ENGINEERING MODELING
(WHERE PRACTICE HAS TO EQUATE WITH THEORY)

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Abstract

Engineering mathematics should be more concerned with generating a meaningful model than with equation manipulation. Toward this end, various forms and features of modeling are discussed. Those emphasized fall into a two-by-two matrix of categories: Glass Boxes/Black Boxes and Cloak/Mold. In Black Boxes, the response to certain stimuli are known, but the elements and their interconnections are hidden. In Glass Boxes, the elements and interconnections are visible (the elements are Black Box-described). The Cloak form forces the model to take on the relevant features of the system being modeled, whereas the Mold form forces the model to take on the features of the analysis method being applied.

When the problem being modeled is a network, then Block Diagrams, Flow Charts, and Schematics often become the language of model description. These models serve for parameter optimization, performance analysis, property diagnosis, parameter sensitivity analysis, and instrumentation.

Further, some common fallacies concerning mathematical models are illuminated.

Engineering Design

The engineer's ideal output should be a useful product or service without exhaustive deduction (tracing out all possible alternatives in detail). It should:

1. Produce an optimum design (efficient, versatile, accurate, feasible, and economical).
2. Produce the result quickly at low cost.
3. Produce physically realizable designs.
4. Be applicable to a large variety of design tasks (variations on the theme).
5. Use available tools and methods.
6. Be amenable to variation in "Utility" (benefit/cost values).

Engineering Design is the synthesis of optimum means for fulfillment of needs in the face of limited resources. It is a "People Process," typified by iterative decision making. An important part of the design is the set of available alternatives and choice mechanisms encountered as the design evolves. In addition, a worthwhileness function which equates relative cost values of utilities and relative merit of the performance parameters is needed. In producing an optimum design from an idea, a model can become a valuable tool. A flow chart model of such a Design synthesis is shown in Figure 1.

Most optimization procedures are similar in that system design is trial-and-error manipulation of pieceparts, a sequence of engineering guesses and evaluations. First, a model is characterized in an effort to get to a feasible solution or within easy walking distance of same. Second, the performance and cost of the model are evaluated. Third, the models structure and parameters are walked to a more nearly optimum set. Fourth, steps two and three are iterated until the models cost/performance are close enough to the optimum to mitigate against the cost of redesign. Two models of an analytic optimization are presented in Figures 2 & 3.

Model Building

To model is to translate from a physical context to the language of the model. Models can be formed at any level of abstraction: from reality through three-dimensional working mockups and isometric or perspective drawings; and charts, diagrams, schematics and equations. This was stated very well by R. H. McKim (see Figure 4).

Models come in many forms (i.e., Verbal Symbolic, Logical, Mathematical, Analogy, Stochastic, Computer, Interactive, etc.). They are a hypothetical simplification of the real system. They are further assumed to retain the relevant characteristic behavior of the system being modeled. The simplification gained in the modeling is both a blessing and a damnation. As a blessing, it reduces complexity to a point which is amenable to understanding and computation; as a damnation, it masks the subtle interactions and characteristic anomalous behavior such that the model predictions can be more believable than real world experience.

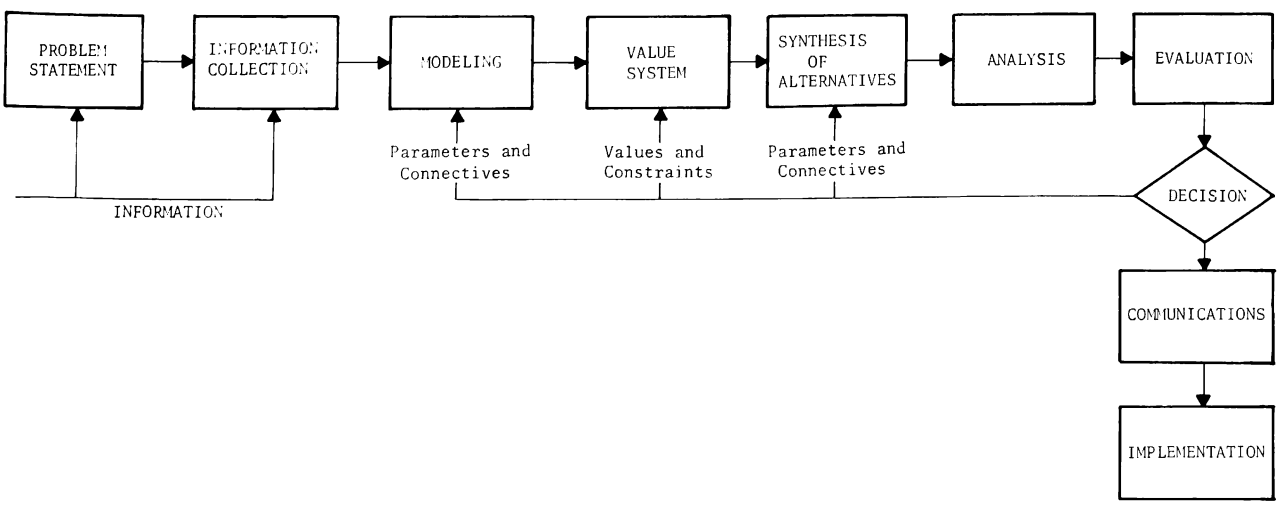


Figure 1 - A Design Process Flow Chart

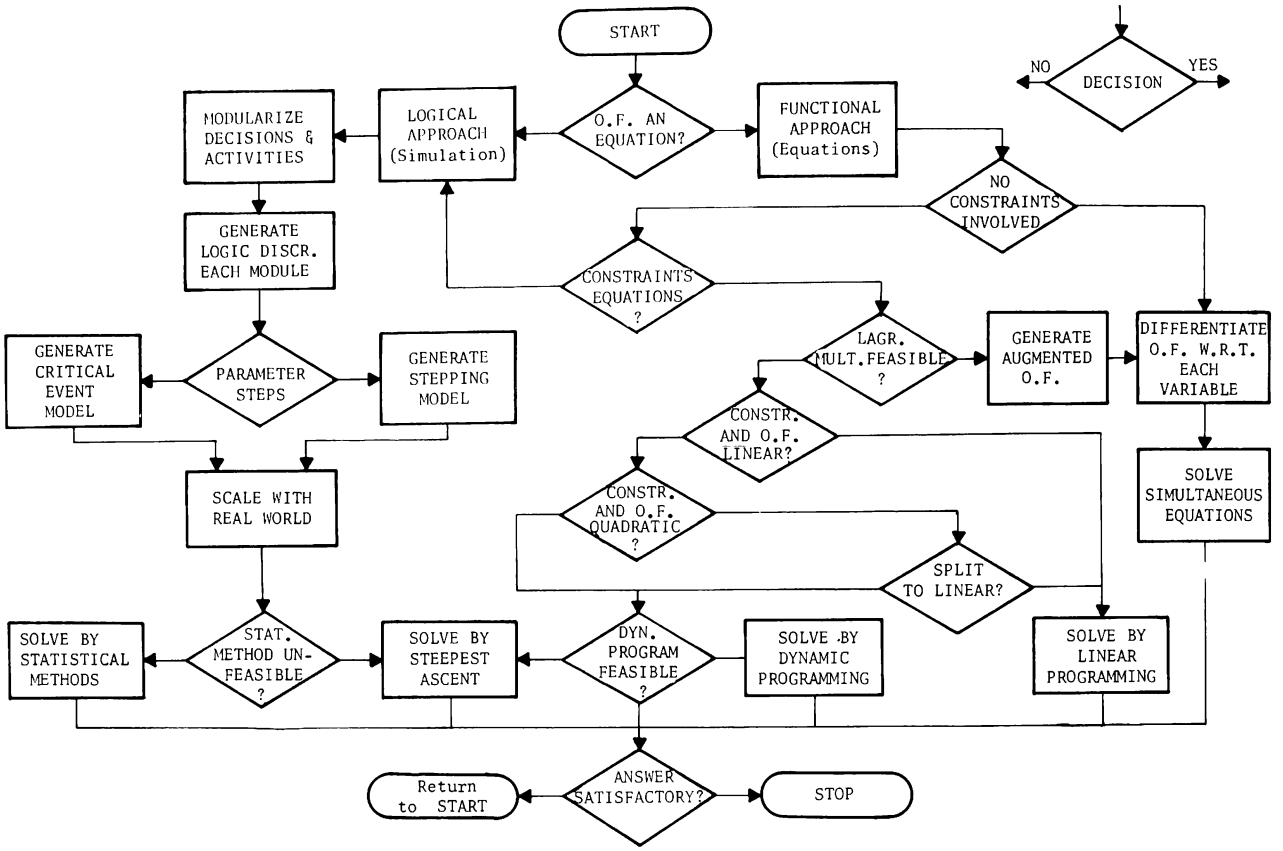


Figure 2 - Flow chart of analytic optimization

Table 1. Decision Table Guide to Optimization

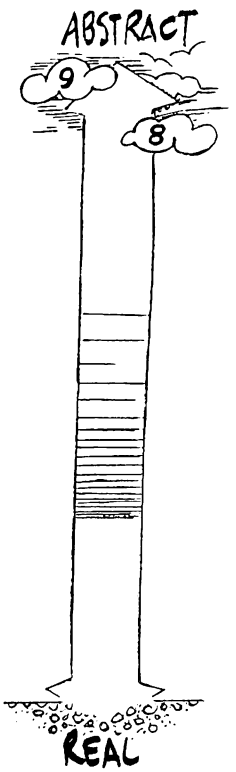
Are O.F. equations	Y	Y	Y	Y	Y	Y	Y	N						
Are constraints invalid	Y	Y	Y	Y	Y	Y	Y							
Are the constraints equations	Y	Y	Y	Y	Y	Y	N							
Are Lograngion Mult feasible	Y	N	N	N	N	N								
Are constraints & O.F. linear		Y	N	N	N	N								
Are constraints & O.F. quadratic			Y	Y	Y	N								
Will problem split to linear			Y	N	N									
Is dynamic progr feasible				Y	N	Y	N						Y	N
Will parameters vary each time step									Y	N				
Are statistical methods feasible											Y	N	N	
Write augmented O.F. equations	X													
Differentiate O.F. wrt each variable	X													
Solve the simultaneous equations	X													
Solve by linear programming		X	X											
Solve by dynamic programming				X		X							X	
Solve by steepest ascent					X		X							X
Take the simulation approach								X	X					
Modularize decisions & activities								X	X					
Develop logic description for ea module								X	X					
Solve by statistical methods											X			
Build stepping model										X				
Build critical events model											X			
Scale with real world										X	X			
Return to this table next column								X	X	X	X			
Go to Table 2	X	X	X	X	X	X	X					X	X	X

Table 2.

Is the answer satisfactory	Y	N
Enter Table 1		X
Stop	X	

Figure 3. Decision Table for Analytic Optimization

ABSTRACT VISUAL ANALOGS	Abstract visual analogs make abstract ideas visible (e.g., "yin-yang" which symbolizes unity of opposites; overlapping circles which symbolize overlapping concepts).
CHARTS, DIAGRAMS, AND SCHEMATICS	Charts make quantitative relationships visible (e.g., pie chart). Diagrams and schematics also make basic functional and spatial relationships visible (e.g., circuit diagram; schematic which shows force applied to a cantilever beam).
ORTHOGRAPHIC PROJECTION	Orthographic projection makes functional relationships visible and spatial relationships both visible and measurable. (Orthographics are "real" in the sense that they portray "true shape," but are abstract in that they utilize line to portray shape and require mental coordination of multiple views.)
ISOMETRIC, OBLIQUE, AND PERSPECTIVE PROJECTION	Isometric and oblique projection make spatial configurations visible in a simple image, although optically distorted. Perspective drawings make configurations visible, as a camera would portray them. (2D illusions of 3D appear more real, of course, when shape is portrayed with color and shading rather than with line.)
THREE DIMENSIONAL MOCKUPS AND MODELS	Mockups make a facet of the real thing available to all sensory modes in 3D (e.g., a working mockup, sans appearance, or an appearance mockup, sans function). Models are usually more explicit and complete than mockups (e.g., a working, appearance model).
THE REAL THING	



R. H. McKim Figure 4. Span and Scope of Models

Model Synthesis creates a structure of elements to give a desired system behavior. The elements can be electrical, mechanical, thermal, fluidic, or "what have you." The specific elemental behavior is often of the designer's free choice. This is due to the wide choice of parameters available and the ability to convert easily from one form of energy to another.

It is often said that when one has a clear statement of the problem, he has over 90 percent of the solution. If the problem statement is in the form of a model containing just sufficient detail to insure proper performance, then it is certainty 90% solved. Engineering synthesis problem statements take the form of more than just a model of the world involved. They also require the stimuli, the environment, and the constraints.

Model Language

Engineers use drawings more than they use prose or mathematics to capture and transmit concepts, ideas, requirements, and instructions. Models are very often formed as networks. Block Diagrams, Flow Charts, Bond Graphs, and Schematics seem to be the language of network design. All of these diagramatic representations of network models use transfer functions and interconnections as their only method of denoting character. An engineer's proficiency in these languages determines to a large degree his technological worth (Note Bene to be kept distinct from his salary in this Macro-political, Micro Technological world). This proficiency is in the reading, writing, and understanding of the models.

Black Boxes and Glass Boxes

Models come in many forms, two of which are of particular interest: the "Black Box" and the "Glass Box." The black box is the most usual form of model and is the only one used to describe atomic elements in circuits. This is true whether the circuit is a schematic or a block diagram. The Black Box is behavioristically defined (output described in terms of input, environment, and output). The Glass Box, on the other hand, has all of its internal elements (these are Black Box-defined) and their interconnections in full view. It commonly goes under the name of a circuit or network.

Black Boxes and Glass Boxes form the models each of which are particularly appropriate for certain uses. A few of the uses of models are in Analysis, Design, Diagnosis, Instrumentation, and Synthesis.

Glass Box Analysis is the determination of the inputs which cause the perceived outputs. Each has its element's behavior and connectivity known.

Black Box Design is Black Box Synthesis to an optimum Glass Box. The costs and benefits which are optimized are prescribed.

Glass Box Diagnosis is the determination of the changes in elemental behavior which can cause the observed change in Black Box behavior. This is a deviation from nominal behavior. This results in a list of changes in probability of occurrence order, sometimes with only one entry. It is used for malfunction study and is the inverse of sensitivity analysis.

Glass Box Instrumentation is the determination of the inputs which cause the perceived outputs. Each has its element's behavior and connectivity known.

Black Box Synthesis is the determination of the Glass Box characteristics which will produce the Black Box behavior prescribed. This includes generation of behavior and connectivity of the elements.

Cloaks and Molds

Two general methods of model building (Cloak and Mold) are in broad use. Constant awareness of the method must be maintained by the engineer. The Cloak conforms to the contours and characteristics of the system, and the Mold makes the contents take on the characteristics of the model.

In macroeconomic and other very complex systems there has been a growing trend in the use of the Mold method especially making the model fit a given form of Game theory or a given linear programming technique. There is nothing wrong with molding the problem into conformity with your pet method as long as you do not over represent your solution's meaningfulness in the real world.

Point of View and Objective

In order to give meaning, it is necessary to give point of view and objective. The point of view is that of an engineering (problem solving, goal directed designing with near optimal utilization), and the objective is most often results (practical physical embodyment) though, at times, the objective is academic (understanding).

In this view, before the system comes the problem and before the problem comes the situation and so an attempt to give meaning will start here.

SITUATION: In modern socio-economic system engineering, it often is one look (a snapshot) at the totality of the universe including all known and/or unknown parts, all known and/or interrelations. In less vast tasks, its a snapshot of a small subspace of the universe still containing a multitude of known parts and interrelations, but remember, the unknowns may be significant.

PROBLEM: A subset of the situation with a question attached (i.e., how can the interrelations be controlled to affect the desired results?)

SYSTEM: A subset of the problem including the known parts believed to be significant and the known interrelations believed to be significant. The system under consideration is not a fixed set for as the solution to the problem progresses, the significance of parts and relations change as does the knowledge of many of the previously poorly known portions.

SUBSYSTEM: A subset of the system partitioned out for specific attention. Commonly one in which there is more interrelation ("effect-on" between elements) than interrelations ("effect-on" between the subsystem and the rest of the system).

ELEMENT: The smallest part of the system. This means a part which does not have to be subdivided to get at its "effect-on" the relevant characteristics of the system. Some elements are depicted in Figure 5 for various systems.

A MODEL: The model is intended to portray all relevant interrelations of the system, subsystem, part or element being modeled. There is a known or hoped-for transformation equivalence between the relevant interrelations in the model and the "system" being modeled. There is no implied need for the model being substantive. It can be described in prose or mathematics as well as in metal, information or energy, etc.

Type of Element	Transvariable Source	Pervariable Source	Dissipator	Perstore Element	Transtore Element
Functional Relationship	$E=E(t)$	$I=I(t)$	$E=D\times I$	$E=\int \frac{I}{P}dt$	$I=\int \frac{E}{K}dt$
Rectilinear Mechanical System	Linear Velocity $V=V(t)$	Force $F=F(t)$	Friction $V=D\times F$	Linear Inertia $V=\int \frac{F}{M}dt$	Linear Compliance $F=\int \frac{V}{K}dt$
Rotational Mechanical System	Rotational Velocity $\omega=\omega(t)$	Torque $\tau=\tau(t)$	Friction $\omega=D\times\tau$	Rotary Inertia $\omega=\int \frac{\tau}{J}dt$	Rotary Compliance $\tau=\int \frac{\omega}{K}dt$
Electrical System	Voltage $E=E(t)$	Current $I=I(t)$	Resistance $E=D\times I$	Capacitance $E=\int \frac{I}{C}dt$	Inductance $I=\int \frac{E}{L}dt$
Fluid (Eulerion) System	Pressure $P=P(t)$	Volume Flow $\dot{V}=\dot{V}(t)$	Friction $P=D\times\dot{V}$	Acoustic Mass $P=\int \frac{\dot{V}}{M}dt$	Acoustic Compliance $\dot{V}=\int \frac{P}{K}dt$
Thermal System	Temperature $T=T(t)$	Heat Rate $\dot{Q}=\dot{Q}(t)$	Resistance $T=D\times\dot{Q}$	Thermal Capacitance $T=\int \frac{\dot{Q}}{C}dt$	
Magnetic System	Magneto-Motive Force $F=F(t)$	Flux Rate $\dot{\phi}=\dot{\phi}(t)$	Hysteresis $F=D\times\dot{\phi}$	Permeance $F=\int \frac{\dot{\phi}}{\Gamma}dt$	

Figure 5. Two Terminal Elements

A DETERMINISTIC MODEL: A model in which the relevant interrelations are stated uniquely with certainty.

A STOCHASTIC MODEL: A model in which the relevant interrelationships are stated at least in part as random variables whose action is a stochastic (time varying statistic) process (step-by-step). In a dynamic system for an operational model, the time sequence of the random variables must be tractable in some governing sense is germain to the relevant interrelationship.

FLOW DIAGRAM: Flow diagrams are used in describing computer programs. These are similar, but not the same, as block diagrams because action takes place at only a few places at a time in a flow diagram (usually at only one place at a time). The particular path taken depends upon the results of previous action. It's a geometric portrayal of a decision table, and it is often more fluent when loops are involved. An atypical flow diagram is shown in Figure 6.

BLOCK DIAGRAM: A block diagram is used to set emphasis in a transfer system. For the "snapshop" the diagram gives, it makes each block an "element" with the transfer function as the only relevant property of the block, and then the diagram shows the blocks' place in the community of blocks, with the block-to-block interrelations depicted by the line geometric connectives.

These diagrams can be drawn in a multitude of ways for the same system. These ways show differences in local detail and differences in partition placement. Figure 7 shows a simple system at various levels of complexity. Extreme care must be taken when defining a block's transfer function, as all loading effects need to be accounted for.

These diagrams are very useful in macro-systems, since they allow viewing of the problem with a small span of complexity. Namely, the whole works can be viewed by showing how the systems, or super-systems, relate to one another to make up the macro-system, or they can show how a subsystem operates with respect to its elements. The span of detail can often be limited in a block diagram representation without confusing the issue.

I advocate block diagrams only for macro-systems and for indoctrination lectures as they let many people believe they "understand," when, in truth, they do not even "suspect."

DIRECTED GRAPHS: Network diagrams are directed graphs with nodes and links. Either, or both the nodes and the paths, may have characteristics associated with them, but it is normal to have weights associated with the links (the usual path is the shortest summation weight path from one node to another). In some use of network diagrams, a complex mix of traffic must be determined which does not overload any path, and the problem is to determine the traffic mix.

It is reasonable to depict problems or portions thereof with two or three different diagrams (models), each emphasizing a different point of view. Several different representations of a single concept are shown in Figure 8. These models may be totally separate or intercoupled. If intercoupled, then the models form a whole model using a variety of forms for expression.

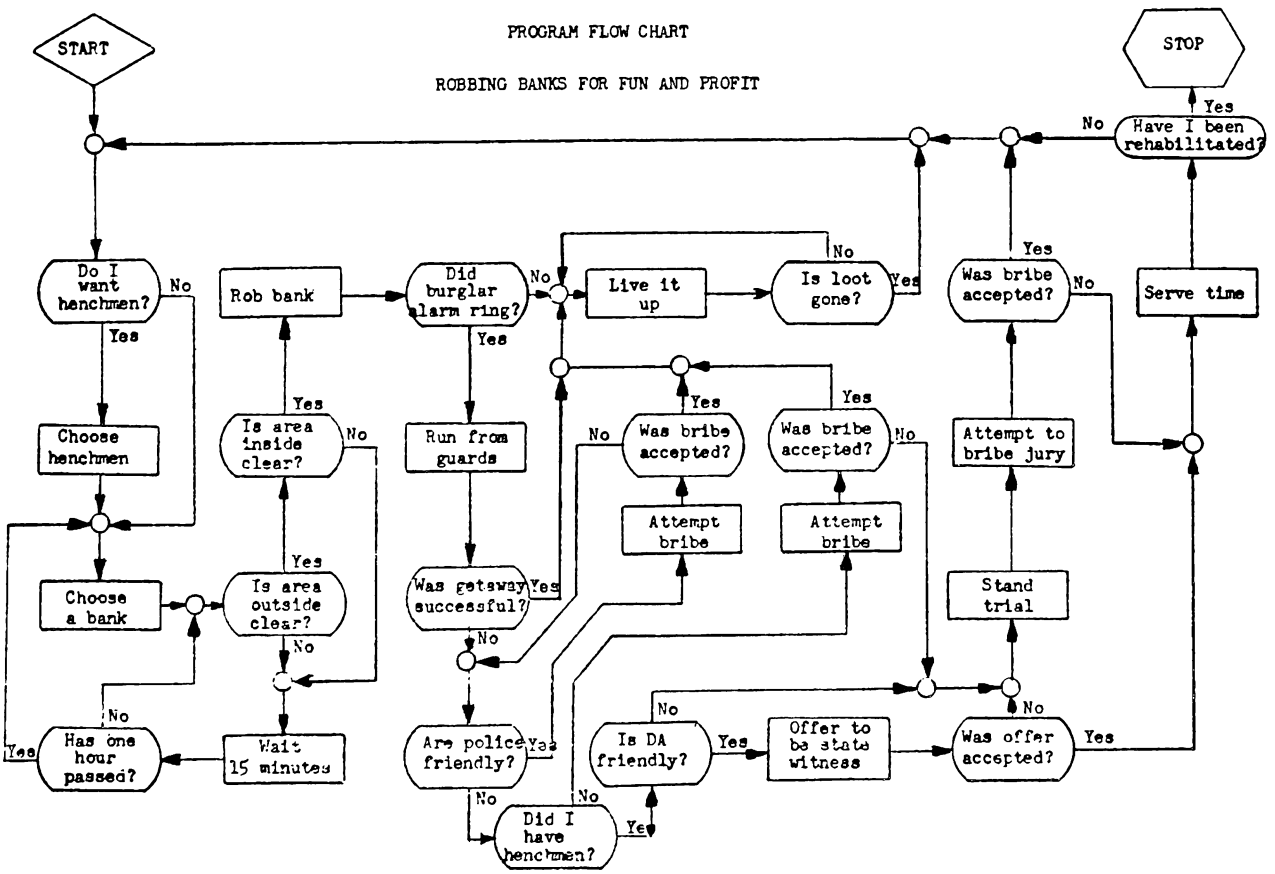


Figure 6. Program Flow Chart of Robbing Banks for Fun and Profit

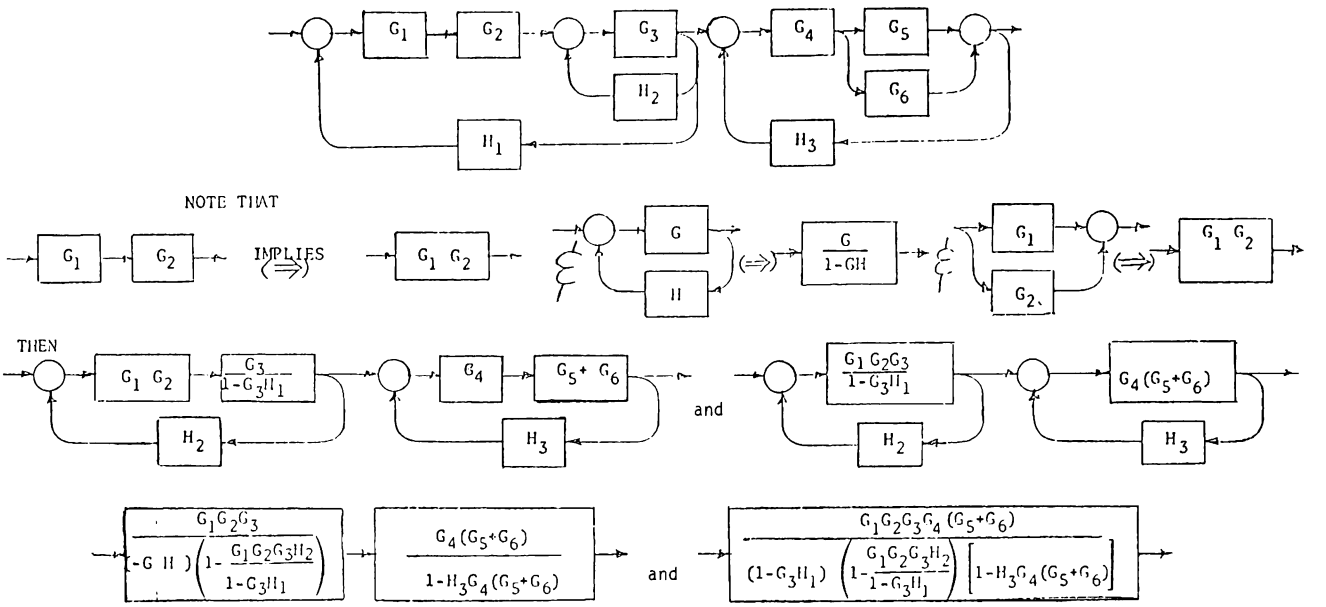


Figure 7. A Block Diagram Transformation

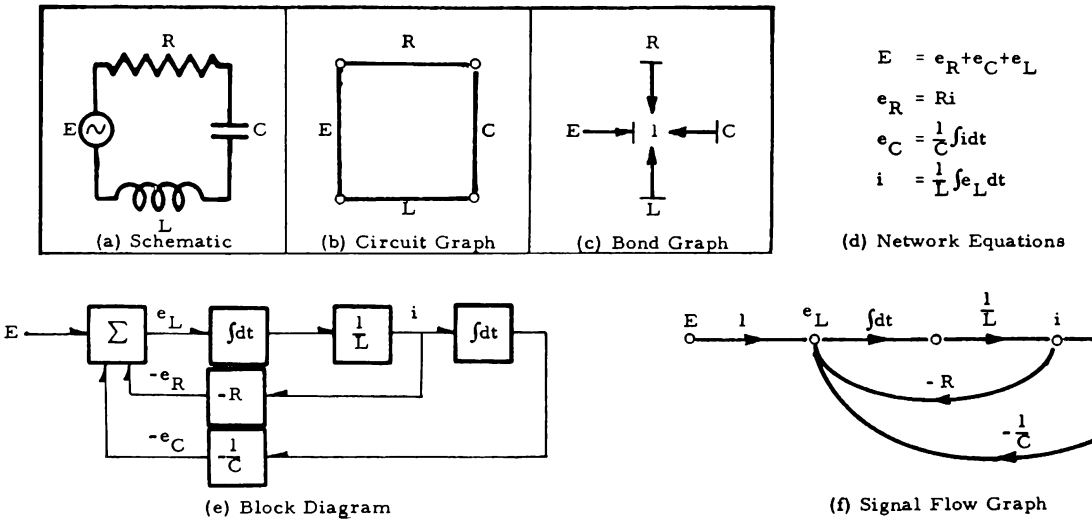


Figure 8. Comparison of Graphic Notations

MATH MODELS

The physics of the problem and the feedback control theory, etc. always seem to define the world as a set of inter-coupled set of constant coefficient second order differential equations. No physically realizable system can ever be described with infinitessimal inaccuracy by such a set of characteristic equations. However, many, if not most, systems can be so described over a limited range (small signal condition) with a reasonable degree of accuracy (a carefully measured degree of vagueness). These differential equations are often represented by D operators, by Laplace transforms, by Fourier transforms, and by matrices of state variables. Such models are extremely valuable, but the equations are interrelational, not causal. To an engineer, the cause effect path is of extreme importance and must be kept prominent in the model.

Some fallacies concerning Mathematical Models are:

1. Mathematics is the dominant aspect of problem solving.
2. Quantitative techniques yield real solutions.
3. Proven techniques are available for all problems.
4. Mathematics has mystic powers (mathemagic).
5. Clients must become mathematically sophisticated.
6. Publications describe solutions to real-life problems.
7. Nonmathematical techniques are necessarily incomprehensible and show a low level of technical expertise.

GAME THEORY

Tactics and strategy are needed in macro-system design whenever there is an opponent. This occurs most often with competition such as between your product and the other similar products in the market place. If the system is expected to have a large impact upon the situation, then Game Theory must be used to gain a meaningful model of the market. Nearly all market surveys fail to take into account the feedback to be brought into play by the competition and the feedforward created by new markets created by the presence of the system in the market place.

PREPARE FOR CHANGE

If a system (hardware and people) is to be designed with a flexible mission during the development phase, then the planning must allow for parametric variation in those transfer functions, boundary values, and objective functions which are considered to have a meaningful probability of changing. The range of change should also be considered. It is very important that expected system performance be communicated to those who may change the mission (the customer), irrespective of whether this performance with respect to the various missions be healthy, sick, or glorious. Simulation is a common technique for bounding the performance dependency upon parameters and connections.

If some variation in mission objective is highly probable after the delivery of the hardware and the training of the people, then general purpose hardware and general purpose staffing is necessary. The general purpose hardware can often be implemented on programmable equipment, such as digital computers, with only the specific terminals being physically rebuilt. The general purpose staff must be educated (not just trained) in the equipment. They must know why (not just how). They must appreciate the equipment (not just recognize it). There must be tightened communication between administrators, designers, and customers. Further budget must be provided for flexible systems and for the additional understanding that must go into them.

SCALING

In scaling a model up (or down) in "size," a great deal of care must be taken that important scale relationships are not disturbed [e.g., A plastic model of a "system" under stress is often used due to the fringe pattern visibility but if length only is the scale being transformed, then dead loading (those due to the structure's own weight) would be in error because the volume would scale by the cube and, at the same density, this gives an error in dead loading]. If the scale is down, it's possible to choose the lineal scale such that the density ratio (plastic model to metal system) compensates. Much care in depicting proper scale in all the things of interest and importance must be exercised, or that which was forgotten will spoil the engineering. It's often necessary to study the system through a set of models, not just one (due to a conflict in scaling). In the plastic stress/strain model, it should be noted that even in the elastic range, it takes a well planned set of scales to keep the stress vs. strain scaled well (Young's modulus and geometry). When Poisson's ratios or complex moment of inertias must also be scaled, beware. The admonition is model only what is pertinent and keep it simple so that you retain visibility.

CONCLUSION

The model should be carefully constructed to just the degree of complexity necessary to support the conclusions that will be drawn from it. Excess complexity contributes to excessive analysis, and insufficient complexity results in unrealistic performance. Models should simplify, linearize, and characterize.