ORIGIN OF SIMULTANEITY IN CORPORATE MODELS

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Corporate planning models are firmly established in the formal planning process of many firms. Of particular popularity is the deterministic corporate model, which represents the entire firm or some subdivision of the firm. This paper defines the nature of simultaneous relationships that are found in these corporate models. Three classes of simultaneity are identified that do not correspond to the actual causal behavior of the firm. Two of these classes are useful simulation devices, while the third is a class of problems common to corporate models.

INTRODUCTION

Recent advances in simulation software and modeling expertise have contributed to the success of the corporate model as a planning tool [4]. Unlike the earlier mission of the corporate model [3] to represent every facet of the firm in great detail, the concept of a corporate model described here is a comprehensive, yet concise, mathematical representation of the firm or some subdivision of the firm. As demonstrated by Warren and Sheldon [7], a simultaneous equation approach is essential if a corporate model is to be both concise and capable of expressing the interdependent characteristics of the firm.

This paper defines the nature of simultaneity found in corporate models as it relates to causal behavior. In particular, this paper identifies three classes of simultaneity that do not correspond to actual causal relationships: simultaneous decision rules, reverse simultaneity, and spurious simultaneity. Simultaneous decision rules and reverse simultaneity are useful computational devices, while spurious simultaneity is a misrepresentation of relationships in a corporate model.

CAUSALITY, SIMULTANEITY, AND CORPORATE MODELS

It is widely acknowledged that in theory dynamic models of social and economic systems are purely recursive [1][8]. That is, actions are caused by previous actions because time delays in the natural stimulus-response processes eliminate the possibility of simultaneous causality. Adhering to this principle, some schools of modeling maintain that derived mathematical models should also consist of only recursive relationships. A notable field in this purist school is Industrial Dynamics [2], which proposes that the time period of a model should be sufficiently small as to dissolve any interdependencies between variables within the same time interval, thereby eliminating any simultaneous relationships. Attempting to preserve these recursive relationships in a mathematical model poses resource problems, and an approach with larger time periods and concise relationships becomes a desirable alternative. By choosing such an alternative, one finds that simultaneous equation models are useful in expressing interdependent feedbacks that are captured within concise relationships and larger time periods.

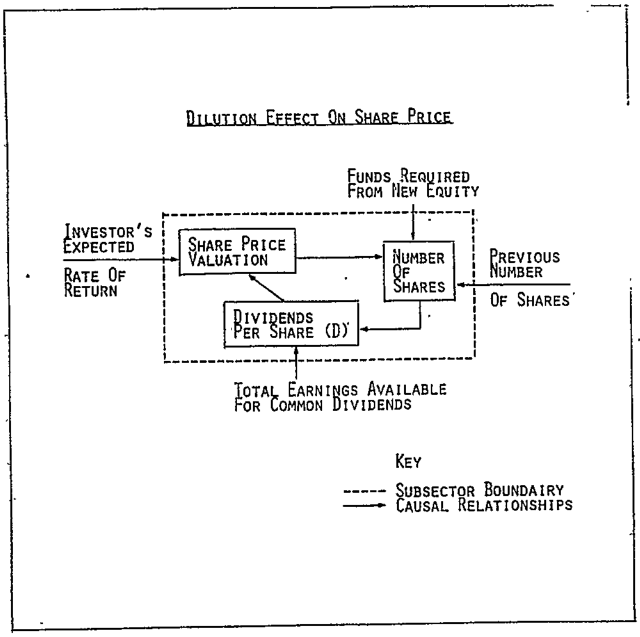
The focal points or "bottom-lines" of social and economic models depend upon their subsystems. Individually, each of these subsystems is sufficiently complex to encumber a model with immense detail. Perpetually faced with this problem, the model designer must balance the tradeoffs of detail versus realism by summarizing the behavior of the subsystem. For instance, dividends and capital structure are customarily two focal points in corporate models. If a corporate model projects dividends per share and new equity issues, then it must also express the valuation of these new equity issues. The process of equity valuation implies the inclusion of equity market behavior, which is a subsystem of tremendous complexity.

A simplification of this subsystem is shown in Illustration 1. The characteristics of this subsector are based on a classical valuation model [5] that reflects the discounted return on investment to the owner of the equity share. Upon issuing new equity shares, the following chain of causes and effects between the number of shares (S), the "per-share" return on investment (R), and the price per share (P) is anticipated.

$$S\uparrow \rightarrow R\downarrow \rightarrow P\downarrow \rightarrow S\uparrow^{1} \ldots$$

[1] $\uparrow$ indicates an increase.
$\downarrow$ indicates a decrease.
$\rightarrow$ indicates dependency.

ILLUSTRATION 1



Apart from these simplified assumptions, building a detailed recursive model of an equity market poses difficult questions. What are the time delays and durations in this sequence of causes and effects? Is the effect of the dilution anticipated far in advance of the formal sale of new shares, or is the market less efficient? In building comprehensive corporate models that must express many complex subsystems, these questions become distracting. Alternatively, the intricate behavior of the equity market can be summarized by a simple simultaneous system as shown in Equations 1, 2, and 3.

$$P_t = \frac{D_t}{(r-g)} \qquad (1)$$

$$S_t = \frac{F_t}{P_t} + S_{t-1} \qquad (2)$$

$$D_t = \frac{E_t - R_t}{S_t} \qquad (3)$$

Endogenous Variables

P_t = Equity share price

S_t = Number of equity shares

D_t = Dividends per share

Exogenous Variables

r = Investor's expected rate of return

g = Dividend growth rate

F_t = Funds required from new equity

E_t = Total earnings available for common dividends

R_t = Desired retention of earnings

The purpose of this example is not to propose a particular price valuation theory or dividend policy, but to demonstrate a simple scheme for expressing the dilution cost accruing to the sale of new equity issues. Building a more detailed recursive model is an unreasonable alternative. The increase in development expense of such an approach is a certainty, while the payoff in increased predictability is doubtful.

In many cases the availability of reported data forces the period size of derived models to exceed that which is necessary to accurately reflect the recursive qualities of a system. Corporate models, however, are an exception. The restrictions on period size of corporate models arise from the long-range perspective of the planning function of a firm. As a planning tool, corporate models generally project a subset of variables over a three- to five-year range of large time periods, requiring the approximation of intraperiod feedbacks as a simultaneous system. The objective of corporate models is not to foretell the future in minute detail, but rather to express the long-range implications of strategic plans over a range of hypothetical economic and competitive environments. In adhering to these objectives the larger time intervals of corporate models prohibits a purely recursive representation of a firm. The example shown in Illustration 1 demonstrates the classical use of simultaneous equations to approximate recursive feedback relationships trapped within the larger time periods of a corporate model.

This simultaneous approximation is an attempt to concisely express actual cause and effect relationships. The following sections are a departure from this attempt. These sections present three classes of simultaneous relationships that do not correspond to causal relationships.

SIMULTANEOUS DECISION RULES

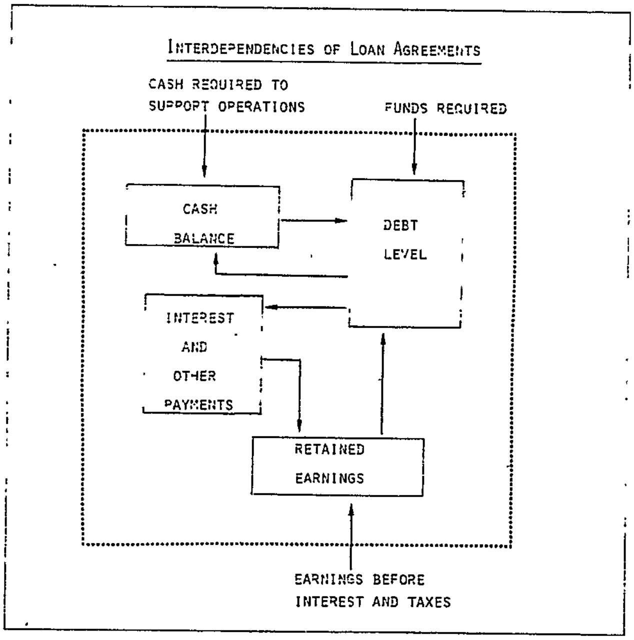
The behavior of a corporation is highly constrained by explicit and implied commitments. As an integral part of the environment of a firm, these constraints include agreements with external agents, industry conventions, stockholder expectations, loan covenants, and so forth. Realistically, an accurate forecast of the financial position of a firm cannot be prepared independently of these restrictions. Within the limitations of a corporate model, simultaneous decision rules are devices that ensure the compatibility of the simulated performance of a firm with these environmental constraints.

To realistically specify a decision process, decision rules and their effects must be jointly considered. Often these decision rules pose problems in identifying the ultimate response dictated by the decision rules of the model. The impact of each decision simultaneously alters the state of the projected firm upon which the decision was based, which consequently may evoke a different decision and so on. As a device, simultaneous decision rules provide a joint solution for the state of the firm, the actions prescribed by the decision rules, and the effects of these actions.

A common example of the use of simultaneous decision rules in corporate models is the problem of accounting for the impact of debt financing. As part of a corporate model, the decision rule that chooses the level of debt financing affects other corporate variables, which in turn determines the state of the corporation upon which the debt-financing decision is based. These feedback relationships are caused by the payment of interest and other fees that the lender requires as inducement to make the loan. Commercial loan covenants specify a package of direct and indirect charges consisting of a combination of interest charges, commitment fees, compensating balances, and inflationary hedges that together create an additional expenditure for which the firm must find more funds.

Illustration 2 demonstrates the mutual dependency between the decision rule specifying the use of debt financing and the resulting effects of a loan agreement.

ILLUSTRATION 2



The projected funds required in this example are presumed to be financed entirely with debt. This debt level is determined by the funds required (independent of the debt balance) and two components that are directly related to debt: funds invested in compensating balances and deductions from earnings.

Given a specific amount of required funds, the debt funding decision combines with the effects of the loan agreement to generate an endless chain of computations. For example, to determine the level of required financing, a firm includes in its calculations money that will be drained by "front-end" fees and cash committed to required balances, both of which are a function of the eventual debt balance. In addition, the level of financing required at the end of the period must be sufficient to cover interest expense accruing during the period. Compution of a solution is vastly simplified by using simultaneous equations to represent the interdependencies of debt financing as shown below.

$$(D_t - D_{t-1}) = XF_t + (C_t - C_{t-1}) - (RE_t - RE_{t-1}) \quad (4)$$

$$C_t = MAX(cD_t, XC_t) \quad (5)$$

$$I_t = iD_t + fMAX(D_t - D_{t-1}, 0) \quad (6)$$

$$(RE_t - RE_{t-1}) = (EBIT_t - I_t)(1-t) \quad (7)$$

Endogenous Variables

C_t = Cash balance at time t
D_t = Debt balance
I_t = Payments as per loan agreement
RE_t = Retained earnings balance

Exogenous Variables

XF_t = Funds required
XC_t = Cash balance required by operations
$EBIT_t$ = Earnings before interest and taxes
c = Compensating balance commitment
i = Interest expense
f = Commitment fee rate
t = Effective tax rate

Equation 4 determines the change in debt balance as a function of the funds required (XF_t), the change in cash balance ($C_t - C_{t-1}$), and the change in retained earnings ($RE_t - RE_{t-1}$). As specified in Equation 5, the corporate cash balance is equal to the greater of the cash requirements for operations or for compensating balance agreements. The change in retained earnings specified in Equation 7 is directly affected by the deductions from earnings summarized in Equation 6.

The simultaneous equation system 4 through 7 is a device that provides a joint solution for the four endogenous variables that satisfies both the debt financing rule and the loan agreements. It is unrealistic, however, for a firm to satisfy all of its funds requirements with long-term debt because shareholders insist on a more balanced capital structure. Therefore, a corporate model must be capable of specifying combinations of several financial sources. With the addition of new financing alternatives, the decision rules for anticipating the behavior of management become complicated. They are confounded by combinations of states, actions, and the presence of interdependencies between these states and actions. A more complete version of the previous example is presented below to illustrate the use of simultaneous decision rules to simplify this added complexity.

The following example includes a subset of decision rules found in the financial sectors of corporate models. The model first determines either a surplus or a deficit of funds and then employs the following hierarchy of decision rules.

Surplus Funds State

1. Retire short-term debt
2. Invest residual surplus funds in short-term securities

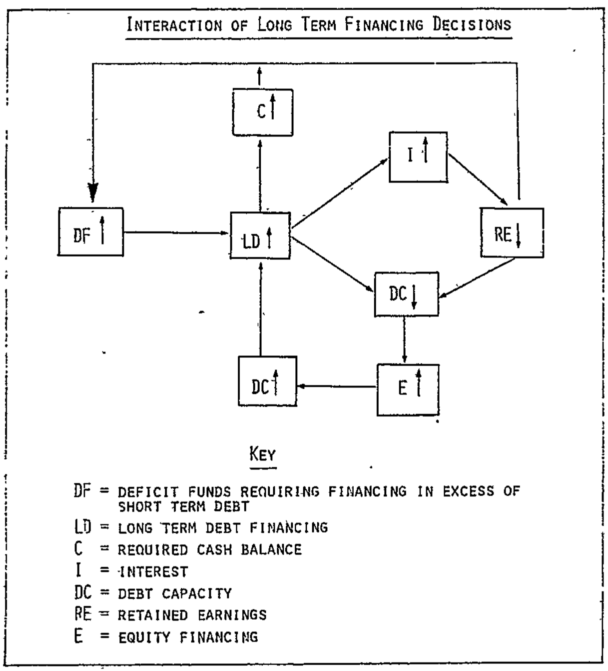
Deficit Funds State

1. Liquidate short-term securities
2. Increase short-term debt to maximum level
3. Increase long-term debt to maximum debt capacity
4. Fund residual deficit with common equity

Additionally, all assets and liabilities must be nonnegative; and long-term debt and equity levels are presumed to be nondecreasing.

Individually, each of these decision rules is simple. However, as part of an interdependent system, each decision leads to an endless sequence of effects, altered states, and, ultimately, a modification of the decision. For example, Illustration 3 demonstrates this interaction in funding a deficit in excess of allowable short-term debt. The deficit is also presumed to exceed the debt capacity of the firm, and therefore, the decision rules specify a mix of debt and equity financing.

ILLUSTRATION 3



This network of effects is an example of the many possible interactions dictated by the decision rule hierarchy. While representing the entire financing sector in a block diagram would be confusing, the total financing sector is completely represented by the simultaneous decision rules 8 through 15.

$$F_t = XF_t - (RE_t - RE_{t-1}) + (MC_t - C_{t-1}) \quad (8)$$

$$DF_t = \mathrm{MAX}(F_t, 0) \quad (8A)$$

$$MC_t = \mathrm{MAX}(XC_t, b(SD_t)) \quad (9)$$

$$C_t = \mathrm{MAX}(MC_t - F_t, MC_t) \quad (10)$$

$$SD_t = \mathrm{MIN}(DF_t, u) \quad (11)$$

$$LD_t - LD_{t-1} = \mathrm{MIN}(DF_t - SD_t, k(E_t + RE_t + SD_t + LD_t) - LD_{t-1}) \quad (12)$$

$$E_t - E_{t-1} = \mathrm{MAX}(DF_t - SD_t - (LD_t - LD_{t-1}), 0) \quad (13)$$

$$I_t = i_1(SD_t + LD_t) - i_2(C_t - MC_t) \quad (14)$$

$$RE_t - RE_{t-1} = (EBIT_t - I_t)(1-t) \quad (15)$$

Endogenous Variables

F_t = Effective surplus or deficit funds
DF_t = Deficit funds
MC_t = Required cash balance
C_t = Cash and short-term securities
SD_t = Short-term debt
LD_t = Long-term debt
E_t = Equity
I_t = Total income deductions
RE_t = Retained earnings

Exogenous Variables

XF_t = Funds required
XC_t = Cash required to support operations
$EBIT_t$ = Income before interest and taxes
b = Average compensating balance requirement
i_1 = Average interest rate on borrowed funds
i_2 = Interest rate on invested funds
t = Effective tax rate
u = Maximum allowable short-term debt
k = Debt to total capital constraint

Equation 8 defines the magnitude of the surplus or the deficit of funds as the difference between projected assets and liability levels for time t. (Auxiliary Equation 8A simplifies the definition of deficit funds in subsequent relationships.) Since Equation 8 computes the funds required net of short-term debt (short-term debt is assumed to mature at year-end), any surplus funds are added to the level of cash and short-term securities. Fund deficits are financed first by obtaining short-term debt in Equation 11 and then by a combination of long-term debt and equity. Equations 12 and 13 determine the mix of long-term debt and equity by the debt to total capital constraint that is controlled by the parameter k. As expressed by Equations 9, 14, and 15, the effects of these decision rules modify the state of the fund (Equation 8) and thus complete the interdependent cycle. The alternative to these simultaneous decision rules is guessing combinations of decisions that would satisfy the financial requirements of the firm. This would be a difficult task that would involve a lengthy and undesirable sequence of trials and errors.

The previous example demonstrates the use of simultaneous decision rules as a device to ensure the compatibility of the simulated performance of the firm with environmental constraints. This class of simultaneous relationships serves to constrain simulation results and does not correspond to actual cause and effect relationships. For instance, the interaction between long-term debt, debt capacity, and equity as shown in Illustration 3 is merely a device that maintains the simulated capital structure of the firm within a realistic tolerance. This notion of causal representation is completely absent from the classes of simultaneity presented below.

REVERSE SIMULTANEITY

Typically, the bulk of a corporate model is a straight sequence of nonsimultaneous relationships. Largely due to accounting convenience, this "top-down" flow of relationships starts with key input assumptions. For example, a sales forecast is a key input assumption that starts a sequence of relationships which cascades downward throughout the relationships in an income statement. In addition to simulating the results of these key inputs, corporate models are also used to determine the key inputs that are necessary to achieve given target results. For instance, a corporate model could be used to determine the level of sales that is necessary to achieve an earnings-per-share target. These "backward" simulations are generated by re-oriented corporate models that contain reverse simultaneous relationships, that is, relationships that solve for the cause which will produce a given effect.

In order to determine the key inputs necessary to achieve a target result, a corporate model must be reoriented in a "bottom-up" fashion. This process of reversing a model is accomplished by reversing the original identity relationships. These reversed identities now define the variable that was originally the key input in terms of a variable which was originally dependent upon the key input. For example, the following "top-down" sequence of equations defines gross profit (G_t) in terms of revenue (R_t) and cost of goods sold (C_t). Cost of goods sold is assumed to vary as some proportion k of revenue as shown in Equation 16, which combined with revenue determines gross profit as shown in identity 17.

$$C_t = kR_t \tag{16}$$

$$G_t = R_t - C_t \tag{17}$$

This straight sequence of relationships generates the results (gross profits) resulting from the key input (revenue). Alternatively, a reversed form of Equations 16 and 17 calculates the revenue necessary to achieve a specified target gross profit. This reversed form is created by reversing identity 17 as shown in Equation 18.

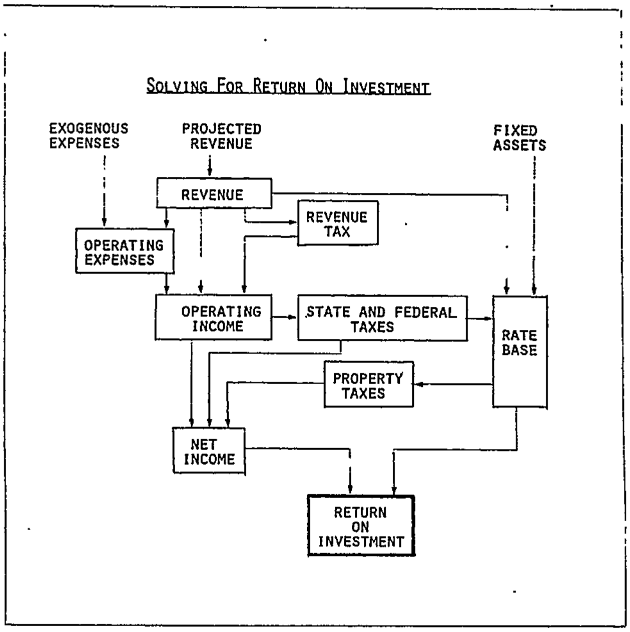
$$R_t = G_t + C_t \tag{18}$$

Equations 16 and 18 now solve for revenue in terms of a gross profit target. Note that Equations 16 and 18 are simultaneously related; revenue both determines cost of goods sold (Equation 16) and is defined by cost of goods sold (Equation 18). The simultaneous relationship is produced by the reversed identities that redefine a variable in terms of components which are dependent upon the variable. This simultaneity, however, does not describe a causal relationship, but rather a reverse-causal relationship; contrary to the actual cause and effect relationship, revenue is now dependent upon cost of goods sold and gross profit. This reverse simultaneity provides a means of solving for key inputs necessary to achieve a target result, while preserving the original assumptions of the model.

A more complete example of reverse simultaneity is shown below. The example illustrates the use of a reversed corporate model of a utility that solves for the level of revenue (a key input) which is implied by a return on investment fixed by a regulatory commission (a given target result). The original model consists of a "top-down" sequence of accounting identities, regulatory rules, and behavioral relationships. In solving for revenue implied by return on investment, however, the objective of the model flows "bottom-up". Reverse simultaneity results from this flow of objects contrary to the flow of relationships in the original model.

The downward flowing dependencies in the original model are demonstrated in Illustration 4 and in Equations 19 through 26.

ILLUSTRATION 4



The sequence of dependent relationships begins with projected gross revenue for a given year (referred to as the "test year")[2], which determines revenue taxes in Equation 19 and operating expenses in Equation 20[3] and which defines operating income in identity 21. Net income is defined in Equation 24 as the excess of operating income over income and property taxes. Referred to as the rate base, the value of the net investment of the firm as expressed in Equation 25 is dependent upon Federal taxes[4] and gross revenue.[5] Finally, net income and the value of the rate base combine to yield return on investment in Equation 26.

$$RT = t_1 R \tag{19}$$

$$OE = XOE + eR \tag{20}$$

$$OI = R - OE - RT \tag{21}$$

$$PT = t_2 XRB \tag{22}$$

$$FT = t_3 OI \tag{23}$$

$$NI = OI - PT - FT \tag{24}$$

$$RB = XRB - r_1 FT + r_2 R \tag{25}$$

$$ROI = \frac{NI}{RB} \tag{26}$$

Endogenous Variables

RT = Revenue tax

OE = Operating expenses

OI = Operating income

PT = Property tax

FT = Federal and State tax

NI = Net income

RB = Rate base

ROI = Return on rate base

[2]For convenience, the time subscripts have been omitted.

[3]Operating expenses such as collections are dependent upon gross revenue.

[4]The rate base is adjusted for the Federal loan implied by tax accruals.

[5]Working capital such as receivables are dependent upon gross revenue.

Exogenous Variables

R = Gross revenue

XOE = Operating expenses unrelated to revenue

e = Relationship of operating expenses to revenue

t_1 = Revenue tax rate

t_2 = Property tax rate

t_3 = Effective Federal and State tax rate

XRB = Exogenous rate base components (e.g., fixed plant)

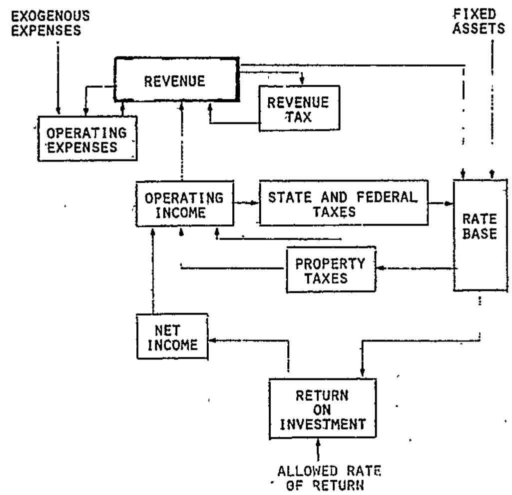
r_1 = Adjustment for accrued taxes

r_2 = Relationship of working capital to revenue

The corporate model above is incomplete. Though the flow of dependencies is clearly a "top-down" sequence, the objective of the analysis is to solve for revenue as opposed to determining a return on investment resulting from a revenue estimate. Illustration 5 is a block diagram of the reversed version of the original corporate model, which now solves for revenue.

ILLUSTRATION 5

SOLVING FOR REVENUE



The presence of reverse simultaneity is clear in Illustration 5. Observe the circular flow of relationships resulting from the "top-down" flow of relationships from the original model (Illustration 4) and the "bottom-up" flow of the objective in the reversed model.

The following equations represent the reversed version of the original Equations 19 through 26.

$$RT = t_1 R \tag{19}$$

$$OE = XOE + eR \tag{20}$$

$$R = OI + RT + OE \quad (27)$$

$$PT = t_2 XRB \quad (22)$$

$$FT = t_3 OI \quad (23)$$

$$OI = NI + PT + FT \quad (28)$$

$$RB = XRB - r_1 FT + r_2 R \quad (25)$$

$$NI = RB * ROI \quad (29)$$

<u>New Endogenous Variable</u>

R

<u>New Exogenous Variable</u>

ROI

Note that the only changes to the original system of equations are the reversed identities 27, 28, and 29; the equations 19, 20, 22, 23, and 25 remain unchanged.

The reversed identities 27, 28, and 29 specify simultaneous relationships by defining a variable in terms of components that are dependent upon the variable. For example, Equation 27 defines revenue (R) in terms of revenue tax (RT) and operating expense (OE), both of which are dependent upon revenue itself. In addition, these simultaneous relationships are reverse-causal. For instance, Equation 27 does not correspond to the actual causal relationships of the utility, as revenue is not determined by expense items. These simultaneous relationships result from relationships that are exactly opposite to the actual cause and effect dependency of the utility.

Despite this lack of correspondence to the actual relationships of a firm, reverse simultaneity is useful for solving for the key inputs necessary to achieve target results.

<u>SPURIOUS SIMULTANEITY</u>

Corporate models often contain spurious simultaneous relationships that do not correspond to causal behavior; they, in fact, result from a distortion of the true cause and effect relationships of a system. These false interdependencies occur when a flow, determined as a function of a level, also determines this level. For instance, Equation 30 determines a flow (F_t) during period t as a function of a level (L_t). The level (L_t) is then determined by the flow (F_t) in Equation 31.

$$F_t = f(L_t, \ldots) \quad (30)$$

$$L_t = f(F_t, \ldots) \quad (31)$$

The intended process underlying spurious simultaneity is one of assignment; the flow during period t is computed and then assigned in the equation describing the level at the end of period t. To imply that a flow during an interval of time is dependent upon the level it determines at the end of the time period is a misrepresentation of time in a model, which causes spurious simultaneity.

Spurious simultaneity is eliminated by removing the dependency of the flow on the level it determines. This dependency is removed either by changing the time notation of the level to represent the beginning of the period (as shown in Equation 32) or by excluding the level altogether from the relationship describing the flow.

$$F_t = f(L_{t-1}, \ldots) \quad (32)$$

The interpretation of the flow determines the correct alternative. If the level is determined by the flow and the previous value of the level, then the flow should be dependent upon the previous level. Alternatively, if the level is dependent upon the flow alone, and not the flow plus the previous value of the level, then the level should be excluded from the relationship describing the flow.

An example of spurious simultaneity is shown below in Equations 33 and 34. Invested cash (C_t) is presumed to grow with interest earned during period t (I_t), assuming an annually compounded interest rate k.

$$I_t = kC_t \quad (33)$$

$$C_t = C_{t-1} + I_t \quad (34)$$

Equations 33 and 34 are simultaneously related. However, this simultaneous relationship is not equivalent to the "intended" growth in the cash balance. Equation 35 is the reduced form of Equations 33 and 34. When compared to the "intended" relationship shown in Equation 36, the spurious simultaneity problem in Equations 33 and 34 becomes clear as Equations 35 and 36 are not equivalent.

$$C_t = \frac{1}{(1-k)} C_{t-1} \quad (35)$$

$$C_t = (1 + k)C_{t-1} \quad (36)$$

The problem is found in Equation 33 where the flow of interest earnings during period t is dependent upon the level of cash at the end of period t. The period-end level of cash is determined by adding interest earnings during period t to the cash level at the beginning of period t, which modifies the flow of interest earnings in Equation 33 and so on. Since the cash level is determined by the flow of interest earnings and the previous cash level (Equation 33), the spurious simultaneity is eliminated by changing the time notation for the level of cash in Equation 33 to represent the level at the beginning of the time period t. Equation 37 correctly specifies the flow of interest earnings during period t. Note that the reduced form of Equations 37 and 34 restores the intended expression for the level of cash shown in Equation 36.

$$I_t = kC_{t-1} \quad (37)$$

The complexity of corporate decision rules is the primary cause of spurious simultaneity. When faced with this complexity, the model designer often specifies each relationship independently. When the perspective of the model as a total system is lost, spurious interdependencies are overlooked. Since the financing sector of a corporate model consists of the most complex relationships, it is the most probable candidate for spurious simultaneity. In accordance with this popularity, the spurious simultaneity problem in the financing sector is given special treatment below.

The spurious simultaneity problem in the financing sector originates with the specification of funds required. When the funds requirement for a period (a flow) is dependent on a level that is assigned all or a portion of the funds required, the model will contain spurious simultaneity. This problem is illustrated in Equations 38, 39, and 40, where the funds required during period t is computed in Equation 38 as a function of period-end asset and liability levels. This funds requirement is assumed, then, to be financed with debt as shown in Equation 40. Note, however, that the flow of funds required is dependent upon the period-end level of debt because of the definition of the liability level in Equation 39.

$$F_t = A_t - L_t \tag{38}$$

$$L_t = XL_t + D_t \tag{39}$$

$$D_t = F_t + D_{t-1} \tag{40}$$

Endogenous Variables

F_t = Required funds

L_t = Projected liabilities

D_t = Projected debt balance

Exogenous Variables

A_t = Asset levels

XL_t = Other liability levels

The equation system 38, 39, and 40 contains a spurious simultaneous relationship between the flow of funds required (F_t) and the period-end level of debt (D_t). Solving for the period-end level of debt in the reduced form of this equation system is shown in Equation 41.

$$D_t = \frac{A_t - XL_t + D_{t-1}}{2} \tag{41}$$

When contrasted with the intended reduced form shown in Equation 42, the spurious simultaneous relationship between Equations 38, 39, and 40 becomes clear.[5]

$$D_t = A_t - XL_t \tag{42}$$

The intended specification of this financing sector is restored by replacing the period-end debt level in Equation 39 with the beginning period debt level as shown in Equation 43.

$$L_t = XL_t + D_{t-1} \tag{43}$$

Avoiding spurious simultaneity is difficult in financing sectors more complex than the previous example. Recall the expression for required funds (F_t) in Equation 8.

$$F_t = XF_t - (RE_t - RE_{t-1}) + (MC_t - C_{t-1}) \tag{8}$$

Equation 8 is properly specified because the flow F_t is not dependent upon levels that are determined by F_t.[6]

A misspecified version of Equation 8 is shown in Equation 44.

$$F_t = XF_t - (RE_t - RE_{t-1}) + (C_t - C_{t-1}) - (E_t - E_{t-1}) - (LD_t - LD_{t-1}) - (SD_t - SD_{t-1}) \tag{44}$$

Observe that in Equation 44 the flow of funds required (F_t) is dependent upon the level of cash (C_t), equity (E_t), long-term debt (LD_t), and short-term debt (SD_t). Since each of these levels is dependent upon F_t, a spurious simultaneous relationship exists

[5] The factor "2" in the denominator of Equation 41 is found in the reduced form of a spurious simultaneous equation system when the forms of the flow and level equations are:

$F_t = f(cL_t, \ldots)$

$L_t = f(-cF_t, \ldots)$

where c equals either 1 or -1.

[6] There is a subtle difference between the spurious simultaneity described above and the dependency of the flow of interest expense on the level of debt (Equation 13). In the latter case, the debt balance is assumed to be changing during the period as a function of the changing requirement for financing. The flow of interest expense is dependent on an average level of debt during the period.

between Equation 44 and each of the Equations 10, 11, 12, and 13.

Often spurious simultaneity deceptively appears to be a logical representation of causality. Through a misrepresentation of time, spurious simultaneity specifies the dependency of a flow on a level that is determined by the flow. What is intended to be an assignment of the flow to the level becomes a simultaneous relationship that distorts the actual causal behavior of the system.

CONCLUSION

Models of social and economic systems employ simultaneous equations to express feedback relationships captured within the concise relationships and large time intervals. Corporate models also contain three classes of simultaneous relationships that do not correspond to the actual cause and effect relationships of the firm. This paper identifies these three classes as simultaneous decision rules, reverse simultaneity, and spurious simultaneity. In the first two classes, simultaneous equations are employed as a balancing device. Simultaneous decision rules provide a balance between the simulated performance of a firm and an anticipation of environmental constraints. Reverse simultaneity balances the process of solving for the key inputs necessary to achieve target results, while preserving the original assumptions of the corporate model. The third class of noncausal simultaneity, spurious simultaneity, results from a distortion of time in a level and flow relationship.

Simultaneous decision rules and reverse simultaneity are simple devices for generating answers to questions that are otherwise complex. Conversely, spurious simultaneity is an elusive pitfall that destroys the intended relationships in corporate models.

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