

A GENERALIZED COMPUTER PROGRAM FOR STEADY-STATE  
SIMULATION OF COMPRESSIBLE FLOW IN PIPELINES

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INTRODUCTION

A good design of a pipeline system not only relies on the pressure drop calculations, but also on the estimation of flowing gas temperatures as well. Through extensive investigations of gas pipeline performance by the Pipeline Research Committee of American Gas Association (referred as AGA hereafter) over a nine year period of time, a set of flow equations has been developed for accurate determination of the flow behavior of steady-state gas pipelines. These equations were developed under the following assumptions:

1. Isothermal flow
2. Constant compressibility factor
3. Negligible kinetic energy change

In order to obtain accurate calculation of pressure drop or flow rate, properly averaged temperature and pressure must be estimated. Usually, the temperature profile along pipelines is not well known in the design stage. How to arrive at the best average condition is the most essential, yet probably the most difficult part in applying the AGA developed equations. It would totally rely on the engineer's judgement and the values once estimated will remain unjustified unless a proper heat transfer calculation coupled with flow equations is applied. The temperature effect on the gas properties and overall pipeline performance is complicated and interrelated as shown in Figure 1.

The flowing gas temperature can affect the design of a pipeline system as follows:

1. The accuracy of the pressure drop or flow rate calculation depends on the estimation of the average temperature or the entire temperature profile along the pipeline. In certain case the deviation may be as high as 30%.
2. In pipeline system design, the flowing gas temperatures are often subjected to economical and/or physical constraints of the operating system, especially for a large project where the range of flowing gas temperatures must be optimized. A good example is the proposed Arctic gas pipeline. Walker and Stuchly<sup>(1)</sup> reported that for the large part of the Arctic gas pipeline, the flowing gas temperature must be maintained below 32° F to maximize the economical and environmental benefits.
3. The gas temperature is a prime factor which must be considered in determining the cooling requirements for compressor stations or the duty for heating or cooling to maintain the flowing gas temperatures in a desirable range.

A comprehensive computer program was thus developed to calculate the pressure and temperature profiles along the pipelines. The program performs a rigorous heat transfer calculations, as well as enthalpy evaluation which includes the Joule-Thomson effect. The effect of changes in elevation and kinetic energy were also taken into account. Although the discussion here focuses on the natural gas pipeline system, the program is applicable to other compressible flow piping systems as well.

MATHEMATICAL MODEL

The energy balance equation for fluid flowing in pipes can be expressed by

$$m dH + \frac{m}{J} d\left(\frac{V^2}{2g_c}\right) + \frac{m q}{J g_c} dE + \frac{m}{J} dW = d q \quad (1)$$

Differentiating Equation (1) with respect to a differential pipe length,  $dX$ , yields

$$\frac{dH}{dX} + \frac{V}{J g_c} \frac{dV}{dX} + \frac{q}{J g_c} \frac{dE}{dX} + \frac{1}{J} \frac{dW}{dX} = \frac{1}{m} \frac{d q}{dX} \quad (2)$$

The enthalpy of the gas is a function of temperature, pressure and composition. For a given gas mixture, the partial derivative of enthalpy can be written as

$$\frac{dH}{dX} = \left(\frac{\partial H}{\partial T}\right)_P \frac{dT}{dX} + \left(\frac{\partial H}{\partial P}\right)_T \frac{dP}{dX} \quad (3)$$

Substituting Equation (3) into Equation (2) and after rearrangement we obtain

$$\frac{dT}{dX} = \frac{1}{\left(\frac{\partial H}{\partial T}\right)_P} \left[ \frac{1}{m} \frac{d q}{dX} - \frac{1}{J} \frac{dW}{dX} - \frac{q}{J g_c} \frac{dE}{dX} - \frac{V}{J g_c} \frac{dV}{dX} - \left(\frac{\partial H}{\partial P}\right)_T \frac{dP}{dX} \right] \quad (4)$$

In order to evaluate  $dV/dX$  we need to express the change of velocity in terms of density variation. From the continuity equation

$$V = \frac{m}{A \rho} \quad (5)$$

Thus, for a given mass flow rate,  $m$ , the velocity change can be expressed as

$$\frac{dV}{dX} = \frac{-m}{A \rho^2} \left[ \left(\frac{\partial \rho}{\partial P}\right)_T \frac{dP}{dX} + \left(\frac{\partial \rho}{\partial T}\right)_P \frac{dT}{dX} \right] \quad (6)$$

For a real gas the density of gas is calculated by

$$\rho = \frac{PM}{ZRT} \quad (7)$$

Differentiating  $\rho$  with respect to temperature and pressure yields

$$\left(\frac{\partial \rho}{\partial T}\right)_P = \frac{PM}{R} \frac{\partial}{\partial T} \left(\frac{1}{ZT}\right) = -\frac{\rho}{T} - \frac{\rho}{Z} \left(\frac{\partial Z}{\partial T}\right)_P \quad (8)$$

$$\left(\frac{\partial \rho}{\partial P}\right)_T = \frac{M}{RT} \frac{\partial}{\partial P} \left(\frac{P}{Z}\right) = \frac{\rho}{P} - \frac{\rho}{Z} \left(\frac{\partial Z}{\partial P}\right)_T \quad (9)$$

The mechanical energy balance equation can be written as

$$dP + \rho d\left(\frac{V^2}{2g_c}\right) + \rho dW + \rho dF + \frac{\rho g}{g_c} dE = 0 \quad (10)$$

Again, differentiating Equation (10) with respect to a differential pipe length,  $dX$ , results

$$\frac{dP}{dX} = -\rho \left[ \frac{g}{g_c} \frac{dE}{dX} + \frac{V}{g_c} \frac{dV}{dX} + \frac{dW}{dX} + \frac{dF}{dX} \right] \quad (11)$$

The terms inside the bracket in the right hand side of Equation (11) represent the head losses due to changes in elevation and kinetic energy and external work done on the system respectively. The last term accounts for the head loss due to friction which is expressed by the Fanning equation as

$$\frac{dF}{dX} = \frac{2fV^2}{g_c d} \quad (12)$$

Substituting Equation (6) into Equations (4) and (11) and solving for  $dP/dX$  and  $dT/dX$  we finally obtain

$$\frac{dT}{dX} = \frac{C_1 \left[ \frac{1}{m} \frac{dg}{dX} - \frac{g}{Jg_c} \frac{dE}{dX} - \frac{1}{J} \frac{dW}{dX} \right] + C_2 \left[ \left( \frac{\partial H}{\partial P} \right)_T - C_3 \left( \frac{\partial \rho}{\partial P} \right)_T \right]}{C_1 \left[ \left( \frac{\partial H}{\partial T} \right)_P - C_3 \left( \frac{\partial \rho}{\partial T} \right)_P \right] + \left( \frac{\partial H}{\partial T} \right)_P \left[ \left( \frac{\partial H}{\partial P} \right)_T - C_3 \left( \frac{\partial \rho}{\partial P} \right)_T \right]} \quad (13)$$

and

$$\frac{dP}{dX} = \frac{J \left( \frac{\partial H}{\partial T} \right)_P \frac{dT}{dX} - \frac{1}{m} \frac{dg}{dX} - \frac{dF}{dX}}{C_1} \quad (14)$$

where

$$C_1 = \frac{1}{\rho} - J \left( \frac{\partial H}{\partial P} \right)_T \quad (15)$$

$$C_2 = \frac{J}{m} \frac{dg}{dX} + \frac{dF}{dX} \quad (16)$$

$$C_3 = \frac{mV}{AJg_c \rho^2} \quad (17)$$

Evaluating the rate of heat transfer and the partial derivatives of enthalpy and compressibility factor at each increment, Equations (13) and (14) can be integrated simultaneously to give temperature and pressure profiles along the pipeline.

#### HEAT TRANSFER

The rate of heat transfer between the flowing gas and the outside surrounding medium per unit length of pipe is calculated by

$$\frac{dq}{dX} = 2\pi r_i U (T_s - T_{gas}) \quad (18)$$

The overall heat transfer coefficient,  $U$ , based on the pipe inside area is given by

$$U = \frac{1}{\alpha + \beta + \gamma + \delta} \quad (19)$$

where

$$\alpha = \frac{1}{h_i} \quad (20)$$

$$\beta = r_i \sum_{j=1}^n \frac{1}{k_j} \ln \left( \frac{r_{j+1}}{r_j} \right) \quad (21)$$

$$\gamma = \frac{r_i}{r_{m+1} h_o}, \quad (\gamma = 0 \text{ For buried pipelines}) \quad (22)$$

$$\delta = \frac{r_i}{S k_o}, \quad (\delta = 0 \text{ For pipelines surrounded by either air or water}) \quad (23)$$

and the shape factor  $S$  is given<sup>(2)</sup> by

$$\frac{1}{S} = \ln \left( \frac{2l + \sqrt{4l^2 - d_o^2}}{d_o} \right) \quad (24)$$

where  $l$  is the distance from the surface of the ground to the axis of the buried pipe and  $d_o$  is the outside diameter of the pipe including insulations.

The equations for calculation of inside and outside wall heat transfer coefficients  $h_i$  and  $h_o$  are available in References (3) and (4) and will not be listed here.

#### FRICITION FACTORS

Among the current available pipeline flow equations, the most often used are: General, Panhandle A, Panhandle B (New Panhandle), and Weymouth equations. These equations are expressed in integrated forms (5, 6) as follows:

##### General

$$Q = 38.774 \left( \frac{T_b}{P_b} \right) \left[ \frac{P_1^2 - P_2^2 - \frac{0.0375 G (E_2 - E_1) P_m^2}{Z_m T_m}}{G T_m L Z_m f} \right]^{0.50} D^{2.5} \quad (25)$$

##### Panhandle A

$$Q = 435.87 \left( \frac{T_b}{P_b} \right)^{1.0788} \left[ \frac{P_1^2 - P_2^2 - \frac{0.0375 G (E_2 - E_1) P_m^2}{Z_m T_m}}{G^{0.9539} T_m L Z_m} \right]^{0.5394} D^{2.6182} \eta \quad (26)$$

##### Panhandle B

$$Q = 732.2 \left( \frac{T_b}{P_b} \right)^{1.02} \left[ \frac{P_1^2 - P_2^2 - \frac{0.0375 G (E_2 - E_1) P_m^2}{Z_m T_m}}{G^{0.961} T_m L Z_m} \right]^{0.51} D^{2.53} \eta \quad (27)$$

##### Weymouth

$$Q = 433.49 \left( \frac{T_b}{P_b} \right) \left[ \frac{P_1^2 - P_2^2 - \frac{0.0375 G (E_2 - E_1) P_m^2}{Z_m T_m}}{G T_m L Z_m} \right]^{0.50} D^{8/3} \eta \quad (28)$$

A careful examination of each equation reveals that the only difference between each is the method of evaluation of the friction factor.

For the General equation, the friction factors are calculated according to two equations developed by the Pipeline Research Committee of AGA and we will refer to these equations as the AGA equations. One of these equations pertains to partially turbulent flow which is based on Prandtl's smooth-pipe law modified with a drag factor. The other is for fully turbulent flow which is based on Nikuradse's rough-pipe law, using an effective roughness instead of an absolute roughness. The friction factor is expressed in terms of the transmission factor,  $\sqrt{\frac{1}{f}}$ , which is an index of gas carrying capacity of a

pipeline under given conditions, and is directly proportional to the flow rate.

For partially turbulent flow, the transmission factor is calculated as

$$\sqrt{\frac{1}{f}} = \left[ 4 \log \left( \frac{Re}{\sqrt{\frac{1}{f}}} \right) - 0.6 \right] F_f \quad (29)$$

In Equation (29) a drag factor,  $F_f$ , is introduced to account for the resistances resulting from drag-inducing elements such as bends, valves, fittings and weld beads. If the drag factor equals to unity, Equation (29) reduces to the smooth pipe law and the friction factor is a function of Reynolds number only. The drag factors range from 0.900 to 0.985 with an average of 0.96(5).

For fully turbulent flow, the transmission factor is calculated according to

$$\sqrt{\frac{1}{f}} = 4 \log \left( \frac{3.7d}{\epsilon} \right) \quad (30)$$

Note that Equation (30) requires an effective or operating roughness,  $\epsilon$ , instead of the absolute roughness, which is commonly used for liquid flow calculations. The effective roughness represents the composite frictional resistance resulting from all the flow-disturbing elements, such as pipe wall, bends, fittings, weld beads and deposits, etc. Therefore, the magnitude of the effective roughness should be higher than the normal absolute roughness found in the literature. The approximate range of the effective roughness for natural gas pipelines is from 450 to 1850 micro-inches for bare steel pipe, and from 200 to 500 micro-inches for lined pipes or sandblasted or pig-burnished pipes(5).

The type of flow or flow regime in given pipe conditions can be determined by calculating a transition Reynolds number  $(Re)_T$ . Equating Equation (29) to (30) and solving for  $Re$ , the transition Reynolds number can be obtained as

$$(Re)_T = 1.413 \left( \frac{3.7d}{\epsilon} \right)^{\frac{1}{F_f}} \left( 4 \log \frac{3.7d}{\epsilon} \right) \quad (31)$$

No equation was provided for the transition flow in AGA equations. In actual gas pipe flow, the transition from partially to fully turbulent flow was reported to be rather abrupt, as opposed to the wide smooth transition shown in the Moody diagram. In fact, the range of the Reynolds numbers in transition regime is so narrow that it would only cause a maximum deviation of about one percent by applying Equations (29) and (30) when compared with smoothed curves. For practical purposes, Equation (29) and (30) are accurate enough for engineering design.

For the Panhandle A and B equations, the transmission factors are correlated as a function of Reynolds number alone which may be expressed as

$$\left. \begin{aligned} \sqrt{\frac{1}{f}} &= 6.872 Re^{0.07305} \eta^{0.92695} \\ \text{or} \quad \sqrt{\frac{1}{f}} &= 7.211 \left( \frac{QG}{D} \right)^{0.07305} \eta^{0.92695} \end{aligned} \right\} \text{Panhandle A} \quad (32)$$

$$\left. \begin{aligned} \sqrt{\frac{1}{f}} &= 16.49 Re^{0.01961} \eta^{0.98039} \\ \text{or} \quad \sqrt{\frac{1}{f}} &= 16.70 \left( \frac{QG}{D} \right)^{0.01961} \eta^{0.98039} \end{aligned} \right\} \text{Panhandle B} \quad (33)$$

An efficiency,  $\eta$ , which is defined as the ratio of the actual flow rate to the predicted flow rate, is employed in each Panhandle equation. It's effect is somewhat equivalent to AGA's drag factor in partially turbulent flow and effective roughness in fully turbulent flow, but with less significance of physical meaning.

The Panhandle A equation is a resemblance to the smooth-pipe law and the Panhandle B equation is an approximation to the rough-pipe law. Figure 2 shows comparisons between the Panhandle equations and AGA equations with various efficiencies and a drag factor of 0.96. Normally, the natural gas flow in pipelines will fall into the range plotted. As can be seen, the Panhandle A equation, with an efficiency of 1.0 approximates the smooth pipe law. It approaches the AGA partially turbulent flow equation with an efficiency of about 0.94. Using the proper efficiency, the Panhandle A equation can be used satisfactorily for partially turbulent flow. It becomes totally inadequate for the fully turbulent flow, and tends to predict pressure drops which are too low.

The Panhandle B equation, on the contrary, predicts low pressure drops in the partially turbulent flow regime. Unlike the AGA's fully turbulent flow equation which includes the effect of pipe relative roughness, the efficiencies in the Panhandle B equation must be varied to accommodate the use of different interiors of pipes for given pipe size and flow conditions. With an efficiency of about 0.82, it approximates the AGA's fully turbulent flow equation with a relative roughness of 0.0001. The Panhandle B equation was originally intended for use with a range of Reynolds numbers from  $4 \times 10^6$  to  $40 \times 10^6$ (7). The normal range of efficiencies for the Panhandle equations is from 0.75 to 0.98.

The simplest flow equation is the Weymouth equation, where the transmission factors are calculated as a function of pipe inside diameters only. With an efficiency of 1.0, the Weymouth equation for a 20 inch pipe, approximates the AGA fully turbulent flow equation with a relative roughness of 0.0001, as shown in Figure 2. Depending on the flow range, the efficiencies for the Weymouth equation can be from 0.9 to 1.2.

Note that all friction factors discussed here are the Fanning friction factors which are one-fourth the magnitude of the Darcy friction factors.

#### PHYSICAL PROPERTIES ESTIMATION

The accuracy of the temperature calculation depends largely on the evaluation of enthalpies of the gas mixture. Curl-Pitzer enthalpy (8,9) was used for the natural gas system. The partial derivatives of enthalpies with respect to temperature and pressure were evaluated numerically. It is a good practice to check the program by running an adiabatic flow problem for a pure component, to insure the enthalpy calculations and mathematical equations are formulated properly. Figure 3 shows the adiabatic temperature changes for methane. It checks very well when compared with an enthalpy diagram for methane.

The compressibility factor,  $Z$ , has a significant effect on the system pressure drop calculation. It may be evaluated from several equations of state, corresponding state correlations, or input in forms of tabular data or polynomials. The most reliable compressibility

factors are determined by laboratory test for the gas being considered of the probable temperature and pressure ranges. AGA's publication "Supercompressibility Factors for Natural Gas", Vols. 1-7(10) are probably the most extensive correlations currently available. The supercompressibility factor is defined as  $\sqrt{1/Z}$ . The data are correlated with gas gravity, and CO<sub>2</sub>, and N<sub>2</sub> contents, or gas gravity, heating value, and CO<sub>2</sub> content, over wide ranges of temperature and pressure levels.

Two equations of state were considered for calculating the compressibility factor, Z. One is the Beattie-Bridgeman equation of state(11) and the other is an equation of state which correlates the Standing-Katz Z-factor chart by Yarborough and Hall(12). The compressibility factors calculated from the Yarborough and Hall correlation will be referred as Standing-Katz Z-factors. The Standing-Katz Z-factor is a function of reduced pressure, Pr, and reduced temperature, Tr, while the Beattie-Bridgeman Z-factor calculation requires a third parameter, critical compressibility factor, Zc.

In general, the Standing-Katz Z-factors check more closely with the generalized corresponding state Z-factor chart, as expected. However, the Beattie-Bridgeman Z-factors were found to be more accurate when compared with AGA supercompressibility factors (13, 14) in this study. It was reported that the supercompressibility factors from the existing corresponding state correlations were not satisfactory for accurate flow calculations. Both Beattie-Bridgeman and Standing-Katz Z-factors seem lower than the AGA Z-factors.

Figure 4 shows pressure drops calculated using Equation (25) and AGA Z-factors (X-axis) comparing with that using Equation (14) and Beattie-Bridgeman Z-factors (Y-axis) for isothermal flows. AGA friction factors were used in both equations. The AGA Z-factors were evaluated at average pressure determined by

$$P_m = \frac{2}{3} \left( \frac{P_1^3 - P_2^3}{P_1^2 - P_2^2} \right) = \frac{2}{3} \left( \frac{1 - P_1 P_2}{P_1 + P_2} \right) \quad (34)$$

The average deviation of the absolute error of pressure drop is 0.96% with Beattie-Bridgeman Z-factors and 2.72% with Standing-Katz Z-factors. The maximum deviation is 2.78% with Beattie-Bridgeman Z-factors. The Standing-Katz Z-factors show as high as 10% deviation for a 956 psi pressure drop line. The ranges of data covered are given in Figure 4. For the compressibility factors, the average deviations are 0.2% and 1.2% for the Beattie-Bridgeman and Standing-Katz Z-factors respectively.

#### TEMPERATURE EFFECT

The changes in flowing gas temperatures in a pipeline are proportional to the rate of heat transfer between the flowing gas and the outside surrounding medium, the gas specific heat, and the Joule-Thomson effect. The rate of heat transfer relates to the gas flow rate, pipe insulation, and the type and condition of the surrounding medium which can be calculated by Equation (18). The gas specific heat and Joule-Thomson effect can be evaluated numerically from the enthalpy changes with respect to temperature and pressure. For most natural gas systems, the adiabatic temperature change can be roughly evaluated using an enthalpy diagram for methane for an estimated pressure drop. However, the overall change of temperature must be obtained by integrating Equations (13) and (14) simultaneously.

Figure 5 shows the possible calculated temperature profiles of a natural gas pipeline, 160 miles long, buried underground with two different materials of insulation,

and ground temperatures of 60°F and 10°F. The temperature drop due to the Joule-Thomson effect contributes from one-third to two-thirds of the overall temperature drop depending on the insulation and the ground temperature. Table 1 gives the pressure drops for the pipeline with the various temperature profiles shown in Figure 5. As can be expected, the isothermal flow at the inlet temperature of 120°F, results in the highest pressure drop. The deviations range from 11% to 31.7% as compared with the one that has the largest temperature drop (Curve 6).

If the temperature profile is known the pressure drop can be accurately determined by applying AGA equations with a weighted average temperature calculated by the one of the following equations:

$$T_m = \frac{T_{1,2} + T_{2,3} + T_{3,4} + \dots + T_{n-1,n}}{n} \quad (35)$$

where T<sub>ij</sub> is the temperature at the midpoint of the interval between i and j, or

$$T_m = \left( \frac{T_1 + T_2}{2} + \frac{T_2 + T_3}{2} + \dots + \frac{T_{n-1} + T_n}{2} \right) / n \quad (36)$$

where the average temperature at each interval is taken as the arithmetic average of the temperatures at the beginning and end of each equally divided interval.

Table 2 shows the pressure drops resulting from isothermal flow calculations for various average temperatures assuming the actual temperature profile represented by Curve 6 in Figure 5. The average temperature, calculated by Equation (35) or (36), gives almost the same pressure drop as the one using rigorous temperature calculations. Using an arithmetic average temperature, results in a 4.4% higher pressure drop. As indicated in Reference 5 the arithmetic average temperature is good for a small temperature change (less than 20°F). For a larger temperature change Equation (35) or (36) should be used.

TABLE 1

Temperature Profile Curve	Pressure Drop, PSI	Pressure Drop Ratio	Remark
1	974.5	1.318	Isothermal Flow
2	911.6	1.233	Adiabatic Flow
3	866.2	1.172	Normal Flow
4	821.2	1.111	Normal Flow
5	824.5	1.116	Normal Flow
6	739.1	1.00	Normal Flow

Gas gravity = 0.693  
 Pipe inside diameter = 28 inches  
 Pipe length = 160 miles  
 Gas flow rate = 1050 MMSCF/D  
 Inlet pressure = 2400 psia  
 Inlet temperature = 120°F  
 Depth of pipe buried = 3 feet

TABLE 2

Isothermal Flow Temperature, °F	Method of Calculation	Pressure Drop, PSI	% Deviation
120	Assumed	974.5	31.8
90	Assumed	854.1	15.6
77.36	Arithmetic mean	771.9	4.4
71.02	Eqn. (35)	743.0	0.5
70.86	Eqn. (36)	742.3	0.4
65	Assumed	716.0	-3.1
Rigorous calculation (Curve 6)		739.0	0.0

## CONCLUSION

For design of a compressible flow pipeline system, the pressure drops or flow rates, can be accurately determined by applying the AGA flow equations, provided that reasonably accurate average temperature and pressure are used. Both the compressibility factors and flowing gas temperatures have profound impact on the overall pipeline performance. The compressibility factors calculated from Beattie-Bridgeman equation of state were shown to be accurate for natural gas systems in the temperature and pressure ranges studied. The temperature profile along a pipeline can best be determined by rigorous calculations of enthalpy and heat transfer, coupled with flow equations. The inclusion of the temperature effect will greatly enhance the program's capability of optimum design of gas pipeline systems.

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## NOMENCLATURE

A.	Pipe inside cross-sectional area, FT <sup>2</sup>
D	Pipe inside diameter, IN
d	Pipe inside diameter, FT
d <sub>o</sub>	Pipe outside diameter, FT
E	Elevation of pipe, FT
F	Head loss due to friction
F <sub>f</sub>	Drag factor
f	Fanning friction factor
G	Gas gravity (Air=1.0)
g	Acceleration of gravity, 32.17 FT/SEC <sup>2</sup>
g <sub>c</sub>	Newton's law conversion factor, 32.17 (FT) (LB)/(LB <sub>f</sub> ) (SEC <sup>2</sup> )
H	Enthalpy per unit mass, BTU/LB
h <sub>i</sub>	Pipe inside wall heat transfer coefficient, BTU/(HR) (FT <sup>2</sup> ) (°F)
h <sub>o</sub>	Pipe outside wall heat transfer coefficient, BTU/(HR) (FT <sup>2</sup> ) (°F)
J	Mechanical equivalent of heat, 778.1 (FT) (LB <sub>f</sub> )/BTU
k	Thermal conductivity, BTU/(FT) (HR) (°F)
L	Pipe length, MILES
l	Depth of pipe buried, i.e. the distance from the surface of ground to the axis of the pipe, FT
M	Gas molecular weight, LB/MOLE
m	Gas mass flow rate, LB/HR
Re	Reynolds number
(Re) <sub>T</sub>	Transition Reynolds number
P	Pressure, LB <sub>f</sub> /FT <sup>2</sup>
P <sub>b</sub>	Base pressure, PSIA
P <sub>1</sub>	Pipe inlet pressure, PSIA
P <sub>2</sub>	Pipe outlet pressure, PSIA
Q	Gas volumetric flow rate, SCF/D
q	Rate of heat transfer, BTU/HR
R	Gas constant, 10.731 (PSIA) (FT <sup>3</sup> )/(LB-MOLE) (°R)
r	Radius of pipe, FT
S	Shape factor defined by Eqn. (24)
T	Temperature, °R
T <sub>b</sub>	Base Temperature, °R
U	Overall heat transfer coefficient based on pipe inside surface area, BTU/(FT <sup>2</sup> ) (HR) (°F)
V	Gas linear velocity, FT/HR
W	Mechanical or shaft work, (FT) (LB <sub>f</sub> )/LB
X	Distance along pipeline, FT
Z	Compressibility factor

## GREEK LETTERS

α	Defined by Eqn. (20)
β	Defined by Eqn. (21)
γ	Defined by Eqn. (22)
δ	Defined by Eqn. (23)

ρ	Gas density, LB/FT <sup>3</sup>
μ	Gas viscosity, LB/(HR) (FT)
ε	Pipe effective roughness, FT
η	Pipe flow efficiency

## SUBSCRIPTS

b	Base value
i	Pipe inside wall
m	Average value
o	Pipe outside wall
r	Reduced property

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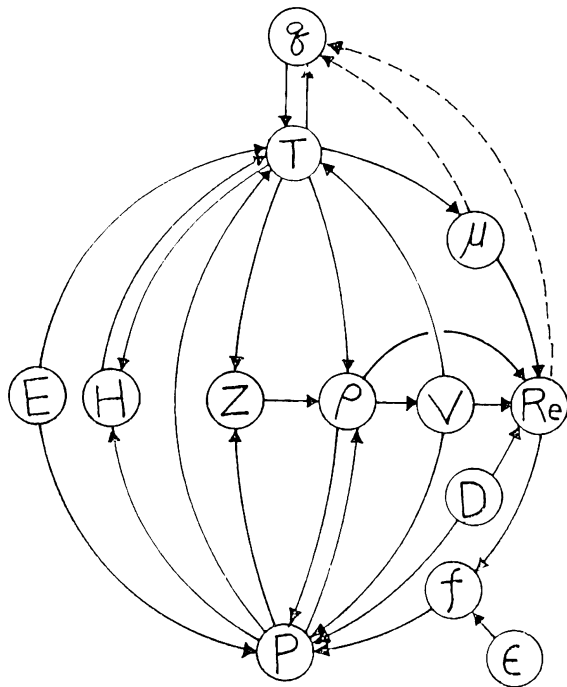


FIGURE 1

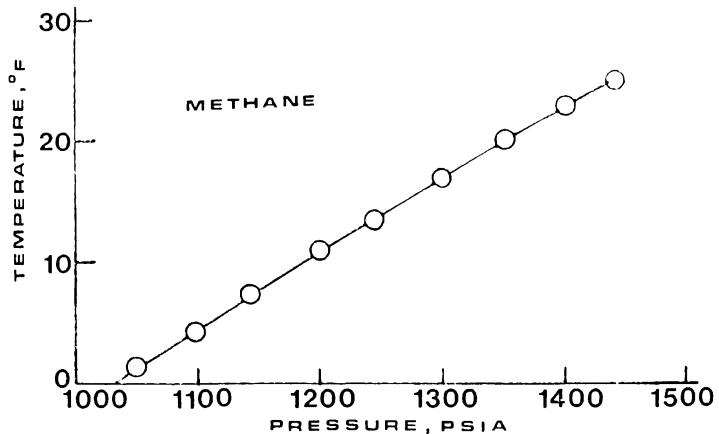


FIGURE 3

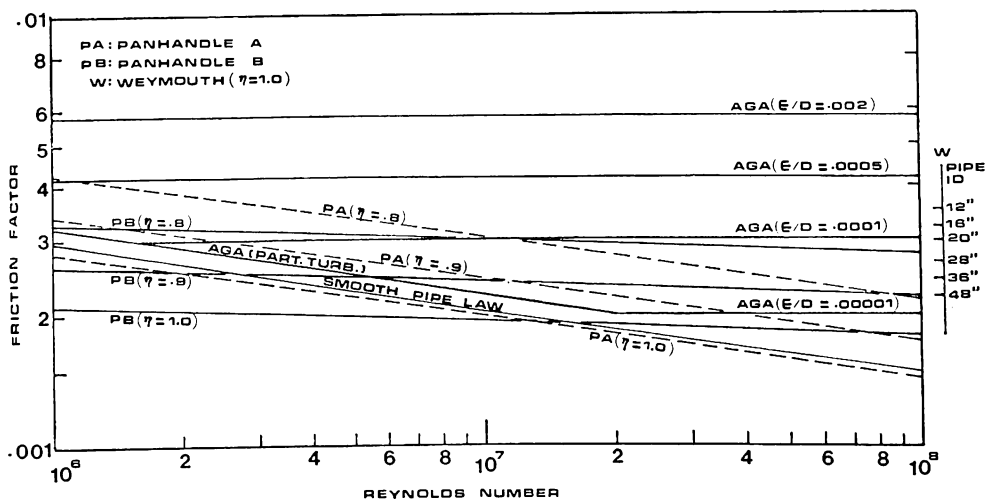


FIGURE 2

