ECONOMIC EFFECTS OF FLUCTUATIONS IN CATCHES AND POPULATION LEVELS, UPON A POPULATION OF ALASKA KING CRAB[1]

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ABSTRACT

A multiple-size-class model of the Bering Sea king crab fishery is presented, including a nonlinear, observation-based reproductive submodel, as well as measures of long-run and short-run economic benefit. The model can simulate a wide variety of equilibrium and non equilibrium fishing strategies. Demonstration cases show the importance of natural mortality parameters to the relationship between fishery management strategy and equilibrium yield, and give an example of alternating between two fishing strategies, to produce long-run yields exceeding those of either strategy implemented alone.

I. INTRODUCTION

The fishery for king crab (Paralithodes camtschatica) is of long standing in Alaska, and of major economic importance. For example, during the recent 1974-75 fishing season, about 91×10^6 lbs of king crab were taken [7, table 2; taken from 1]. At prices of about $0.42/lb paid to fishermen, that represents about 38×10^6 dollars; at wholesale prices, after processing, the value is substantially higher. The majority of this catch was taken in the Bering Sea, Kodiak, and Dutch Harbor statistical areas of Alaska, with 17.75×10^6, 10.15×10^6 and 5.17×10^6 dollars paid to fishermen in 1974-75.

Previously, Balsiger [2,3] has formulated a sex-differentiated, multiple-age-class simulation model of Bering Sea king crab, based upon population dynamics and fishing parameters obtained by National Marine Fisheries Service in 1966-1969 surveys, and upon Powell's [9,10] studies of the crabs' mating process. Balsiger's model was dynamic (discrete-time) in nature, but its applications analyzed differences between effects of various steady-state fishing strategies. This is because the fishery's management has historically been based upon sustainable yield strategies (maximum sustainable yield, optimum sustainable yield). These equate management of the fishery, for long-run benefit, with maintenance of the fishery at an optimum equilibrium yield, and

with returning the fishery to equilibrium if perturbed. Here, we present a modification of Balsiger's model, which retains the biological detail characterizing the crab population, while allowing simulation of a wide variety of fishing strategies (equilibrium or non-equilibrium, finely or coarsely size selective; catching males only as in the present fishery, or males and females). The model includes a measure of long-run economic benefit, whose parameters can be related to "real world" fishery economic data.

II. MODEL FORMULATION

The following features of the population and the fishery motivate the model's design:
(1) The molting/breeding season occurs annually at a predictable time. Growth can occur only in conjunction with molting. Not all crabs of a given size grow the same amount.
(2) Mature males tend to skip-molt (molt in alternate years) though not with probability one. This further reduces the direct correspondence between age and size.
(3) The number of eggs produced, in any year, is a complicated function not only of numbers of males and females, but of current male and female size distributions, vis-a-vis each other. The function is most complicated when appropriate-sized males are in short supply (say, through severe fishing).
(4) Growth and mortality data [2] is by size.
(5) Fishing parameters (catchability coefficients, weight per crab) depend upon crab size.
(6) Crabs can be size-sorted aboard fishing vessels, and prohibited sizes returned alive to the sea, with minimal damage. Thus, fishing effort can be exerted differentially, by size.

Because of the above features, the model is discrete time (annual time step), and the state variables are number of males and females by size. This is a modification upon Balsiger's original formulation in which age-classes were the state variables, and conversion computations between size and age were required at each time step. The model equations are as follows:

$$x(n+1) = e^{-F_f(n)} e^{-M_f} A_f x(n) + \xi R[s(x(n), y(n), F_f(n))]$$

$$y(n+1) = e^{-F_m(n)} e^{-M_m} A_m y(n) + \xi R[s(x(n), y(n), F_f(n))]$$

where n is the year, and
$x(n), y(n)$ (column vectors) = numbers of crabs in each size class of females, males, in year n <u>before</u> the molt/breeding season. Size classes (states) are defined in Table 1. For adults, size classes are taken at 5 mm intervals in shell length, to match available growth data.

TABLE 1

<u>State Variables and Fishing Groups</u>

<u>Females:</u>
x_0 = first juvenile form (1 yr after mating)
$x_1, \ldots x_5$ = 1, ..., 5 yr old juveniles
$x_6, \ldots x_{10}$ = 92, 97, ..., 117mm adults
x_{11}, x_{12}, x_{13} = 122, 127, 132mm adults
$\qquad\qquad\qquad\qquad$ (Fishing Group 1)
x_{14}, x_{15} = 137, 142mm adults (Group 2)
x_{16}, x_{17} = 147, 152mm adults (Group 3)
x_{18}, x_{19} = 157, 162mm adults (Group 4)

<u>Males:</u>
y_0 = first juvenile form
$y_1, \ldots y_5$ = 1, ..., 5 yr old juveniles
$y_6, \ldots y_{11}$ = 92, 97, ... 117mm adults
y_{12}, y_{13}, y_{14} = 122, 127, 132mm adults (Group 5)
y_{15}, y_{16} = 137, 142mm adults (Group 6)
y_{17}, y_{18} = 147, 152mm adults (Group 7)
y_{19}, y_{20}, y_{21} = 157, 162, 167mm adults (Group 8)

M_f, M_m (diagonal matrices) = size-specific natural instantaneous mortalities of females, males (so that e^{-M} are annual survivorships).

$F_f(n), F_m(n)$ (diagonal matrices) = yearly totals of size-specific <u>instantaneous</u> fishing mortalities of females, males (so e^{-F} are annual survivorships and $1-e^{-F}$ are annual mortalities). These may be defined as control variables, with one independent fishing mortality for each fishing group (comprising 2 or 3 state-variable size classes) defined in Table 1. Alternatively, note that mortality is actually inflicted by lifting crab pots (traps) and the cost of fishing depends on number of pots lifted, not (directly) upon fractional mortality inflicted. Accordingly, elements of the F matrices may be related to potlifts $F_i(n) = q_i f_j(n)$ where q_i is a size dependent catchability coefficient and $f_j(n)$ is the number of potlifts allowed to be exerted on the corresponding fishing group. For example, $f_8(n)$ = number of potlifts exerted upon group 8 (class 19,20,21) males (cf. Table 1). Then

$$F_i(n) = q_i \, f_8(n), \quad i = 19, 20, 21$$

gives the last 3 elements of the $F_m(n)$ matrix
A_f, A_m (20 x 20, 22 x 22 matrices) are Markov growth matrices.
ξ (column vector) has a 1 in the top position, the rest zeros. It reflects the fact that (surviving) eggs become the smallest size crabs next year.
R (scalar function of scalar variable) is a Beverton-Holt larvae-recruit function which estimates fractional survivorship from egg hatching until beginning maturity (x_5 or y_5). Thus natural mortalities (in the model) of sizes 0-5 males and females are set to zero. The R function is

$$R(L) = C[1.7688 \times 10^{-8} + 2.61175 \times 10^5/L]^{-1}$$

where the numerical values were obtained by Balsiger from data [2]. The constant C allows adjustment of steady state values to suit any present or past conditions being used as a "nominal" solution.
$s(x, y, F_f)$ (scalar-valued function of two vector and one matrix variable) computes the number of larvae released at the end of year n, as a function of numbers and sizes of males and females at the beginning of year n, and of the natural and fishing mortality (if any) exerted upon females (who carry the eggs on their body for 10 months after fertilization). The complicated functional form of s, and the underlying biological assumptions based on Powell's observations are in the Appendix.

The economics of the fishery are modelled in terms of present value of present and future catch [4]. Thirty years is chosen as "long run" from a fisheries standpoint. Present value is given by

$$PV = \sum_{n=1}^{30} \Delta^n [RTVI(n)]$$

where $1-\Delta$ = annual discount rate and RTVI(n) is return to vessel investment (for the whole fleet). Assuming a fleet of equal-size fishing boats for simplicity, Katz and Lee [8] model RTVI(n) by

$$RTVI(n) = \rho \cdot GR(n) - c \cdot TPL(n) - FC$$

where ρ, c, and FC are constants, derived [8] from fuel cost, fraction of a trip spent running into port, share allocated to crew and skipper, etc. In our model, gross return in year n is given by

$$GR(n) = p\{w_f[I-e^{-F_f(n)}]A_f x(n) + w_m[I-e^{-F_m(n)}]A_m y(n)\}$$

where p is price per pound of crab in the shell, w_f and w_m are row-vectors of size-specific weight per crab (females, males).

Total pots lifted are given by

$$TPL(n) = \max[f_1(n), \ldots, f_4(n)] + \max[f_5(n), \ldots, f_8(n)]$$

The form of the above function arises as follows. In the present day (males only) fishery, males are fished when they are separate from females; should females be fished, a similar practice would be expected. Thus the two terms for separate fishing of males and females. The origins of the max function are illustrated by example. Suppose 1000 potlifts were to be allowed on #8 size-group crabs and 1500 potlifts on #7 size-group crabs. In the field, this would be implemented by allowing 1500 actual potlifts, but requiring #8 size-group crab to be thrown back after 1000 potlifts. Thus f_8 would be 1000, f_7 would be 1500, but the number of pots physically lifted, and causing cost to fishing boats, would be max(1000,1500) = 1500.

III. DEMONSTRATION CASES

Two sorts of demonstrations of the model's performance are presented. First, the importance of variations in key biological parameters, upon steady state fishing yield will be demonstrated. Secondly, the model's dynamic behavior will be displayed by

comparing its outputs for certain equilibrium and non-equilibrium fishing strategies.

A DISCLAIMER

Any simulation model, no matter how thoroughly calibrated and studied, can never be more than a tool to suggest strategies for consideration by fishery managers. Besides this general caution, we warn that the following results depend upon many biological parameter assumptions, some of which are unverified. A few parameters were chosen ad hoc, without extensive discussions with experienced fishery sources. We hope, then, that the type of results presented here will stimulate interest, on the part of fishery management personnel. It is imperative, however, that the demonstration results not be construed as explicit or implicit management recommendations.

PARAMETER ASSUMPTIONS

Obviously not all parameters could be varied in these demonstrations. Certain ones were fixed as follows, for all cases contained in this paper.

The growth matrix for males, A_m, was produced [2] from Bering Sea data of 1966-69. It incorporates probabilities of skip molting and of growing various amounts, as these depend on size. The female growth matrix was estimated by Katz and Balsiger [6] from Gray's [5] data on female growth.

Female natural mortalities (matrix M_f) for which good data is not yet available, were taken to be 10% annually, except 30% annually for the two largest female size classes. These are ad hoc estimates based on casual discussions with Alaska crab specialists. Effects of varying these estimates will be discussed in a later work.

The internal parameters in the reproductive submodel (cf. Appendix) included a 30-day mating season, a 5-day grasping time from start of female molt until copulation, and a conservative maximum of 4 females/male, for the season.

Catchability coefficients q_i, though available from data [2] were not used. Instead, annual fractional fishing mortalities were used as inputs. An independent mortality was allowed for each fishing size group. Females were not fished, in keeping with present fishery policy.

Because of ongoing interaction with fisheries personnel regarding dependence of cost constant c (in the present value index) upon trip length, the return to vessel investment was taken, for the time being, as proportional to gross return. This allowed short and (undiscounted) long-run performance to be represented, respectively, by annual and 30-year-total yield.

STEADY STATE COMPARISONS

A question of continuing importance in the king crab fishery is the allowable severity of fishing, if maximum fishing yield is to be sustained. At one extreme, if all the crabs were caught one year there would be no more reproduction and no yield sustained. At the other extreme, catching no crabs would result

in the maximum standing stock of crabs, but no yield. The maximum lies in between. The situation is complicated by the oldest male crabs containing the most meat, and being most likely to die naturally before next fishing season (mitigating in favor of catching them). However no other males can mate the largest (most fecund) females - cf. Appendix (mitigating in favor of letting tnem survive).

The lower are male natural mortalities, the greater is the fraction of males surviving to be large, the greater is the fraction of larger females mated, the greater is the dependence of total reproduction on large females' eggs, and thus the greater is the loss in population if a high fraction of large males are fished. This is well known. Here we demonstrate the model's ability to quantify the phenomenon.

Two sets of male annual natural mortalities were defined. For size classes 6 through 15, both sets are respectively equal to (10%, 10%, 10%, 10%, 10% 10%, 7%, 7%, 7%, 7%) which are approximately equal to those reported by Balsiger [2]. For male size classes 16-21, the "high" natural mortalities were defined to be (10%, 10%, 12.5%, 15%, 20%, 30%) whereas the "low" natural mortalities (16-21) were all defined equal to 7%.
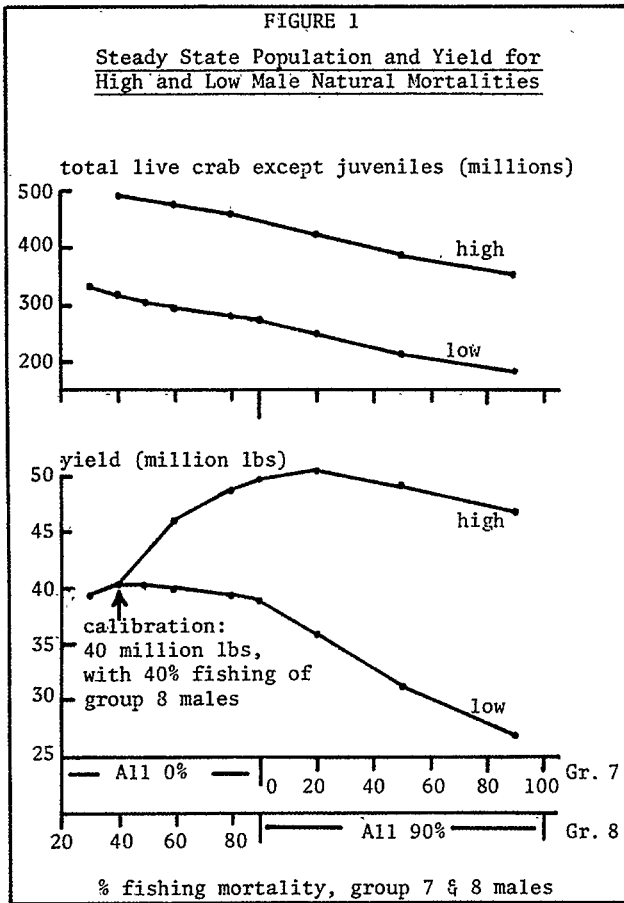
To provide a standard of comparison, a value of reproductive constant C was chosen, for each set of male natural mortalities, so that 40% annual fishing mortality of group 8 (largest three size classes - 19, 20, 21) males would result in 40×10^6 lbs yield, at steady state. The 40×10^6 lb at 40% figure is in the ballpark of present-day management policy. Thus we have abandoned all pretext of predicting absolute population and yield levels; instead we scale the population to approximate present policy and levels.

We sought to compare the effects of assuming high and low natural mortalities, upon the changes from 40×10^6 lb yield which would result if more severe fishing than (40% annually, group 8 only) were applied to the model. To this end, we established a sequence of increasingly more severe fishing policies. First we increased fishing on the group 8 males until 90% annually; then we began fishing group 7 (size classes 17-18) males with increasing intensity. The chosen sequence of (group 7 fishing mortality, group 8 fishing mortality) pairs was (0%,40%), (0%,60%), (0%,80%), (0%,90%), (20%,90%), (50%,90%), (90%,90%). The horizontal scales of figures 1, 2, and 3 depict these pairs, with increasing severity, from left to right. A single horizontal coordinate defines a point on each of the "group 7" and "group 8" scales, and thus a member of our sequence (or a pair in between).

The crab model was run repeatedly to steady state, applying each fishing mortality pair above, with each set of "high" and "low" male natural mortalities. The C values that resulted in (40×10^6 lbs, at 40% group 8 fishing) were retained for their respective sets of male natural mortalities. Thus, in each of the "high" and "low" series of runs, no model parameters were changed except fishing mortality.
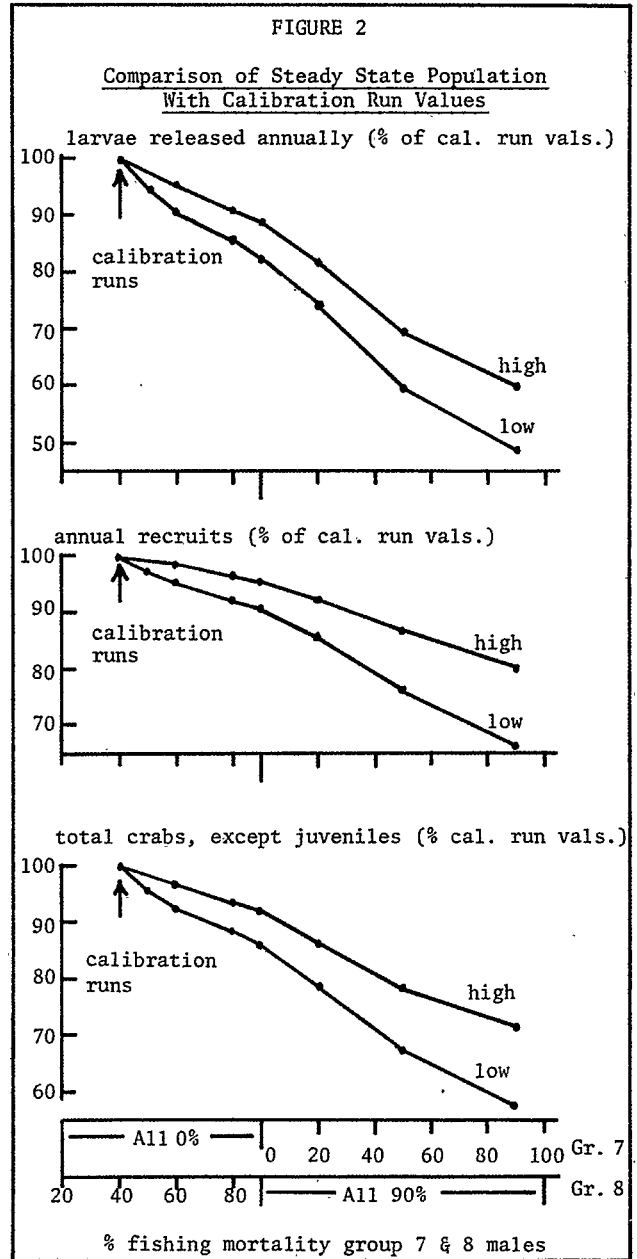
Figure 1 displays the steady state standing stock and fishing yield for the two series. The upper graph shows nothing extraordinary. For both series, the more severe the fishing, the fewer crabs are

left to reproduce, and the lower the steady state population. The standing stock is higher, across the board, for the "high" natural mortality series because high natural mortalities mean that a smaller fraction of males reach the largest (group 8) sizes; thus more crabs must be hatched to allow 40% of group 8's to equal 40 million lbs, than would be required under "low" natural mortality assumptions. The lower graph, however, displays strikingly different behavior, with increased severity of fishing, for the two series; the "high" natural male mortalities result in increased yield, peaking at (90% group 8, 20% group 7) male fishing mortalities, whereas the "low" series results in decreased yield.

In the "high" series, this tendency is counterbalanced, at least up to (90% group 8, 20% group 7) fishing (cf. Fig. 1), by taking an increased fraction of crabs before they die natural deaths, whereas in the "low" series that tendency is not sufficiently counterbalanced.

FIGURE 1

Steady State Population and Yield for
High and Low Male Natural Mortalities

total live crab except juveniles (millions)

yield (million lbs)

calibration:
40 million lbs,
with 40% fishing of
group 8 males

% fishing mortality, group 7 & 8 males

FIGURE 2

Comparison of Steady State Population
With Calibration Run Values

larvae released annually (% of cal. run vals.)

annual recruits (% of cal. run vals.)

total crabs, except juveniles (% cal. run vals.)

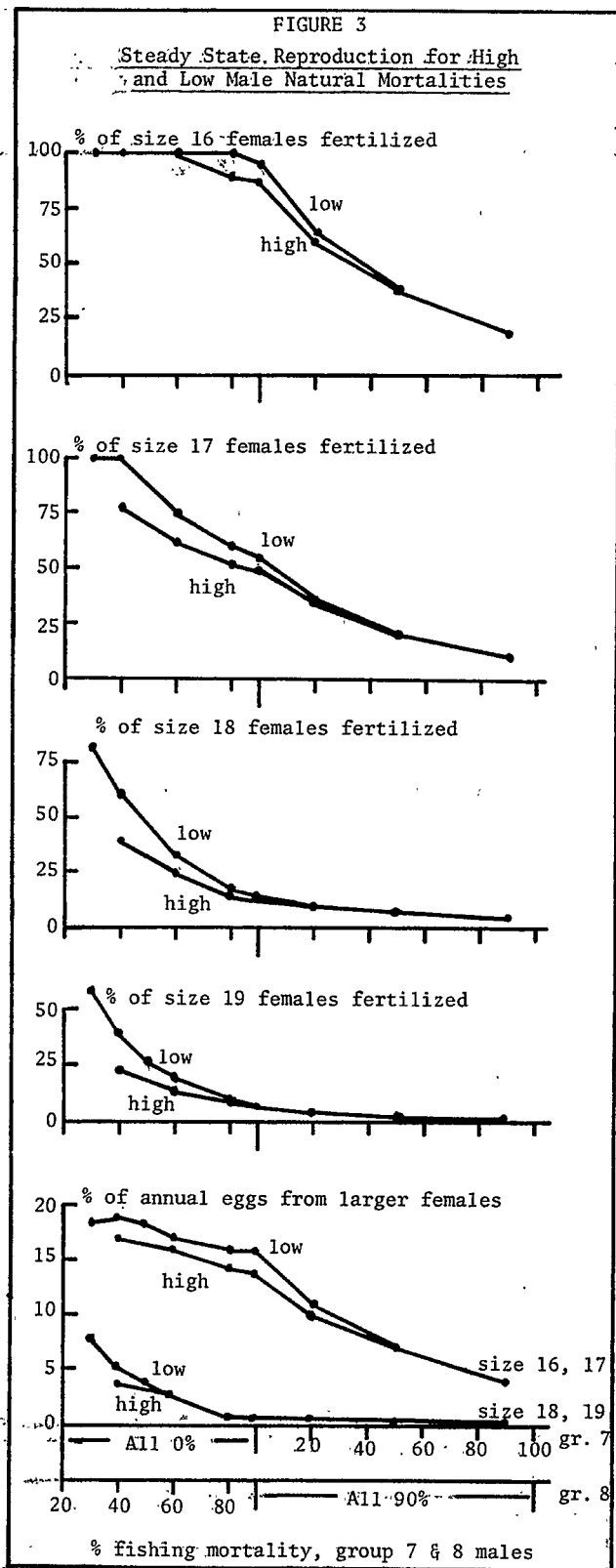% fishing mortality group 7 & 8 males

To elucidate the causes of these yield differences, figure 2 displays released larvae, annual recruits, and total crabs, varying with fishing severity. To "wash out" the effect of differing total populations, values are displayed as percentages of those generated in their respective series' calibration runs; thus all curves go through 100% at (40% group 8, 0% group 7) fishing. All these population-reproduction measures fall faster in the "low" series, relative to their original values, than they do in the "high" series. Evidently steady state recruitment and population fall, with increasing fishing severity, for both "low" and "high" natural mortalities, tending toward decreased yield.

The reproductive submodel assumes that no male can fertilize a larger female. The size classes are defined so that size 18 and 19 females can be fertilized only by group 8 males, and size 16 and 17 females only by groups 7 and 8. Figure 3 displays the fractions of these sizes of females fertilized, and their percentage contribution to total egg pro-

duction. It can be seen that at the (low fishing severity, 40% group 8 fishing) calibration point, group 17, 18, and 19 females are substantially

FIGURE 3

Steady State Reproduction for High and Low Male Natural Mortalities

% of size 16 females fertilized

% of size 17 females fertilized

% of size 18 females fertilized

% of size 19 females fertilized

% of annual eggs from larger females

% fishing mortality, group 7 & 8 males

better fertilized, and contribute a greater percentage of total eggs, in the "low" series than in the "high". Thus in the "low" natural mortality case, the population has more to lose than in the "high" case, as fishing severity is increased and group 8 males become unavailable for mating big females. This is most pronounced at the low end of the fishing severity scale, where the yield rates contrast the most; once fishing severity reaches (90%, 20%) the differences virtually cease, and yield decreases in both the "high" and "low" series (Figure 3).

Observe, then, that in the model, the "low" male natural mortalities indicate that if steady state yield is to be maximized, a multiple-year-class fishery (i.e., 40% taken annually) ought to be applied to group 8 (largest) males, and no smaller males should be taken. Conversely, the "high" male natural mortalities mitigate for a "recruit" fishery (fishing almost all, annually) on group 8 males, and for taking a few group 7 males, as well.

Here we recall the earlier disclaimer. Particularly, we caution against taking the demonstration cases' specific numbers (e.g., 7% vs 10%, 15%, 20% male natural mortalities, 40% vs 90% fishing mortalities) too seriously. A few model tests indicate, for example, that decreasing female natural mortalities (which, after all, were fixed ad hoc) would shift all results in the direction of less severe fishing.

If the results are dependent upon numerical parameters, and some of those parameters are not backed by good data, then what can we have achieved? We believe that we have demonstrated a model which qualitatively embodies present beliefs about the actual fishery, vis-a-vis behavior of the reproductive process and dependence of results upon natural mortalities. Thus we hope that experienced fishery managers and participants will be motivated: (1) to collect and analyze data to fill in "gaps" in parameters whose importance is elucidated by the model; (2) in the interim, to provide us with their best "guesstimates" of such parameters, and to consider the implications of model results arising from these "guesstimates".

NON-EQUILIBRIUM FISHING

In the previously described, "low" natural male mortality model tests, equilibrium yield decreased for fishing severities beyond (40% of group 8, 0% of group 7). This appeared to result from the dependence of reproduction upon large females (and hence upon the large males that mate them). However, given the substantial loss of large crabs to natural mortality, over the long run, one may wonder whether more large crabs than 40% may be taken occasionally, without overly large losses to the population.

Fishing pulses of (90% of group 8; 0% of group 7) were applied to the model every 7 years, with 40% of group 8 applied in interim years. All other parameters remained identical to those in the equilibrium 40%-of-group 8 run of the "low" series.

The 30-year yield of the pulsed strategy exceeded the yields of the relevant equilibrium strategies as follows:

| strategy	30 yr yield
equilibrium, 40% group 8	1.208×10^9 lbs
equilibrium, 90% group 8	1.171×10^9 lbs
pulsing between 40%, 90%	1.230×10^9 lbs

Figure 4 summarizes the model's 30 year behavior. Peaks in the yield curve evidently more than compensate for the dips. Large male and female mating indicators fluctuate widely, but recover rapidly. Overall recruitment and adult population fluctuate much less.

Apparently the model's population can sustain substantial temporary decreases in the largest brood stock because of longevity, so that the model has many younger crabs "in the pipeline" ready to become brood stock, even if one year class of large males is cropped. Note that the pulsing here is quite different from overfishing continuously or for several years, which have been known to decimate commercial fish stocks.

The results are interesting but incomplete. There are many types of pulsing strategies of which this is but one. Other pulsing strategies have, indeed, decreased long-run yield in the model. An overall pattern of increases and decreases has yet to be ascertained. For at least one set of parameters, however, the model has demonstrated increased yield due to deliberately induced fluctuations. Again, however, the earlier disclaimer should be recalled.

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FIGURE 4

Comparison of Equilibrium and Pulsed Fishing Strategies
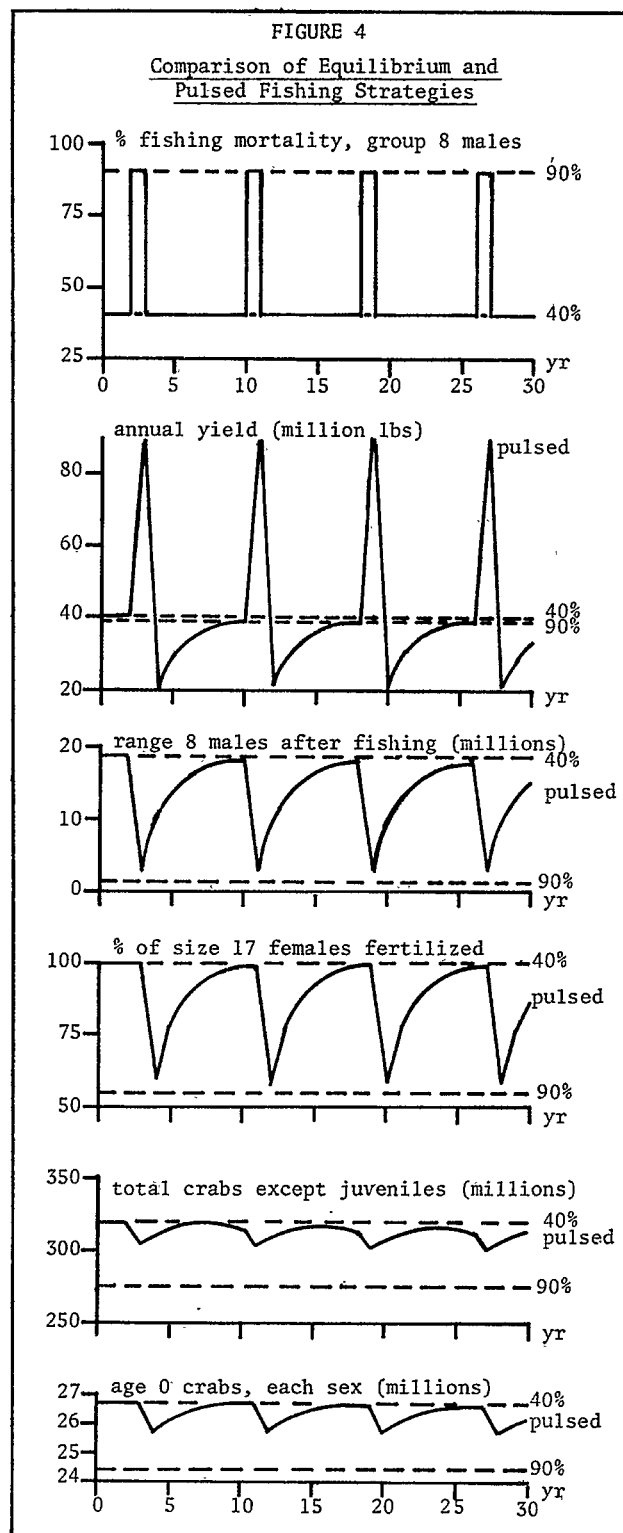
APPENDIX - REPRODUCTIVE SUBMODEL

Powell has extensively studied king crab reproductive biology [9, 10, and unpublished reports and personal communications]. From these, Balsiger (1974) modelled the king crab mating process. The present reproductive model simplifies the equations, retaining essentially all of Balsiger's detailed biological assumptions. The mating model's inputs are numbers of males and females (by size) before the molt-spawning season, as well as death rates to females while carrying eggs. The output is number of eggs hatched (larvae released) 10 months after copulation.

The assumptions about king crab mating behavior which Balsiger and Katz consider to best abstract and model the actual reproductive process, are listed below. Obviously other possible assumptions may be substituted. A major impetus for Powell's original crab-mating studies was a series of observations of partially or completely barren females, in areas where males were depleted. Accordingly, since the problem may have been insufficient male capacity, the assumptions were conservative about male mating capacity.

(1) Copulation occurs when females molt, if appropriate males are available; males encounter no difficulty in locating molting females. This is supported by the crabs' definite migratory patterns to the spawning grounds; encounter is not left to chance.
(2) Males must be at least as large as the females with which they mate. Males may be either pre-molt, post-molt, or skipping molt (but not during molt) at the time they grasp a female for mating. Females grasped for mating are always pre-molt, since the molt initiates copulation and fertilization. Thus the relevant sizes are the males' actual size at time of mating and the females' size before mating.
(3) Males hold females an average of 5 days prior to copulation, waiting for the females to molt. Males are capable of mating no more than about 4 females each, for the season. These limit the capacity of males for multiple breeding.
(4) Larger males have precedence over smaller males in holding females prior to mating, so that males have priority by size. However, larger males, faced with a variety of sizes of females, apparently show no preference for larger females.
(5) A single copulation results in fertilization.
(6) Male size does not affect the fraction of the egg clutch which is fertilized. (The total number of eggs fertilized is size dependent, since fecundity of the female is related to size.)
(7) If a female dies within 10 months after copulation (prior to releasing her eggs), the eggs die.

Katz and Hamilton [6] transformed the above assumptions into a single complicated equation. The algebra was non-trivial, but probably uninteresting now that it's done.

$$s(x,y,F_f) = \varepsilon_6 \, z_6 \, e^{-F_{f6}} \, \text{sat}\left(\sum_{i=7}^{21} \lambda_i/Z_i \right)$$

$$+ \ldots + \varepsilon_{19} \, z_{19} \, e^{-F_{f19}} \, \text{sat}\left(\sum_{i=20}^{21} \lambda_i/Z_i \right)$$

In the above, the 6 through 19 terms represent larval production by all sexually mature size classes of female. The ε_k are (size-dependent) eggs/female, corrected for 10 months' female natural mortality; the z_k are number of sexually mature females, by size; the e^{-F} terms correct for females that die by fishing (if any) while carrying eggs; the λ_i are male mating capacities (number of males x # females servicible by each male), by male size, taking into account length of mating season, grasping time, fraction of males molting, mating incapacity due to male molting, and overall season maximum females/male; the Z_i are total number of females small enough to be held by size i males (whether they are so held, or not). The summations determine whether there are large enough males to service the various sizes of females. The sat functions give the fraction of each female size class fertilized.