific estimating techniques for crop production factors particularly related to water management practices. This paper presents a methodology for relating the selection of irrigation system design parameters to the crop production profitability. It is not our purpose to present an exhaustive array of design examples, however, field based crop production and economic data will be used to illustrate the technique.

## INTERACTION OF THE IRRIGATION SYSTEM WITH CROP YIELD

Information describing crop response to environmental factors must be identified before prediction of the effects of irrigation on crop growth and yield can be made. A conceptual basis for handling the resulting complex relationships is presented by Keller, et al (5). They suggest that the crop response to environmental conditions can be expressed as the interaction of two multi-dimensional vectors: agricultural environment,  $E$ , and production materials,  $M$ . The crop response vector,  $R_c$ , can thus be expressed as  $R_c = f(E, M)$ . Husbandry practices serve to modify the environment of the plant by changing such factors as

fertility, planting dates, variety, control of pests, etc. The design and operation of an irrigation system is for the express purpose of modifying the soil water status to be more favorable for plant growth and thus to increased yield.

Water production functions are useful as a link between the agronomic aspects of crop growth and the irrigation system operations. The functions must be determined in the field to find the response of particular varieties to the environmental factors experienced during the tests. Changes in irrigation frequency also bring about changes in seasonal yield even though the total water applied remains the same. The nature of such crop responses to varying irrigation scheduling schemes has been the subject of many research studies.

Because of the many interactions between scheduling of irrigation water and crop yield the design of an irrigation system and its subsequent management have a strong influence on net farm income. The nature of these interactions and a suggested framework for incorporating them into the design and management of irrigation systems was discussed by Keller et al. (6). They emphasized the influence of economics on the design and management of irrigation systems, and attempted to identify and discuss some factors associated with irrigation which influence both the cost and quality of vegetative production. They recognized a need for selecting management options based on a fundamental understanding of the physical and economic processes interacting in the system as a whole.

Varlev (8) examined a component of the system design--crop interaction with the development of second-degree polynomial equations relating crop yield to system uniformity on a theoretical basis without including the economic components of design and operation in his technique.

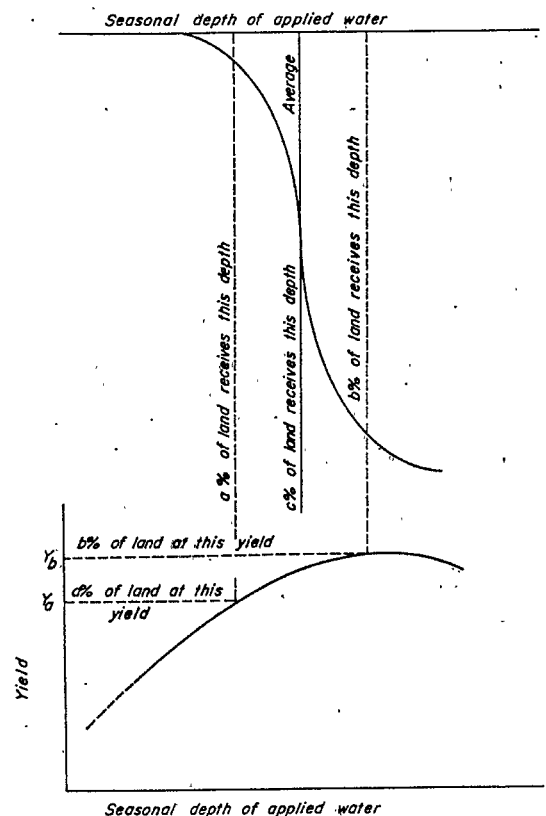
#### SYSTEM SELECTION TECHNIQUE

For any system some portions of the field will receive more and some positions less water than the average depth. If water is plentiful and inexpensive and there is no detrimental effect on crop yield from overwatering, then the average depth of applied water can be increased until all portions of the field receive enough to maximize yield, as Varlev (8) suggests. However, this is seldom if ever the case and uniformity and economic concepts should be integrated into the design process.

To do this some means must be identified for relating the depth of water application on each fraction of the area irrigated as a function of system design. The resultant "distribution function" can thus be used in

conjunction with the crop-water production curve, as shown in Fig. 1, to estimate the contribution to the total field yield of each incremental area-depth combination. Hart and Reynolds (3) demonstrated this for sprinkle irrigation systems using the Christiansen Uniformity Coefficient, UC, and assuming a normal distribution for the uniformity of applied water depths, as depicted by the distribution curve in Fig. 1. In this paper these concepts were applied to sprinkle, trickle, and furrow irrigation systems to develop a methodology for relating crop production economics to the selection of system design parameters.

FIGURE 1



A computerized optimization technique was developed which finds the amount of seasonal water to apply for a given system to maximize profit from the crop. The technique requires data related to system capital and operation costs, crop production costs, water charges, a crop-water production function, and some parameters which permit determination of the incremental depth-area distribution function.

## SYSTEM AREA-APPLICATION DEPTH INCREMENTS

The parameters which permit determination of the incremental depth-area distribution function are arrived at rather directly for sprinkle and trickle systems, but are somewhat difficult to develop for surface systems.

### Sprinkle irrigation

The depths of application from sprinkle irrigation tend to assume a normal distribution around their average, and can be plotted as a function of the percentage of land receiving a given depth. Because of the distribution this tends to give an S-shaped pattern similar to Fig. 1, as described by Hart and Reynolds (3). The shape of this pattern can be determined from the application uniformity as described by the Christiansen Uniformity Coefficient, UC. Hart and Reynolds (3) divided the area irrigated into 20 equal increments, used UC, and developed a method to determine the ratio of the depths received by each area increment to the average depth applied.

Thus, from the average application depth and the crop-water production function, total yield, Y, can be determined by

$$Y = (A/100) \sum_{i=1}^m P_i y(d_i) \quad (1)$$

where

- Y = total farm yield, units
- A = total area, ha(acres)
- m = total number of area increments
- $P_i$  = percent of land receiving  $d_i$  depth of irrigation water
- $d_i$  = depth of application received by the  $i^{\text{th}}$  area increment, mm (inches)
- y = functional relationship between yield, and depth of applied water, units.

When the irrigation system and crop production costs are known, Eq. 1 can be used to determine the profit associated with the considered average depth of application.

### Trickle Irrigation

The depth of application in trickle irrigation varies due to pressure changes in the main line; manifold and laterals and from variations in emitter characteristics. In large systems, the flow rates from individual emitters can be assumed to be normally distributed. The use of emission uniformity (Keller and Karmeli, 4) provided a basis for determining the distribution function for water applied by a trickle system. The Emission Uniformity, EU, is

$$EU = 100 q_n / q_a \quad (2)$$

where

EU = emission uniformity expressed as a percentage

$q_n$  = average of the lowest 1/4 of the emitter flow rates, lph (gph)

$q_a$  = average of all emitter flow rates, lph (gph)

The EU definition given by Equation 2 is identical to the definition of distribution uniformity DU, and as such can be related to UC by

$$DU = 1.6 UC - 60 \quad (3)$$

From Equations 2 and 3 and assuming that the point discharge of trickle emitters is normally distributed, the application depths ( $d_i$ ) for m area increments can be determined. Thus a calculation similar to that for sprinkle systems, using Equation 1, can estimate total yield and profit.

### Furrow irrigation

The uniformity of application in furrow irrigation depends on the furrow stream size, advance rate, intake characteristics, and time of application. The advance rate and intake are dependent upon soil conditions which are influenced by the soil type as well as by cultivation practices and previous irrigation history. Thus the depth of water stored varies not only with distance along the furrow but from furrow to furrow throughout the field. The prediction of crop yield as influenced by this variation requires a mathematical description of the interacting effects. For this study, a modification of the furrow irrigation model developed by Stringham and Hamad (2, 7) was used. Water distribution along the furrow is predicted by using intake characteristics of the soil and furrow channel geometry. Thus the time of advance is expressed by

$$t_x = f(q, x, K, n, c) \quad (4)$$

where

$t_x$  = time to advance a given distance, x, measured from the head of the furrow, minutes

f = a functional relationship

q = furrow stream flow rate, lps (gpm)

x = a given distance measured from the head of the furrow, m (ft)

c = a parameter to describe furrow characteristics

K and n are empirical coefficients

The accumulated depth of infiltrated water is estimated from field data by

$$d = K t^{n+1} / n+1 \quad (5)$$

where

d = accumulated depth of infiltrated water, mm (inches)

t = opportunity time, minutes

However, the intake function given by Eq. 5 may be valid for a given furrow only at a given time. For furrows which happen to be wheel

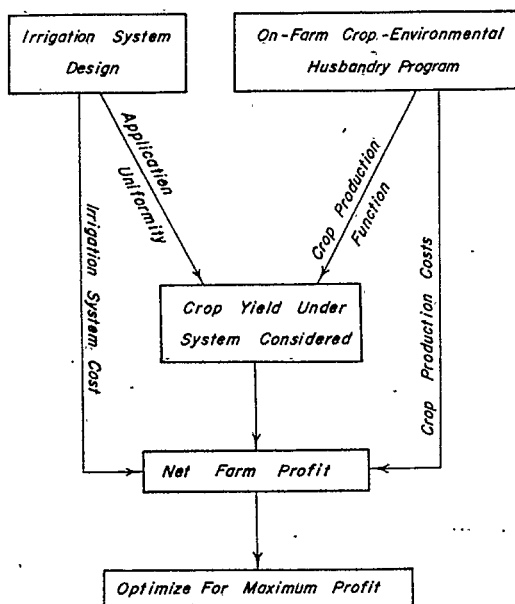
rows, intake rates will be appreciably lower than non-wheel rows although in some soils this difference decreases after the first irrigation following a cultivation. Due to the difficulty of measuring this variation in intake rates, and thus accumulated depth of infiltrated water, it was assumed that the measured values could represent an average intake function and that the actual values represent a typical distribution around the average. From the time of application per irrigation, the furrow advance function and average intake characteristics of the soil, an estimation of the water distribution along the furrow and the percentage of land receiving a given depth can be obtained as representing the average furrow. We assumed that the variation across different rows around this average was normally distributed.

Thus, if furrow length is divided into several reaches, not necessarily equal in length with each reach representing a percentage,  $P_i$ , of the land and receiving depth,  $d_i$ , of infiltrated water, then Equation 1 can be used to estimate total yield.

OPTIMIZATION

The flow diagram for the selection process which includes system uniformity and crop production factors is shown in Fig. 2. Both the physical and economic aspects must be considered for correct assessment of the best design. The system selection process as programmed on a digital computer permits relative ease of computation, minimizes analysis cost and thus simplifies examination of several different design alternatives and price-cost variations in the crop production components.

FIGURE 2



The actual optimization is accomplished by varying the seasonal depth of application between specified upper and lower limits in ten percent increments of the difference between limits. When the application depth giving a maximum profit point is located, the limits are redefined to this point plus and minus one increment value. The process is repeated until the upper and lower limits on the seasonal water depth are within a pre-established error. This process is conceptually simple as well as requiring minimal computer time.

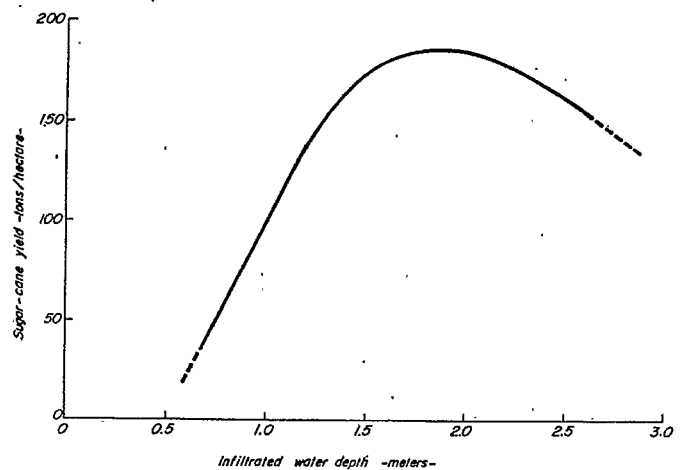
SAMPLE APPLICATION

The selection process is demonstrated for a given crop for each of the three system types under conditions approximating a given typical field situation.

Crop Production

The intent of this paper dictated that we use a crop which has been irrigated by all three system types under field conditions and for which some water production function type data were available. Sugar cane was picked as a suitable crop because it has favorable economics for the more expensive irrigation systems. Fig. 3 represents a water-production function for sugar cane yield as a function of water applied by rain and/or irrigation. This graph is a composite of the results from Chang, et al. (1) in Hawaii and data from South Africa (9). The Hawaiian data was manipulated to simulate a one year crop by dividing the yield and water quantities in half. When this was done the data for the two locations gave similar results which we feel are satisfactory for demonstrating the optimization techniques herein.

FIGURE 3

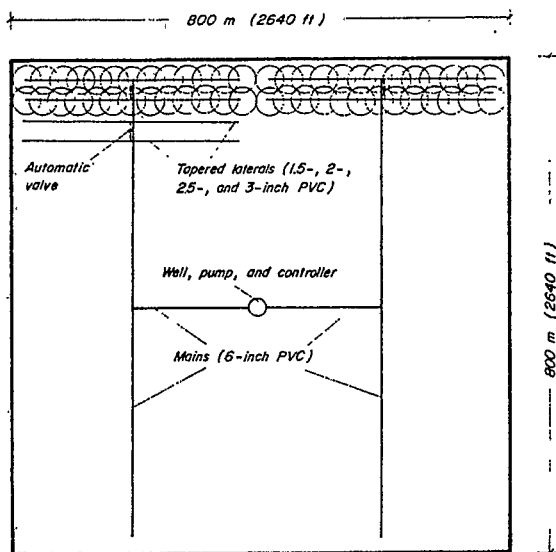


## System Layouts

A 65 hectare (160 acre) field with cross slopes of 0.1% and 0.32% was used as an appropriate sized unit to demonstrate the system uniformity-cost interactions. The field was assumed to have no extreme variation in the medium heavy soil with good conservation practices applicable to the particular method. The center of the field served as the water source for all systems.

The layout for the permanent sprinkler system is shown in Fig. 4. This system uses an H shape main line of 6 inch PVC pipe with tapered PVC laterals controlled in blocks by automatic valves. The blocks are operated such that the flow is split at the pump and again at the ends of the crossbar, thus minimizing mainline headloss. The trickle system in Fig. 5 has 6 and 8 inch PVC mainlines, with tapered manifold lines and a split flow operation similar to the sprinkle layout. The emitter line hoses are replaced annually for each new crop of sugar cane. Fig. 6 shows the components of a furrow irrigation system with a runoff recovery pump back system. Water is delivered at the top center of the field and near the well to open ditches which flow left and right from the center, thus minimizing the required mainline and pump-back piping.

FIGURE 4



## Capital costs

Different designs giving a range of application uniformities were developed for each respective system type. The capital costs of each design as a function of distribution uniformity (DU) are given in Fig. 7 for solid set sprinkle, trickle, and furrow irrigation systems. Land planing from time to time as required was assumed to be part of the annual costs for the furrow system. The data points

FIGURE 5

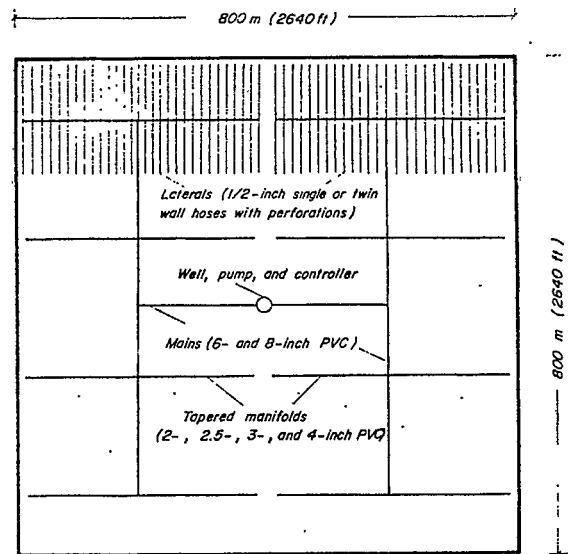
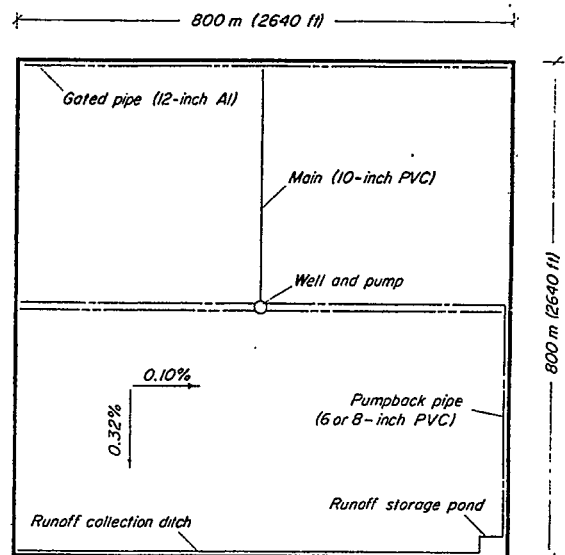
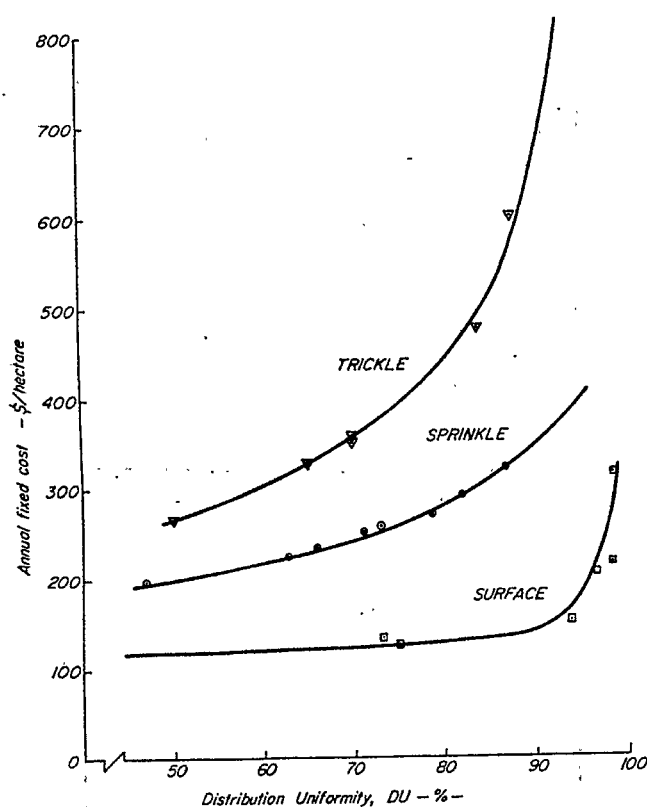


FIGURE 6



shown represent particular system designs. The common tendency for each cost vs. uniformity curve to asymptotically approach some base cost at low uniformities is attributed to the fact that a certain minimum amount of hardware is required for even a very poor system.

FIGURE 7



#### Adaptation of Designs to Field Conditions

The cost versus uniformity curves shown in Fig. 7 are based upon conditions assumed for design which may not be practicably attainable in the field. The sprinkle system uniformities were based on can catch data from the interior of an infinitely extending pattern with uniform nozzle pressure, and the surface uniformities came from intake and advance functions for an assumed average furrow. Sprinkle systems under field conditions have an appreciable edge effect without mirrored overlap and are subject to pressure differences due to elevation and friction headless in the pipeline which produces a "2nd level" reduction of application uniformity. A similar 2nd level reduction effect occurs in furrow irrigation as a result of soil variability along the furrow as well as intake differences between wheel row and non-wheel rows and from variations in stream size, and furrow shape and other miscellaneous losses. Design uniformities for trickle systems used herein already include the effects of pressure difference and flushing and the edge effects are negligible. Thus we assumed no 2nd level reduction for trickle.

The 2nd level effect was related to DU such that the estimated field DU was obtained by multiplying the design DU by the 2nd level DU. This estimated field DU was then used to obtain the irrigation application depth distribution for estimating total field yields from equation 1. Values of the assumed 2nd level DU are given in Table 1.

Results from field experiments have indicated an increase in sugar cane yield when shifting from furrow irrigation to sprinkle and a further increase from sprinkle to trickle. The increase could be a result of better uniformity as well as from more frequent irrigations associated with sprinkle and trickle. The portion of this increase estimated to be due to irrigation frequency is shown in Table 1.

#### System selection

The design process as programmed on a digital computer was used to evaluate the relative effects on the "best" irrigation system for sugar cane as influenced by market price, water cost, water lift in the well, individual system costs versus uniformity and crop production costs. The actual crop production and other costs used for this study are given in Table 1. The interaction of these variables for the previously discussed physical conditions are given in Tables, 2, 3 and 4, respectively, for sprinkle, trickle and furrow irrigation systems.

The included effective rainfall of 0.5 m (20") tended to moderate the system uniformity effects on profit, as about 30% of the required water came from the rain. Generally, the amount of water to apply from irrigation decreased as the cost of water (purchase or pumping) increased, which shifts the system uniformity distribution function into the region of the crop production curve (see Figs. 1 and 3) giving the highest marginal yields per unit of applied water. As might be expected, for a given crop sale value, the maximum profit decreased as water cost increased.

The sprinkle system (Table 2) had its maximum profit for most cases at the highest available design DU (95%). The selection of the high design uniformity is attributed to the relatively moderate cost increase to achieve higher uniformities shown in Fig. 7 for sprinkle systems as well as to the effect of the 2nd level uniformity reduction. Additional effort could be attempted to improve the field attainable uniformity with even further gains in profit.

Maximum profits for the trickle systems (Table 3) were obtained at design uniformities

Table 1. Basic data used in the analysis

System	2nd level uniformity factor (expressed as DU%)			% Yield Advantage
Sprinkle	Pressure Differences	Edge <sup>a/</sup>	Total	5 <sup>b/</sup>
	92	87-95	80-87	
Trickle	No effect		Total	15 <sup>b/</sup>
			100	
Surface	Intake function variability	Furrow size, q, etc.	Total	-3 <sup>c/</sup>
	81	84	68	

<sup>a/</sup> Edge effect varies with nozzle spacing dimensions

<sup>b/</sup> Estimated as a result of more frequent irrigations.

<sup>c/</sup> Estimated from amount of land lost to ditches and runoff pond.

Additional Information

Sugar cane sale value, two levels, \$10.90 and \$16.35/ton of cane  
 Crop production cost =  $600 + (Y - 119) \times 1.60$  \$/ha; Y = yield, tons/ha  
 =  $243 + (Y - 48) \times 1.60$  \$/acre; Y = yield, tons/acre

Effective rainfall = 508 mm (20 inches) at 100% uniformity.

Operation Maintenance Cost:

Sprinkle; 2% of capital cost per year.

Trickle; 2% of capital cost of pumps plus system excluding hoses per year. Emitter lateral hoses only last 2 years.

Furrow; 2% of capital cost per year, plus one full time man @ \$10 per hour including benefits, etc.

Diesel fuel at \$0.16/liter (\$0.60/gallon)

Land Grading Cost = \$371/ha (\$150/acre)

Money costs at 10% interest, 15 year term, CRF = 0.1315

Pumping plant capital cost = \$150/installed BHP

Pump efficiency = 74%

Irrigation efficiency =  $0.95 \times UC_{\text{field}}$   
 =  $0.59 \times (DU_{\text{field}} + 60)$

IRRIGATION SYSTEM . . . Continued

Table 2. Interaction of crop value, water cost, water lift, and uniformity under sprinkle irrigation for the sugar cane production function of Figure 3.

Crop Production Value = \$10.90/ton										
Water Cost \$/ha m	0 Lift					61 m Lift				
	Profit <sup>a/</sup> \$/ha	DU% Design	Yield Field	Irrigation <sup>b/</sup> ton/ha	mm	Profit <sup>a/</sup> \$/ha	DU% Design	Yield Field	Irrigation <sup>b/</sup> ton/ha	mm
24	803	90	79	189	1549	588	95	84	189	1397
162	600	95	84	189	1372	403	95	84	186	1321
406	284	95	84	181	1219	106	95	84	177	1143
Crop Production Value = \$16.35/ton										
3	1843	95	84	192	1524	1625	95	84	191	1473
162	1635	95	84	191	1473	1428	95	84	189	1397
406	1294	95	84	188	1346	1099	95	84	189	1321

<sup>a/</sup>Maximum profit from all systems available in selection process

<sup>b/</sup>Gross application over the season.

Table 3. Interaction of crop value, water cost, water lift, and uniformity under trickle irrigation for the sugar cane production function of Figure 3.

Crop Production Value = \$10.90/ton										
Water Cost \$/ha m	0 Lift					61 m Lift				
	Profit <sup>a/</sup> \$/ha	DU% Design	Yield Field	Irrigation <sup>b/</sup> ton/ha	mm	Profit <sup>a/</sup> \$/ha	DU% Design	Yield Field	Irrigation <sup>b/</sup> ton/ha	mm
24	963	70	70	202	1651	766	70	70	201	1575
162	741	75	75	204	1524	561	75	75	202	1448
406	393	75	75	198	1346	232	75	75	194	1270
Crop Production Value = \$16.35/ton										
24	2077	75	75	206	1626	1885	80	80	208	1549
162	1857	80	80	208	1549	1672	80	80	207	1499
406	1497	80	80	206	1448	1321	80	80	204	1397

<sup>a/</sup>Maximum profit from all systems available in selection process

<sup>b/</sup>Gross application over the season.



Table 4. Interaction of crop value, water cost, water lift, and uniformity under furrow irrigation for the sugar cane production function of Figure 3.

Crop Production Value = \$10.90/ton										
Water Cost \$/ha m	0 Lift					61 m Lift				
	Profit <sup>a/</sup> \$/ha	DU% Design	Yield Field ton/ha	Irrigation <sup>b/</sup> mm		Profit <sup>a/</sup> \$/ha	DU% Design	Yield Field ton/ha	Irrigation <sup>b/</sup> mm	
24	627	97	66	168	1753	405	97	66	166	1575
162	400	97	66	165	1549	195	97	66	162	1448
406	47	97	66	158	1346	--	--	--	--	--
Crop Production Value = \$16.35/ton										
24	1541	97	66	168	1753	1312	97	66	167	1676
162	1307	97	66	167	1651	1092	99	67	166	1549
406	929	99	67	165	1473	731	99	67	162	1422

<sup>a/</sup>Maximum profit from all systems available in selection process

<sup>b/</sup>Gross application over the season.

(70-80% DU) less than the largest (95%) allowed in the selection process. The selected design uniformities increased with water cost and with increases in crop sale value. The trickle system also shows higher profits under all conditions than did sprinkle. This is a result of the higher yield advantage for trickle and correlates with field experience.

The furrow systems differed in that higher design DU's were available for selection, 99% versus 95%. However, a harsher 2nd level DU was imposed to account for field variability, etc., as indicated in Table 1. Inspection of the furrow system cost curve in Fig. 7 would indicate that the great improvement in uniformity to that point without much increase in capital cost. The results in Table 4 support this suggestion, in that a 97% DU system is selected for all but three economic settings. Even for those three the difference in profit between the 97% and 99% DU systems was less than \$5.00/ha. The profit under furrow was less than under sprinkle by some \$200/ha to \$370/ha depending on the situation. Although this is a result of the 2nd level DU, this again supports the observation that the growers are shifting away from surface systems under these economic constraints in sugar cane production.

To be fair in the evaluation one must include all operational and capital costs for each system, i.e.: water, labor, field shaping, land lost to ditches, etc. In the analysis of furrow irrigation land was taken out of production for the ditches and the tail water recovery system, whereas for all systems

the area lost to pipelines and pump units was taken to be of negligible extent.

#### Summary and Conclusions

The systematized design process presented herein provides a tool for the designer which integrates the effects of system, water and crop production costs and returns. With this capability the design engineer has a higher probability of identifying adequate irrigation systems for the specific economic setting of the user. The inclusion of more relevant variables into a workable design process should prove helpful in increasing the designers' awareness of the interaction of crop production with irrigation systems.

The crop yield response to water use was provided by a seasonal crop-water production function. The influence of changes in irrigation frequency, depth of application per irrigation, soils and other related factors can be readily incorporated into this design process, when more reliable crop yield prediction techniques (models) are available.

Increasingly higher costs of applying irrigation water caused by escalating energy and capital costs will require more attention to the total environment in which a designer must operate. The sensitivity of the profit to uniformity and system operating point poses some serious implications for owners of systems installed in past years. If these systems are operated as if yesterday's economics still hold, then quite likely money is being lost without their knowledge. The methodology

presented here provides a mechanism for analysis and presents results suggesting that increased attention to improved irrigation system uniformity, at reasonable costs, may be mandatory to maintain a favorable profit picture.

9. Rhodesia Sugar Association Experiment Station, Chiredzi, Rhodesia. Miscellaneous Reports of field trials. Courtesy of T. L. Pearse.

#### ACKNOWLEDGMENTS

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