FLOWGRAPH TUTORIAL: GUIDELINES FOR CONSTRUCTING NETWORK MODELS OF COMPLEX, PROBABILISTIC PROCESSES

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ABSTRACT

Network models consist of:

- Nodes or vertices which represent probabilistic events.
- Directed edges or transmittances, which represent transition probabilities between any two events.

The construction of a network is intended as a conceptual aid prior to simulation, which allows insight into the adequacy of correlation between the model and the "real world."

A systematic account is presented of:

- Guidelines to construct network models.
- A sequence of examples of increasing complexity.
- Algorythmic statements of evaluation procedures.
- Use of time, cost, and similar tags.

Evaluation of representative examples to provide benchmarks for programmers, including:

- Games of chance and stochastic processes.
- Patterns of promotion in an organization.
- Machinery repair processes.

INTRODUCTION

Flowgraph concepts for systems analysis originated with C. E. Shannon and were developed by Lorens, Mason, Huggins and others. Although the flowgraph model originated within electrical engineering, Huggins (19,1960) notes that it has emerged into many technical fields. This more general acceptance is explained by the ability of the flowgraph to:

- Allow systems structures to be easily perceived (the system's complexities as well as its interelationships).
- Be relatively easy to manipulate.
- Be less abstract than a set of equations.
- Be solvable with basic algebra.

This survey intends to explore representative uses of flowgraphs:

(i) for model validation and,
(ii) as a management - analyst communicative tool.

(i) It is the analyst's task to formulate a model of the problem presented. Flowgraphs provide an aid for constructing a quantitative model and for validating its approximate performance, prior to extensive and costly machine runs. The flowgraph lends itself well to pre-run analysis of simulation models. In this manner, flowgraphs can serve as benchmarks.

(ii) The analyst and the administrator must communicate about complex problems, at an adequate level of detail. A compatible vehicle of communication is necessary to allow managers to interact with technical experts. The flowgraph offers a precise language in which the technical expert is comfortable, while it offers elegant graphics to the administrator, who can accept the displayed logic, without a systems background.

The need for tools such as flowgraphs is dramatized by TABLE 1., which is adapted from Churchman and Scheinblatt (79, 1965). It is a matrix of the opposing ends of two continuums of understanding.

ADMINISTRATOR-ANALYST FUNCTIONS, BASED ON UNDERSTANDING

TABLE 1.		ANALYST UNDERSTANDS ADMINISTRATOR:	
		WELL	POORLY
ADMINISTRATOR UNDERSTANDS ANALYST:	WELL	MUTUAL UNDERSTANDING FUNCTION	COMMUNICATION FUNCTION
	POORLY	PERSUASION FUNCTION	SEPARATE FUNCTION

- Communication Function - Administrator has to communicate, precisely, with analyst, if he expects to realize any significant technical assistance.
- Persuasion Function - Analyst has to persuade manager to utilize his technical expertise for other than suboptimal uses.
- Separate Function - Analyst and administrator both live in their "own little worlds;" they do not understand each other's functions, nor do they offer much, if anything constructive to each other.
- Mutual Understanding Function - The ideal - when the administrator and the analyst are able to carry on a productive dialogue, respect each other as professionals and form a synergistic relationship which enables both to do better jobs.

Luecke (127, 1973) describes the uneasiness in some administrators when they are exposed to technical "jargon", "cryptic" printouts, and technology in general. The flowgraph may be just nonthreatening enough to allow effective communication, even when dealing with "magic black-box" technology such as digital simulation.

To demonstrate the versatility of flowgraphs, several examples of increasing complexity are examined. To facilitate a more detailed study, an extensive literature survey is provided. This survey, when combined with the bibliography of C. S. Lorens (72, 1964), represents the majority of the publications in the field, to date.

FLOWGRAPH CONCEPTS

The following definitions, tables and examples will attempt to develope an algorythm for the formulation and evaluation of flowgraphs.

Flowgraphs are network models which consist of:

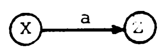
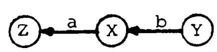
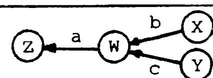
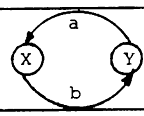
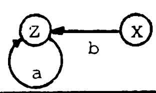
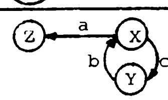
- Nodes - represent variables which may be dependent, interdependent or independent depending on their location with respect to transmittances.
- Transmittances - represent functional relationships between variables, from the independent node to the dependent node. Transmittances have direction and a value equal to the contribution of the independent variable upon the de-

pendent variable.

- Loops - are sequences of transmittances, such that each node of the sequence serves once and only once, in both functions, as independent and dependent variable.
- Paths - are sequences of transmittances which emanate from the independent variable and terminate at the dependent variable.
- Stochastic Processes - are probabilistic sequences of events, which describe events with uncertain outcomes. While each individual trial or realization of such event is independent, large numbers of occurrences are governed by laws of statistics.

EXAMPLES OF FLOWGRAPH NETWORKS

Table 2.

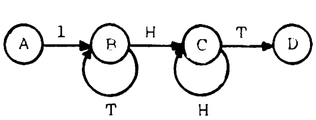
DESCRIPTION	EQUATIONS	FLOWGRAPHS
DEPENDENT & INDEPENDENT NODES	$Z = aX$ $\frac{Z}{X} = a$	
MULTIPLICATION OF TRANSMITTANCES	$Z = abY$ $\frac{Z}{Y} = ab$	
ADDITION OF TRANSMITTANCES	$Z = abX$ $Z = aW$ $W = bX+cY$	
A CLOSED SYSTEM OF INTERDEPENDENT NODES	$X = aY$ $Y = bX$ $ab = 1$	
A FEEDBACK LOOP MODIFIES A TRANSMITTANCE	$Z = aZ+bX$ $\frac{Z}{X} = \frac{b}{1-a}$	
TWO TRANSMITTANCES FORM A FEEDBACK LOOP	$Z = aX$ $Y = cX$ $X = bY$	

In table 2., Z is the dependent variable, W, X, Y are the independent and interdependent variables, and the coefficients a, b, c are the transmittances.

Feedback in a stochastic process is present when in an assembly of transmittances, one or more loops are formed. This is often referred to as a closed system.

Figures 1, 2, and 3 are examples of networks containing feedback loops. The figures are flowgraph representations of coin tossing games, and depict the possible outcomes of each toss, while a player attempts to toss specified sequences of Heads and Tails.

FIGURE 1: TOSSING A HEAD, FOLLOWED BY A TAIL

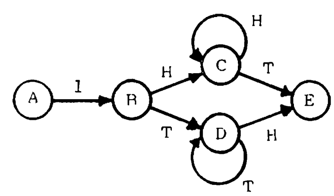


The flowgraph in Figure 1 represents a game: Tossing a coin to obtain the sequence, Head (H), followed by a Tail (T).

The unit transmittance 1 from node A to node B represents the independent variable of one coin entering the system to be tossed, at node B, the dependent variable. Node B represents a probabilistic state which may lead to either an H or a T.

The transmittances eminating from node B are the probabilities of tossing an H or T, each equal to 1/2. If an H is realized, the player moves to node C. If a T is realized, the player returns to node B, through loop T. Once node C is reached, the player can win by tossing a T and reaching node D.

FIGURE 2: TOSSING A HEAD, FOLLOWED BY A TAIL OR TOSSING A TAIL, FOLLOWED BY A HEAD



The situation presented in Figure 2 is similar to the previous example; the difference being the player has two different paths through which he can reach node E and "win." The toss at node B can only move the player closer to success, with his second toss at either node C or D being the first toss which may cause him to loop and not gain ground.

FIGURE 3: TOSSING TWO, CONSECUTIVE HEADS, FOLLOWED BY A TAIL

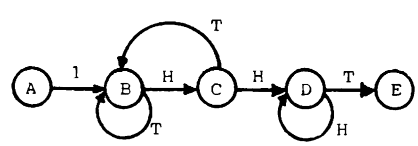


Figure 3 depicts a situation in which the player may loop from node C back to node B. This loop nullifies the player's previously successful toss and forces him to start from the beginning once more.

FLOWGRAPH EVALUATION

This section will deal with the mathematics employed to solve flowgraphs. Because this paper is designed to have meaning for both the technical and non-technical reader, many examples are used to illustrate and clarify the specialized terms and mathematical notation.

Flowgraph evaluation utilizes the values of the loops and paths of a system. The following terms are necessary to identify the elements of the equation used to solve flowgraphs:

- A Loop is a sequence of transmittances, which begin and end on the same node. The value of a loop is the product of its transmittances.
- First Order Loops, L(1), are all the individual loops in the network.

- Second Order Loops, L(2), are all sets of two loops which have no common nodes (sets of two first order loops which do not touch each other).
- Third Order Loops, L(3), are all sets of three loops which have no common nodes.
- Fourth Order Loops, L(4), are all sets of four loops which have no common nodes.
- Nth Order Loops, L(N), are all sets of N Loops, which have no common nodes.

In the Topology Equation, which is used to evaluate flowgraphs, the denominator contains the following terms, defined as:

- Odd Order Loops, L(-), the sum of L(1), L(3), L(5)...
- Even Order Loops, L(+), the sum of L(2), L(4), L(6)...

The numerator of the equation contains terms defined as:

- Paths, the products of the transmittances from the independent node to the dependent node. A path can pass through a node once only. The path value is the product of its transmittances.
- Odd Order, Non-Touching Loops, NTL-(-), all Odd Order Loops, which have no common nodes with a path.
- Even Order, Non-Touching Loops, NTL(+), all Even Order Loops, which have no common nodes with a path.

The Topology Equation, where the probability (Pr) of realizing the dependent node (X) from the independent node (A) is presented in Figure 4.

FIGURE 4: THE TOPOLOGY EQUATION

$$(Pr)\frac{X}{A} = \frac{\sum P\left(1 - \text{ODD ORDER, NON-TOUCHING LOOPS} + \text{EVEN ORDER, NON-TOUCHING LOOPS}\right)}{1 - \sum\left(\text{ODD ORDER LOOPS}\right) + \sum\left(\text{EVEN ORDER LOOPS}\right)}$$

To simplify and prove the Topology Equation, we further define:

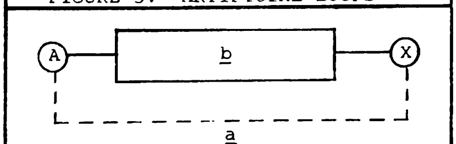
- P as the summation of paths.
- P(-) as the summation of products of the paths times their corresponding, Odd Order, Non-touching Loops.
- P(+) as the summation of products of the paths times their corresponding, Even Order, Non-Touching Loops.

The Topology Equation can now be written:

$$(Pr)\frac{X}{A} = \frac{P - P(-) + P(+)}{1 - L(-) + L(+)}$$

To prove the equation, assume a flowgraph resides inside box b of Figure 5:

FIGURE 5: ARTIFICIAL LOOPS



There exists at least one path from node A to node X. A path is now defined as a First Order Loop from which a real or artificial transmittance has been removed.

The value of b = X/A. We insert now transmittance a, which equals A/X. This generates a closed system, since a relates the nodes outside the box containing the flowgraph. Important results are the equations: ba = 1 and a = A/X.

The following loops contain the transmittance a:

- aP is the sum of all first order loops through a and consists of the sum of all paths from A to X multiplied by a.
- aP(-) is the sum of all Odd Order Loops through a.
- aP(+) is the sum of all Even Order Loops through a.

Therefore, the equation,

$$1 - L(-) + aP(+) + L(+) + aP(-) = 0,$$

is valid for any closed system and is an equation of the type: H = 0; where H is a function of loops only. For the simplest system consisting of only two transmittances, a and b, we have: H = 1 - ab = 0.

A detailed proof and discussion of the Topology Equation is given by Johnson and Johnson (125, 1975).

USING THE TOPOLOGY EQUATION

The first example, using the equation, is the simple coin toss of Fig. 1, which is the probability of tossing a Head followed by a Tail.

There are two loops in the network. When taken singularly, they become L(1)s (H+T); when taken as a set of two, they become an L(2) (HT). Because both loops touch the path, there are no NTLs.

When working with more complex networks, making a table, like Table 3, will help keep track of paths, loops and their transmittances. Also, a table will help set up the arithmetic steps, as will be shown in more complex examples.

TABLE 3: A HEAD FOLLOWED BY A TAIL

NUMERATOR	
PATH	TRANSMITTANCES
BCD	HT

DENOMINATOR		
	L(-)	L(+)
L(1)	H + T	
L(2)		HT

It is now possible to plug the above terms into the equation.

$$(Pr)\frac{D}{A} = \frac{P - P(-) + P(+)}{1 - L(-) + L(+)} = \frac{HT - 0 + 0}{1 - H - T + HT}$$

To solve the equation, replace the Hs and Ts with their probabilities of 1/2, and solve.

$$(Pr)\frac{D}{A} = \frac{(1/2)^2}{1 - 1/2 - 1/2 + (1/2)^2} = \frac{1/4}{1 - 1 + 1/4} = 1$$

This tells us there is 100% probability of realizing node D from node A, given an infinite number of tosses. It also gives us a check on our arithmetic, when dealing with a closed system in which all transactions flow to one node.

Generator Function

By using what is called a Tag, it is possible to utilize the Topology Equation to generate individual probabilities for each toss. The Tag performs several functions, one of which is keeping track of events by "tagging" transmittances.

For example, by redefining the Hs and Ts as being equal to 1/2X, where the 1/2 is the probability and the X has no assigned value, we are able to utilize the Generator Function of the Topology Equation.

$$(Pr)\frac{D}{A} = \frac{(1/2X)^2}{1-1/2X-1/2X+(1/2X)^2} = \frac{1/4X^2}{1-X+1/4X^2}$$

By using long division, we get:

$$1/4X^2+1/4X^3+3/16X^4+1/8X^5+5/64X^6+3/64X^7\ldots$$

$$1-X+1/4X^2 \,\overline{)\,1/4X^2}$$

$$\underline{1/4X^2-1/4X^3+1/16X^4}$$

$$1/4X^3-1/16X^4$$

$$\underline{1/4X^3-1/4X^4+1/16X^5}$$

$$3/16X^4-1/16X^5$$

$$\underline{3/16X^4-3/16X^5+3/64X^6}$$

$$1/8X^5-3/64X^6$$

$$\underline{1/8X^5-1/8X^6+1/32X^7}$$

$$5/64X^6-1/32X^7$$

$$\underline{5/64X^6-5/64X^7+5/256X^8}$$

$$3/64X^7-5/256X^8$$

As you can see, a power series developes, as well as coefficients of decreasing values. The exponents serve as addresses for the Tags. Each exponent value represents an event number. Also, the coefficient represents the probability of realizing the dependent node on the event indicated by the exponent.

In the coin toss example, the Generator Function indicates the probability of reaching node D on the second toss as 1/4. Also, it is impossible to reach node D in less than two tosses, which is indicated by the first exponent being 2.

The coefficient and power of each Tag, denote the probability of reaching the dependent node on that particular toss. To achieve the probability of reaching the dependent node by a particular event, simply add the probabilities through that event. In our example, the probability of reaching node D on the third toss is 1/4; the probability reaching node D by the third toss is 1/4+ 1/4 = 1/2.

The sum of all the individual probabilities equals one, or the same answer provided by the Topology Equation without using Tags.

By using the network in Fig. 2, the tossing of a Head followed by a Tail or the tossing of a Tail followed by a Head, another use of Tags will be shown. That is, the assignment of values to the Tags. Tags can represent units of time, work, money, energy, etc.

First we will identify the paths and loops of the network, using the tabulation method as in the previous example.

In Table 4, two paths from node A to node E are listed. Each has to be multiplied by its non-touching loop; the sum of these products yields P(-).

TABLE 4: TOSSING A HEAD FOLLOWED BY A TAIL OR TOSSING A TAIL FOLLWED BY A HEAD

NUMERATOR			
PATHS	TRANS.	NTL(-) NTL(1)	P(-)
PATH BCE	HT	T	HTT
PATH BDE	TH	H	THH
SUMS:	HT+TH		HTT+THH

	DENOMINATOR	
	L(-)	L(+)
L(1)	H + T	
L(2)		HT
SUMS	H + T	HT

As in the previous example, there are two L(1)s, which also become an L(2) since they have no common nodes. The value of the denominator is the same as the last example because of the same number of loops, of the same values.

By applying the sums from the table, the equation is:

$$(Pr)\frac{E}{A} = \frac{P-P(-)+P(+)}{1-L(-)+L(+)} = \frac{HT+TH-HTT-THH}{1-H-T+HT}$$

We now define:

- H and T as equal to 1/2X
- X as equal to a $2.00 tossing fee or wager

which changes the equation to:

$$(Pr)\frac{E}{A} = \frac{(1/2X)^2+(1/2X)^2-(1/2X)^3-(1/2X)^3}{1-1/2X-1/2X+(1/2X)^2} = \frac{1/2X^2-1/4X^3}{1-X+1/4X^2}$$

The Generator Function yields the following power series:

$$1/2X^2+1/4X^3+1/8X^4+1/16X^5+1/32X^6+1/64X^7+1/128X^8+\ldots$$

The power series displays a geometric curve; after the first toss, the player has a probability of 1/2 of winning with each additional toss. This is typical of the of the power series; generally the individual probabilities will peak in the first few events, then begin approaching zero as the sum of probabilities approach one.

By noting the inverse of the probability of success, we see the player has only a 1/8 probability of not winning by his fourth toss, 1/16 by his fifth toss, 1/32 by his sixth toss,...

With X being equal to $2.00, the cost related to the probability of "winning" can be easily seen.

PROBABILITY	COST
1/2	$4.00
3/4	6.00
7/8	8.00
15/16	10.00

EXAMPLES

The following examples are meant to illustrate the uses of flowgraphs for other than simple coin tossing games.

Machine Repair

The Belch Bottling Company occasionally has to correct a series of three bends, in a bottle guide. This is a manual proceedure where the guide is bent and then tested under operating conditions.

Each succeeding bend, however, can misalign all prior bends. There is a probability of .20 that the second bend will alter bend one, and a probability of .10 that bend three will misalign both bends one and two. In either case, the repairmen must start the entire process over again, until all angles are correct.

Figure 6 is the flowgraph of the Belch bending proceedure. Nodes one, two and three are the bending activities. The results of the bends and their probabilities are presented in Table 5.

TABLE 5: TRANSMITTANCES

OUTCOMES		PROBABILITIES	TAGS
GOOD BENDS	G1	.30	X
	G2	.30	X
	G3	.30	X
BAD	B1	.70	X
	B2	.50	X
	B3	.40	X
BENDS	B4	.20	X
	B5	.10	X
	B6	.20	X

NODE	ACTIVITY
B	BOTTLE GUIDE NON-FUNCTIONAL
1	MAKE FIRST BEND
2	MAKE SECOND BEND
3	MAKE THIRD BEND
S	BOTTLE GUIDE FUNCTIONAL

FIGURE 6: BELCH BOTTLE GUIDE

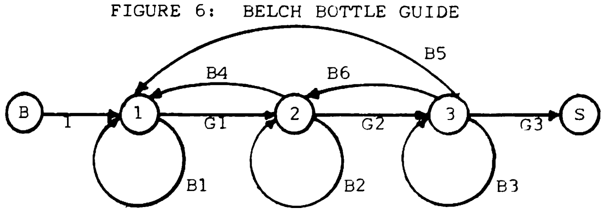


TABLE 6: BELCH BOTTLING CO.

Path 123S	(G1 G2 G3) (.3X)(.3X)(.3X)
P	$.027X^3$ (NO NON-TOUCHING LOOPS)
L(1)	B1 + B2 + B3 + (G1 B4) + (G1 G2 B5) + .7X+.5X+.4X+(.3X)(.2X)+(.3X)(.3X)(.1X)+ (G2 B6) (.3X)(.2X)
L(3)	(B1 B2 B3) (.7X)(.5X)(.4X)
L(-)	$1.6X+.12X^2+.149X^3$
L(2)	(B1 B2) + (B1 B3) + (B2 B3) (.7X)(.5X)+(.7X)(.4X)+(.5X)(.4X)+ (B1 G2 B6) + (B3 G1 B4) (.7X)(.3X)(.2X)+(.4X)(.3X)(.2X)
L(+)	$.83X^2+.066X^3$

The next step is to take the values for P, L(-) and L(+) from Table 6, plug them into the Topology Equation, and solve for the power series.

$$(Pr)\frac{S}{B} = \frac{P-P(-)+P(+)}{1-L(-)+L(+)}$$

$$= \frac{.027X^3-0+0}{1-1.6X-.12X^2-.149X^3+.83X^2+.066X^3}$$

$$= \frac{.027X^3}{1-1.6X+.71X^2-.083X^3} =$$

$$.027X^3+.043X^4+.050X^5+.051X^6+.050X^7+.048X^8+.046X^9\ldots$$

We now define the tag X, as equal to five minutes, that is the mean time required to remove the guide from its place in the bottling line, put a bend on it, and determine whether the bend is good or bad.

The power series tells us the probability of the repairmen fixing the guide within fifteen minutes to be 2.7%. It also tells us that after making nine bends, during a forty five minute period, the probability of the guide being repaired is less than 32% (the sum through X^9).

Organizational Promotions

Every spring the Recall Tire Company hires college graduates (node A) to fill management trainee positions (node B). After one year of employment 30% of the management trainees are promoted to a position of sales manager (node C). This promotion probability implies a 70% chance of staying at the same level (node B). In theory, if a management employee gets a promotion every year, he becomes president (node E) in three years.

In practice, every year about graduation time all the positions within the organization come under consideration for personnel changes. The people in the positions represented by nodes B, C, and D, may be moved to other positions or may be forced to wait a year for another chance to be promoted.

The sales manager (node C) is presented with the most variety of possible changes. He has a 30% chance of promotion to vice president (node D), a 20% chance of demotion to management trainee (node B), a 10% chance of leaving the company (node G), and of course he may remain as sales manager.

Node C also represents the position from which employees are most likely to become ex-employees. The two types of employees who leave the organization from this position are, the upwardly mobile, young executives who accept better job offers from other companies, and the disgruntled ex-vice presidents who have been demoted to sales manager, and decide to quit or retire.

Node F is an unqualified relative of the company president, who infrequently, is made a sales manager.

Table 7 and Fig. 7 will help clarify the above statements.

Although this is obviously a social system, the flowgraph is nearly identical to the one for the bottle guide bending example. In fact, if the values of the transmittances were the same, the denominator would be the same. This is true because the denominator is a function of loops only, and the loops in both flowgraphs are in the same relative positions to each other.

FIGURE 7: UP THE ORGANIZATION

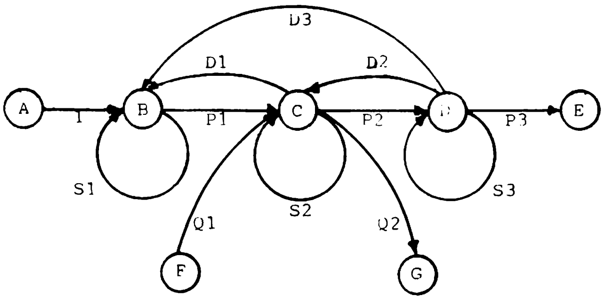


TABLE 7: UP THE ORGANIZATION		
NODE	CAREER POSITION	
A	COLLEGE GRADUATE	
B	MANAGEMENT TRAINEE	
C	SALES MANAGER	
D	VICE PRESIDENT	
E	PRESIDENT	
ORGANIZATIONAL ENTRANCE		
A RECENT COLLEGE GRADUATE IS HIRED AS A MANAGEMENT TRAINEE (NODE A TO NODE B)		
	TRANSMITTANCE:	
PROMOTIONS	SYMBOL	PROBABILITY
MGMT. TRAINEE TO SALES MGR.	P1	.30
SALES MGR. TO VICE PRES.	P2	.30
VICE PRES. TO PRESIDENT	P3	.30
STAY AT SAME POSITIONS		
MANAGEMENT TRAINEE	S1	.70
SALES MANAGER	S2	.40
VICE PRESIDENT	S3	.40
DEMOTIONS		
SALES MGR. TO MGT. TRAINEE	D1	.20
VICE PRES. TO SALES MGR.	D2	.20
VICE PRES. TO MGT. TRAINEE	D3	.10
NEPOTISM		
NEPHEW HIRED AS SALES MGR.	Q1	.10
LEAVE THE ORGANIZATION		
SALES MGR. OUTSIDE SYSTEM	Q2	.10

The denominator, derived from Table 8 is valid for all the combinations of paths from the two input nodes to the two output nodes. The value of L(-) is 1.741; L(+) is .786, therefore the denominator is:

$$1-1.741+.786 = .045.$$

TABLE 8: TOPOLOGY DENOMINATOR	
L(1)	S1+S2+S3+ (P1D1) + (P2 D2)+ (P1 P2 D3) .7+.4+.4+(.3)(.2)+(.3)(.2)+(.3)(.3)(.1)
L(3)	(S1 S2 S3) (.7)(.4)(.4)
L(-)	1.741
L(2)	(S1 S2)+(S1 S3)+(S2 S3)+(S1 P2 D2)+ (.7)(.4)+(.7)(.4)+(.4)(.4)+(.7)(.3)(.2)+ (S3 P1 D1) (.4)(.3)(.2)
L(+)	.786

TABLE 9: TOPOLOGY NUMERATORS					
PATHS	TRANS.	NTL(-)	NTL(+)	P(-)	P(+)
ABCDE ↓	(P1 P2 P3) (.3)(.3)(.3)				
P	.027				
ABCG ↓	(P1 Q1) (.3)(.1)	S3 .4			
P	.03	.4		.012	
FCG ↓	(Q1 Q2) (.1)(.1)	S1 + S3 (.7)+(.4)	(S1 S3) (.7)(.4)		
P	.01	1.1	.28	.011	.0028
FCDE ↓	(Q1P2 P3) (.1)(.3)(.3)	S1 .7			
P	.009	.7		.0063	

Table 9 includes the values for the paths ABCDE, ABCG, FCG and FCDE, their NTLs and the products of the paths times their NTLs.

The path ABCDE = (P1)(P2)(P3) = .027. Since this path has no non-touching loops, .027 is the value of the numerator of the Topology Equation. Therefore, the total probability of the college graduate becoming president is:

$$(Pr)\frac{E}{A} = \frac{.027}{.045} = .60.$$

The path ABCG represents the college graduate leaving the company from the sales manager position. The path value is (P1)(Q2) = .03. There is one NTL, S3(.4), therefore, the numerator P - P(-) + P(+) = .03 - (.03)(.4) + 0 = .018.

The total probability of his leaving Recall Tire is:

$$(Pr)\frac{G}{A} = \frac{.018}{.045} = .40.$$

The nephew becoming president is shown as path FCDE. The total probability of his becoming president is:

$$(Pr)\ \frac{E}{F} = \frac{(.1)(.3)(.3) - (.1)(.3)(.3)(.7) + 0}{.045}$$

$$= \frac{.009 - .0063}{.045} = \frac{.0027}{.045} = .06.$$

For determining the annual probabilities for the above outcomes, a tag equal to one year could be used. There would be a power series generated for each output/input relationship.

For the probabilities of the college graduate becoming president, the generator function is:

$$(Pr)\frac{E}{A} = \frac{.027X^3}{1-1.5X+.6X^2-.055X^3}$$

This yields the power series:

$$.027X^3+.0401X^4+.0456X^5+.0440X^6+.0415X^7+.0383X^8+\ldots$$

Therefore, his one year most likely to become president is his fifth, with the probability of 4.56%. By summing the probabilities, his cumulative probabilities by year are:

- 2.7% by three years
- 6.7% by four years
- 11.27 by five years
- 15.67% by six years
- 19.89% by seven years...

FLOWGRAPH CONSTRUCTION

When drawing the flowgraph, start with the independent variable as the initial node. Draw the transmittances from the independent node, to the dependent nodes, ending with the output node. Be sure to include the loops where they are needed.

Flowgraphs follow a logical sequence of e- ' vents, thereby allowing the user to draw them as he perceives these events occurring. This means, as in computer simulation, the flowgraph model is a representation of the analysts view of the system, and two people may draw two different flowgraphs of the same system.

FLOWGRAPH EVALUATION: AN ALGORYTHM

Flowgraph evaluation has been presented by example until now. Tables have been used, employing different formats. There is no "right" type; the best being the one which the user feels the most comfortable with, for the particular system.

For the more simple networks, and as the user becomes adept with flowgraphs, creating tables may be an extra step, which can be eliminated. At first, however, they are very helpful for structuring the the elements of the flowgraph into a form, which will facilitate the arithmetic of the Topology Equation.

The Denominator

The initial step in flowgraph evaluation is to identify and list the elements of the denominator. Therefore, list all Odd Order Loops together, then make a separate list of Even Order Loops.

The next step is to sum the Odd Order Loops, and then the Even Order Loops, and reduce them to their most simple form, for insertion into the formula as 1-L(-)+L(+).

The Numerator

Identify and list all the paths between the input and output nodes to be analyzed. For each path, identify and list its Odd Order, Non-Touching Loops, list its Even Order, Non-Touching Loops.

Next, multiply the paths times their Non-Touching Loops. Then sum the these products, which will yield the values of P(-) and P(+).

After summing the values of the paths, all of the term values are available for plugging into P-P(-)+P(+).

The Topology Equation

Through the previous steps, all the values necessary for flowgraph evaluation are iden ified and you can now apply the equation:

$$(Pr)\frac{\text{OUTPUT NODE}}{\text{INPUT NODE}} = \frac{P-P(-)+P(+)}{1-L(-)+L(+)}$$

In situations involving very complex systems or for the analyst who finds the mathematics too tedious, Whitehouse (129, 1973) provides, in an appendix, a Fortran program for solving flowgraphs.

CONCLUSION

This survey is an attempt to display the value of the flowgraph technique as an analytical tool, that can be used prior to simulation, and as a tool for expressing ideas between people with various levels of technical expertise.

The flowgraph technique can be an asset to the analyst's "bag of tricks" because it:

- Provides a check of individual and total probabilities of an entire system or a subsystem, before actual simulation; so to speak, a "mini simulation" of the simulation model.
- Provides a language for communication between technical and non-technical people.
- Offers mathematical precision for analysis.
- May serve to reach agreement on the validity of the system or component models.

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