REDUCTION OF THE ORDER OF A NONLINEAR SYSTEM

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ABSTRACT

A steam-driven electrical power generating station was designed with the aid of a hybrid computer simulation. In the process, it was advantageous to reduce the order of the system of nonlinear equations describing the boilers and turbines. Since there is no general procedure for reducing the order of nonlinear systems, each case must be treated individually. However, the methods used in one system are sometimes applicable to others. The approach taken in the above reduction is presented with this in mind.

I. INTRODUCTION

Upon analyzing or designing physical systems by computer simulation it is frequently desirable to form a model of the entire system by obtaining a mathematical expression for each component and then combining these expressions. This can result in a high-order model. It may therefore be advantageous to reduce the order of the model so as to decrease the required computational effort.

Several schools of approach to the model reduction problem have been developed. Chen [1] recognizes six schools and gives corresponding references. Since there is no general procedure for reducing the order of nonlinear systems, each case must be treated individually. However, the methods used in reducing one nonlinear system are sometimes applicable to others. The approach taken in the particular example of this paper is presented with this in mind.

A steam-driven electrical power generating station was designed with the aid of a hybrid computer simulation. After having designed the prime-mover controls, the order of the controlled prime-mover was reduced to free analog computer components for the rest of the station.

The detailed mathematical model is presented in the appendix since the approach used in the reduction can be understood without it. The equations of the model appear in subsets, called 'blocks', which are referred to in the illustrations. A complete list of symbol definitions follows the appendix.

II. SYSTEM DESCRIPTION

The prime-mover consisted of N_B identical boilers and N_T identical turbines, interconnected by a common steam bus, and a power controller manipulating the fuel to the boilers (Illustrations 1 to 3). Nominally there were 6 boilers and 3 turbines each delivering 15 MW at a synchronous shaft velocity of 100π r/s. The other components of the power plant included a power-demand unit, generators, electrical load, governor and auxiliary system (Illustration 4). The purpose of the auxiliary system was to provide a source of stored energy which could be released rapidly to the turbogenerator shafts in such a way as to supplement the relatively slow response of the prime-mover during transients. Alternatively, energy could be taken from the shafts and stored in the auxiliary system to provide braking.

A controller in the auxiliary system acted through the prime-mover by manipulating the power, P_h, required to regulate the stored energy at a desired level. The regulation was purposely sluggish so that it neither interfered with the main and auxiliary-system power control loops during transients nor introduced transients of its own into the net shaft power, P_s, during steady state.

The governor acted mainly via the auxiliary system during transients because the prime-mover could respond only poorly to the relatively high frequencies of shaft-speed variations.

III. PROBLEM DEFINITION

The purpose of the design was to investigate the feasibility of improving the transient response and reliability of steam-driven electrical power plants by means of an auxiliary system (Illustration 4). Four specific tests were used to evaluate the performance:

(1) In test 1, the 'increase test', the plant was subjected to a 10% (or 4.5 MW) increase in power demand.

(2) In test 2, the 'decrease test', the plant was subjected to a 10% decrease in power demand.

(3) In test 3, the 'boiler-dropping test', one of the six boilers was removed discontinuously from operation, leaving the other five boilers with the responsibility of making-up for the loss.

(4) In test 4, the 'turbine-dropping test', one of the three turbogenerator units was removed discontinuously from operation, leaving the other two turbines with the responsibility of making-up for the loss.

Tests 1 and 2 were used to determine if the response time of the plant to a change in power demand could be reduced by means of the auxiliary system, thereby improving the transient response. Tests 3 and 4 were used to determine if the disturbance in net plant-power generation following the loss of a boiler or turbine could be reduced by means of the auxiliary system, thereby improving the reliability of power delivery.

The prime-mover was chosen for reduction over other components of the plant for the following reasons:

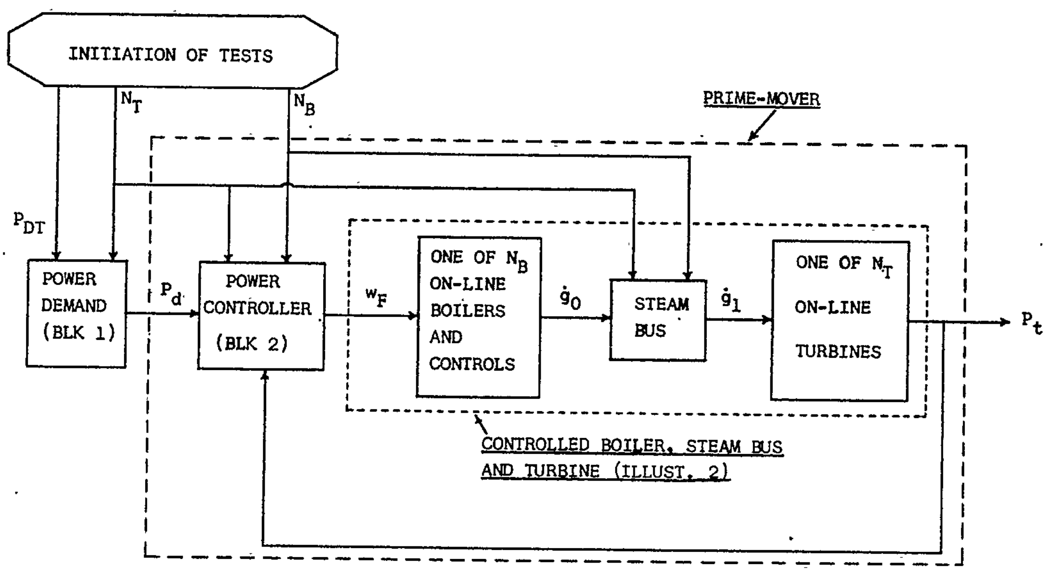
(a) After having designed the prime-mover controllers, the detailed model of the prime-mover and its controls was no longer required since turbine power was the only variable of interest to the rest of the plant.

(b) In order to satisfy the goals of the study, the model had to be representative of typical boilers and turbines for the given tests but did not have to be an accurate representation of a given prime-mover. Moreover the auxiliary system, being an order of magnitude faster-responding, would be capable of coping with performance variations between different prime-movers. Therefore, the required accuracy of reduction was less severe for the prime-mover than it would have been for other components of the plant.

(c) A significant reduction of computational effort was potentially feasible since the order of the prime-mover (including its controls) was fourteen and its implementation occupied approximately two-thirds of the analog computing facility.

ILLUSTRATION 1

Prime-Mover for Identical Boilers and Turbines



IV. METHOD OF REDUCTION

The power demand module and prime-mover were substituted in the simulation by an equivalent power demand module and a reduced prime-mover model (Illustrations 5 and 6). The reduction in Illustration 5 assumed that the principle of superposition was applicable. This assumption introduced inaccuracies since the prime-mover was nonlinear. However, the approach taken was justified by these considerations:

(1) The principle of superposition needed to be invoked only if the two outputs, ΔP_{r1} and ΔP_{r2}, were non-zero simultaneously. But using the principle of superposition introduced negligible errors in the less ideal situation wherein one of the two outputs was negligibly small compared with the other in terms of plant performance. The next two points serve to verify that this situation prevailed for all tests.

(2) The auxiliary-system level controller was purposely designed to be slow-acting so as not to interfere with the transient performance of the plant or to introduce significant transients of its own.

(3) Because the generator was much faster-responding than the prime-mover, the shaft speed was regulated almost exclusively by the auxiliary system. Shaft-speed transients were of such high frequency for the four tests that the prime-mover was relatively insensitive to them. The govenor signal, P_g, was nevertheless fed back to the prime-mover in case of possible non-zero steady-state variations in shaft speed.

(4) The topology in Illustration 5 was desirable since the prime-mover output was well-defined for the four tests while the auxiliary-system level and governor feedback signals were unknown in advance, except qualitatively as explained in (2) and (3).

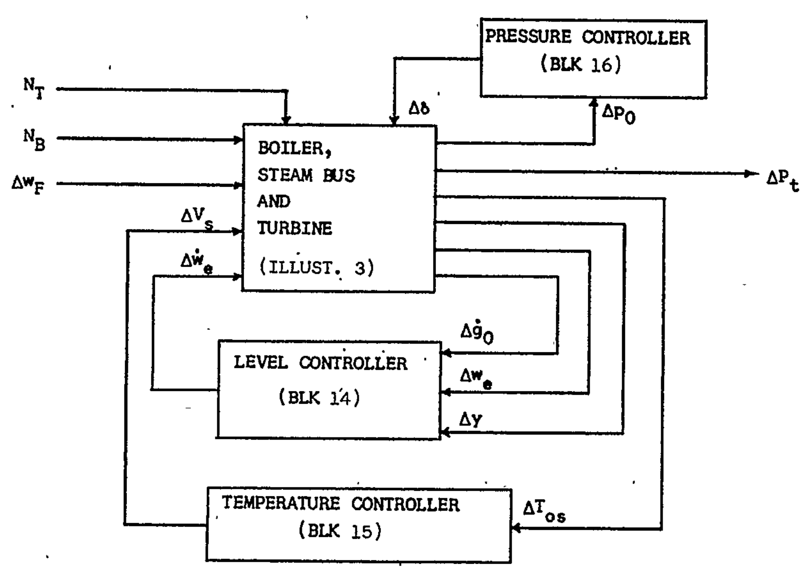
(5) As described in (b) of 'Problem Definition', it was not necessary to have an accurate model of the prime-mover.

The method of reduction for the 'reduced, prime-mover for tests 1 to 4' (Illustration 5) is shown in Illustration 7. The parameters of the reduced model were adjusted by comparing the outputs of both models until they were sufficiently similar according to a subjective performance criterion based on item (b) of 'Problem Definition'. As verified by a separate exercise using a parameter-optimizing routine [2], it turns out that this qualitative approach was roughly equivalent to minimizing an integral of squared-error criterion over the first 20 seconds of response. The structure of the reduced model consisting of a first-order equation in parallel with a second-order was deduced from the shape of the responses of ΔP_t (Illustration 8).

In keeping with consideration (4), a frequency-response test was used to reduce the prime-mover for governor and auxiliary-

ILLUSTRATION 2

Controlled Boiler, Steam Bus and Turbine



system level control purposes (Illustration 5).

V. RESULTS

The output errors, $\Delta P_t - \Delta P_{r1}$, of the 'reduced prime-mover for tests 1 to 4' appear in Illustration 8 along with the actual output, ΔP_t. These results were considered satisfactory as explained in 'Method of Reduction'.

The reduced model for the increase test was also used as the reduced model for governor and auxiliary-system level control. The frequency response of both the reduced and complete models is given in Illustration 9. A better reduction would have been possible but this one was retained as adequate for the indended application based on considerations (2) and (3) of 'Method of Reduction'. Indeed, it was subsequently verified that the natural frequency of the variations in P_h was 0.0022 Hz; therefore, the variations in P_h were slow enough to be in the unity-gain region of both frequency-response curves. On the other hand, the variations in P_g were so fast (0.96 Hz) that their attenuation (by a factor of 0.04, Illustration 9) resulted in a contribution to P_s which was negligible compared with the corresponding contribution coming from the auxiliary system (Illustration 6). In the turbine-dropping test, for example, the latter contribution was 12.3 times larger than the former; thus, a 10% error in prime-mover output would be seen as a 0.813% error in net governor-action contribution to P_s.

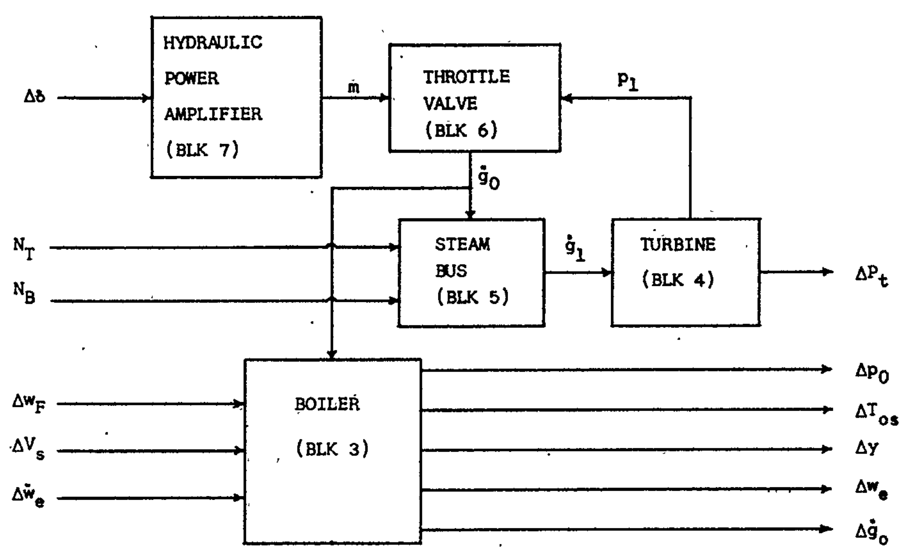
The order of the prime-mover model was reduced from fourteen to six and about one-third of the analog facility was saved.

VI. CONCLUSION

An example of how the particulars of a problem can be exploited in reducing the order of a nonlinear system was presented. Specifically, the order of the prime-mover model of a power plant was reduced using knowledge of four specific tests and of the nature of the pertinent variables fed back to the prime-mover.

ILLUSTRATION 3

Boiler, Steam Bus and Turbine



VII. BIBLIOGRAPHY

1. Chen, C. F. "Model reduction of multivariable control systems by means of matrix continued fractions," Proc. IFAC 5'th World Congress, Paris, June 1972.

2. Birta, L. G. "Parameter optimization in dynamic systems via hybrid computation," AICA-IFIP Conference on Hybrid Computation, Munich, August 1970.

VIII. APPENDIX

The equations are subdivided into subsets, called 'blocks', which are referred to in the illustrations.

BLOCK 1: MODEL OF POWER DEMAND

$$(1 + 0.1D)\Delta P_d = \frac{1}{N_T}(P_{DT} - P_{DT}^*) \quad \text{(A1)}$$

where

$$P_{DT} = P_{DT}^* + \Delta P_{DT} \quad \text{(A2)}$$

$$\Delta P_{DT} = A\, u(t) \quad \text{(A3)}$$

$A = 4.5$ for increase test (A4)
$= -4.5$ for decrease test (A5)
$= 0$ for boiler- and turbine-dropping tests (A6)

$$P_{DT}^* = 45 \quad \text{(A7)}$$

BLOCK 2: MODEL OF POWER CONTROLLER

$$\Delta w_F = 0.335\left(\frac{N_T}{N_B}\right)\left(1 + \frac{1}{19.52D}\right)(\Delta P_d - \Delta P_t) \quad \text{(A8)}$$

BLOCK 3: MODEL OF BOILER

$$\dot{x}_b = A_b x_b + B_b u_b \quad \text{(A9)}$$

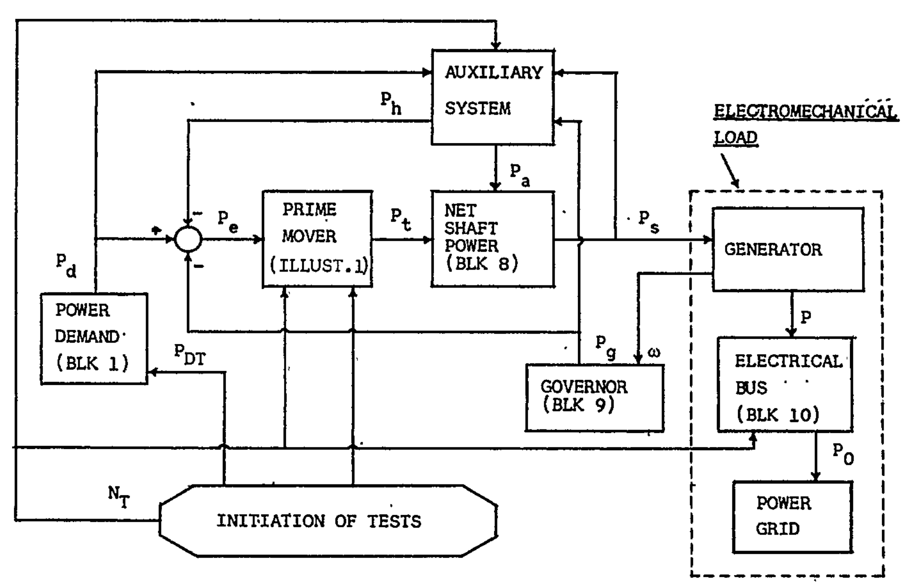
where

$$x_b = [\Delta\rho_o\ \Delta T_o\ \Delta x\ \Delta y]^T \quad \text{(A10)}$$

$$u_b = [\Delta\dot{g}_o\ \Delta w_e\ \Delta\dot{w}_e\ \Delta w_F]^T \quad \text{(A11)}$$

ILLUSTRATION 4

Power Plant for Identical Boilers, Turbines and Generators



$$A_b = \left[\begin{array}{c|c|c|c} -9.61\times10^{-3} & 3.98\times10^{-6} & 3.16\times10^{-1} & 0 \\ 7.08\times10^{-2} & -8.25\times10^{-3} & -1.61 & 0 \\ 3.08\times10^{-4} & 2.04\times10^{-8} & -9.71\times10^{-2} & 0 \\ 1.66\times10^{-2} & 2.33\times10^{-6} & -3.41 & 0 \end{array}\right] \quad \text{(A12)}$$

$$B_b = \begin{bmatrix} -2.34\times10^{-4} & -1.30\times10^{-5} & -1.02\times10^{-7} & 6.36\times10^{-3} \\ -7.57\times10^{-2} & 2.81\times10^{-4} & 1.94\times10^{-6} & 1.36 \\ 3.71\times10^{-5} & -2.15\times10^{-5} & -1.40\times10^{-7} & -9.76\times10^{-4} \\ 1.21\times10^{-3} & -4.89\times10^{-4} & -4.53\times10^{-6} & -3.71\times10^{-2} \end{bmatrix} \quad \text{(A13)}$$

$$\Delta\dot{g}_o = \dot{g}_o - \dot{g}_o^* \quad \text{(A14)}$$

$$\Delta P_o = 747\ \Delta\rho_o \quad \text{(A15)}$$

$$(1 + 20D)\ \Delta T_{oe} = 1.853\ \Delta V_s \quad \text{(A16)}$$

$$\Delta T_{os} = \Delta T_o + \Delta T_{oe} \quad \text{(A17)}$$

$$\Delta w_e = \int_o^t \Delta\dot{w}_e(\tau)d\tau + \Delta w_e(o) \quad \text{(A18)}$$

$$\dot{g}_o^* = 18.38 \quad \text{(A19)}$$

BLOCK 4: MODEL OF TURBINE

$$\left[\frac{44300}{(804+0.0763P_1)^2}\right] DP_1 + \left[\frac{3.52}{(804+0.0763P_1)^{1/2}}\right] P_1 = \dot{g}_1 \quad \text{(A20)}$$

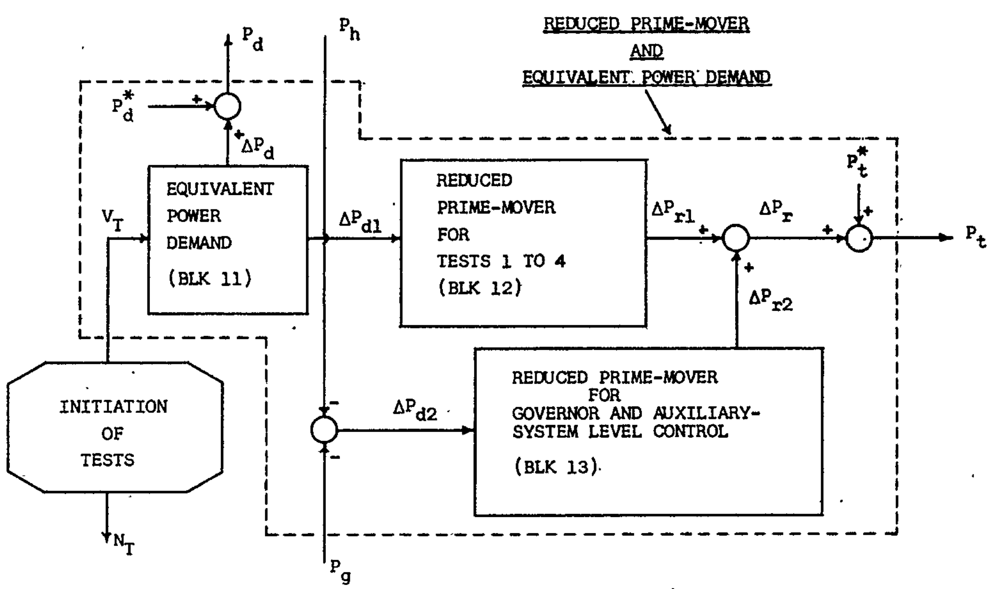
$$P_t = \frac{1.44P_1}{(804+0.0763P_1)^{1/2}} \quad \text{(A21)}$$

$$\Delta P_t = P_t - P_t^* \quad \text{(A22)}$$

$$P_t^* = 15 \quad \text{(A23)}$$

ILLUSTRATION 5

Reduced Prime-Mover and Equivalent Power Demand



BLOCK 5: MODEL OF STEAM BUS

$$\dot{g}_1 = \left(\frac{N_B}{N_T}\right) \dot{g}_o \qquad (A24)$$

BLOCK 6: MODEL OF THROTTLE VALVE

$$\dot{g}_o = 0.181m(4.11x10^5 - P_1^2)^{1/2} \qquad (A25)$$

$$0 \le m \le 1 \qquad (A26)$$

BLOCK 7: MODEL OF HYDRAULIC POWER AMPLIFIER

$$(D^2 + 15.32D + 53.3)\ \Delta m = 70.8\Delta\delta \qquad (A27)$$

$$-0.1348 \le \Delta\delta \le 0.6182 \qquad (A28)$$

$$m = m^* + \Delta m \qquad (A29)$$

$$m^* = 0.179 \qquad (A30)$$

BLOCK 8: MODEL OF NET SHAFT POWER

$$P_s = P_t + P_a \qquad (A31)$$

BLOCK 9: MODEL OF GOVERNOR

$$\Delta P_g = 5.1\Delta\omega \qquad (A32)$$

BLOCK 10: MODEL OF ELECTRICAL BUS

$$P_o = N_T P \qquad (A33)$$

BLOCK 11: MODEL OF EQUIVALENT POWER DEMAND

$$(1 + 0.1D)\Delta P_{d1} = V_T \qquad (A34)$$

where

$$V_T = 1.5\ u(t) \text{ for increase test} \qquad (A35)$$

$$= -1.5\ u(t) \text{ for decrease test} \qquad (A36)$$

$$= 7.5\ u(t-t_B) \text{ for boiler-dropping test} \qquad (A37)$$

$$= 7.5\ u(t-t_T) \text{ for turbine-dropping test} \qquad (A38)$$

$$\Delta P_d = \Delta P_{d1} \text{ for the increase, decrease and turbine-dropping tests} \qquad (A39)$$

$$= 0 \text{ for the boiler-dropping test} \qquad (A40)$$

BLOCK 12: MODEL OF REDUCED PRIME-MOVER FOR TESTS 1 TO 4

$$(1 + T_{d1}D)\Delta P_{r1a} = K_{d1}\ \Delta P_{d1} \qquad (A41)$$

$$(D^2+2\zeta_1\omega_{n1}D+\omega_{n1}^2)\Delta P_{r1b} = K_{f1}\omega_{n1}^2\Delta P_{d1} \qquad (A42)$$

$$\Delta P_{r1} = P_{r1a} + P_{r1b} \qquad (A43)$$

The values of the coefficients for tests 1 to 4 are given in Table 1.

ILLUSTRATION 6

Reduced Power Plant for Identical Boilers, Turbines and Generators

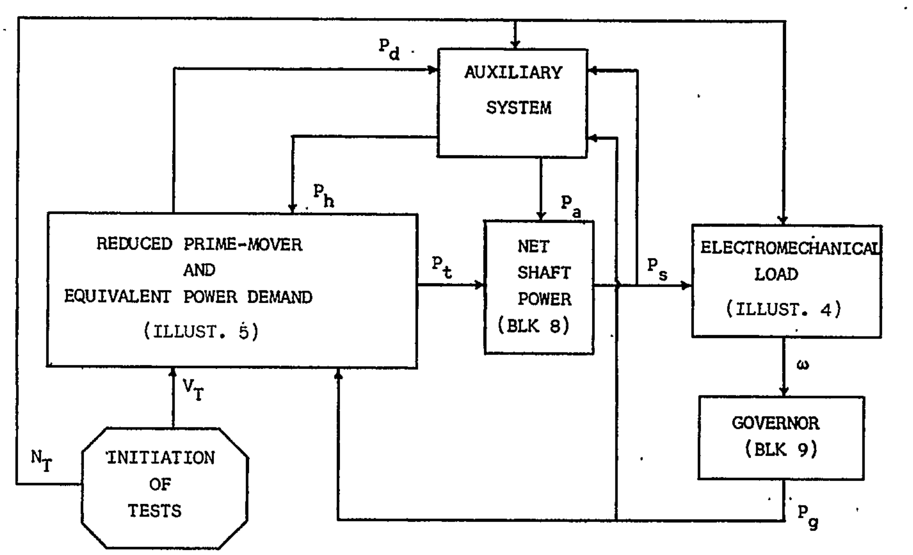


TABLE 1

Coefficient Values for Tests 1 to 4

TEST	K_{d1}	T_{d1}	K_{f1}	ζ_1	ω_{n1}
1. Increase	0.3666	3.205	0.6334	7.589	3.486
2. Decrease	0.3666	3.205	0.6334	7.589	3.486
3. Boiler-dropping	-0.2754	0.345	0.2754	7.589	3.486
4. Turbine-dropping	0.7070	0.345	0.2930	7.589	3.486

BLOCK 13: MODEL OF REDUCED PRIME-MOVER FOR GOVERNOR AND AUXILIARY-SYSTEM LEVEL CONTROL

$$(1 + 3.205D)\Delta P_{r2a} = 0.3666\ \Delta P_{d2} \tag{A44}$$

$$(D^2+52.91D+12.15)\Delta P_{r2b} = 7.697\ \Delta P_{d2} \tag{A45}$$

$$\Delta P_{r2} = \Delta P_{r2a} + \Delta P_{r2b} \tag{A46}$$

BLOCK 14: MODEL OF STEAM-DRUM WATER-LEVEL CONTROLLER

$$\Delta \dot{w}_e = 0.1\Delta \dot{g}_o - 10.1\Delta w_e - (450+ \frac{2.7}{D})\Delta y \tag{A47}$$

BLOCK 15: MODEL OF STEAM-TEMPERATURE CONTROLLER

$$D\Delta V_s = 0.05(D+1)\Delta T_{os} \tag{A48}$$

BLOCK 16: MODEL OF STEAM-PRESSURE CONTROLLER

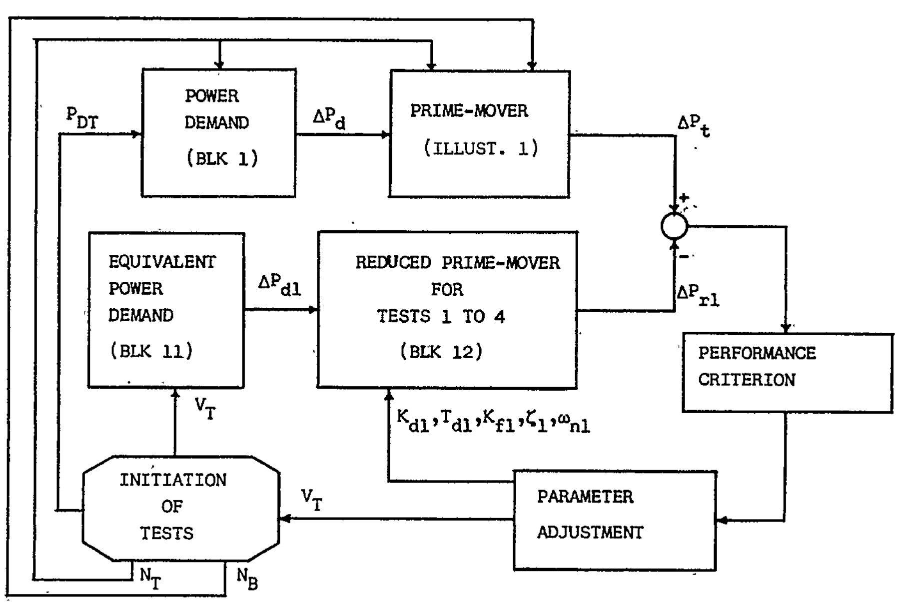
$$(D^2 + 0.578D)\Delta\delta = (7.199D + 3.866)\Delta P_o \tag{A49}$$

IX. LIST OF SYMBOLS

Throughout the mathematical presentation, prefix 'Δ' denotes variations about the nominal (operating) value of a variable and a superscripted asterisk denotes the nominal value. The symbols are, however,

ILLUSTRATION 7

Method of Reduction for Tests 1 to 4



defined without these modifiers. A 'dot' above a variable denotes the derivative with respect to time.

D: Time-derivative operator.

u(t): Unit step function.

t: Time.

POWER DEMAND

P_{DT}: Net power required from all on-line turbines.

P_D: Net power required from all on-line turbines by the power-demand module.

P_d: Power required from each on-line turbine by the power-demand module.

P_{d1}: Power required from each on-line turbine by the equivalent power-demand module.

BOILER

N_B: Number of on-line boilers.

t_B: Value of time when a boiler was dropped.

ρ_o: Density of steam at superheater output.

T_o: Portion of output steam temperature attributable to electrical heating.

x: Quality of mixture at riser output.

y: Water level in steam drum.

$\dot{g}_o$: Mass flow-rate of superheater steam.

w_e: Mass flow-rate of economiser water.

w_F: Mass flow-rate of fuel.

P_o: Steam pressure at superheater output.

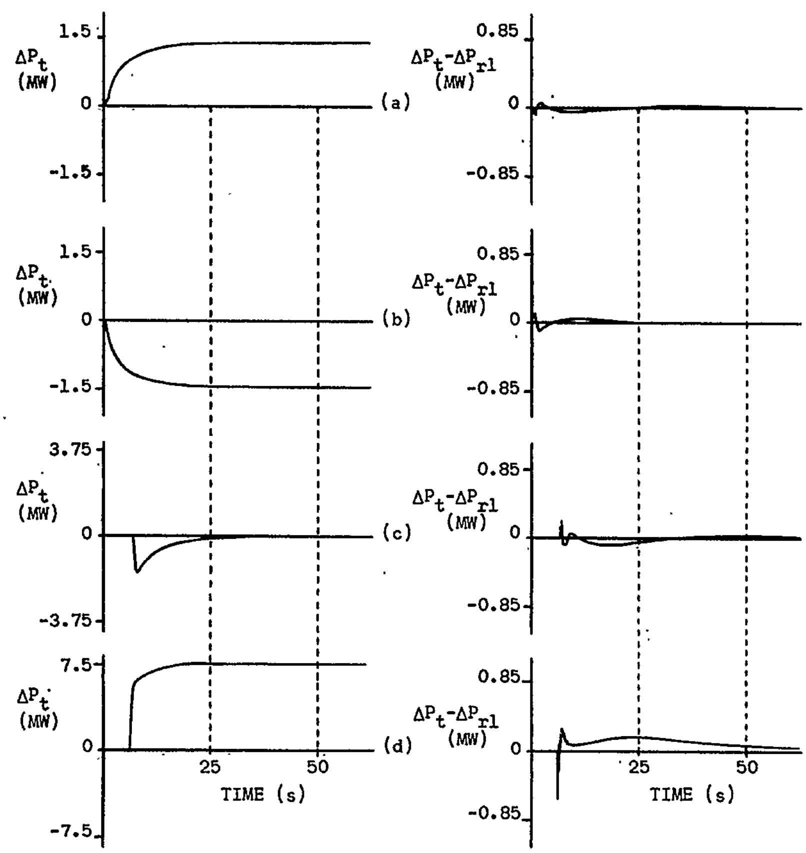
T_{oe}: Portion of output steam temperature attributable to electrical heating.

T_{os}: Temperature of steam at superheater output.

V_s: Manipulated variable to control steam temperature.

ILLUSTRATION 8

Output Errors in the 'Reduced Prime-Mover for Tests 1 to 4': (a) Increase Test, (b) Decrease Test, (c) Boiler-Dropping Test, and (d) Turbine-Dropping Test



TURBINE, THROTTLE VALVE AND HYDRAULIC POWER AMPLIFIER

N_T: Number of on-line turbines.

t_T: Value of time when a turbine was dropped.

P_1: Steam pressure at input to turbine.

$\dot{g}_1$: Steam flow-rate into turbine.

P_t: Power output of turbine.

m: Throttle-valve stem position.

δ: Input displacement of mechanical linkage of hydraulic power amplifier.

SHAFT, GENERATOR, ELCTRICAL BUS AND GOVERNOR

P_s: Net shaft power available to generator.

P: Real power delivered by each generator.

P_o: Total real power delivered by the power plant.

ω: Angular velocity of turbogenerator shaft.

P_g: Power required due to governor action.

AUXILIARY SYSTEM

P_a: Power delivered to turbogenerator shaft by auxiliary system.

P_h: Power required to regulate stored energy in auxiliary system.

ILLUSTRATION 9

Results of Reduction for Governor and Auxiliary-System Level Control