A TWO ASSET CASH FLOW SIMULATION MODEL

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Abstract

We present an *APL PLUS* simulation model of a two asset cash management system. The two assets are cash and an income producing portfolio. The model relates cash transfer policies and futures stocks of cash and income producing assets to restrictions on average and minimum balances, projections of future cash flows, the costs of cash management, and user specified objectives.

I. Introduction

An organization's cash management system acts as a buffer between the organization's internal cash needs and its external financial relations with customers, suppliers, tax collecting agencies, and banks. Three frequently cited reasons for holding cash are to ease anticipated disbursements, to act as a reserve for unexpected cash demands, and to compensate banks for their services. The most important cost of holding cash is the opportunity lost by not using the cash in some alternate way. For example, cash could be used to pay existing loans, as a dividend to shareholders, as a short term external investment, or as an internal capital investment. The opportunity cost for holding cash is the loss incurred by not using the cash in some optimal alternative manner.

Cash flow management involves scheduling the timing and size of cash transfers in order to meet the organization's needs without carrying excessive amounts of cash in nonproductive accounts. The two asset cash management model presented in this paper is based on a forecast of future cash inflows, institutional restrictions on cash stocks, the costs of holding and transferring cash, and a user selected objective. The model is not unnecessarily complicated and includes many essential aspects of the cash management problem. The simplicity and interactive nature of the computer model make it an ideal simulation tool for decision makers. With practice, the input and output of the model can be quickly analyzed. This interactive feature allows the decision maker to simulate the consequences of changes in cash flow forecasts, opportunity costs, compensating balance agreements, and cash management objectives.

Most large organizations have a choice of several income producing assets that are alternatives to holding cash. These assets differ in maturity, risk, and yield. The cash management model presented in this paper aggregates these income producing assets into a single income producing portfolio. The two assets, cash and the income producing portfolio, are similar to a household's checking and savings account. If a firm is constantly in a borrowing position, then the income producing asset can be viewed as a loan portfolio. The two asset assumption leads to considerable simplifcation, and can be considered as a first approximation to the multiple asset model. Cases in which the two asset assumption is inappropriate, will be discussed in the summary.

Similar cash management models have been considered in references (1-3). The models in (2-3) lead to large linear programs that are not suitable for flexible, interactive simulation. The model in (1), is based on the special structure of optimal policies in a dynamic program which includes fixed transaction costs but does not examine average balance constraints.

The next section describes the *APL* simulation package, outlines the reasons for using *APL* as a simulation language, and suggests several uses for the model. Section III formulates the cash management problem and describes several alternative cash management objectives. The fourth section contains an example of marginal analysis under the special objective of minimizing the total cost of cash management. A similar marginal analysis can be developed for alternative objectives. Section V presents some examples, and indicates how optimal policies change in response to changes in external system parameters. The sixth and final section summarizes the results.

II. *APL* Simulation

The simulation is based on an interactive system of *APL*/360 programs. These programs:

(a) Examine a set of globally defined variables.

(b) Construct a linear programming tableau appropriate to the length of the planning horizon.

(c) Insert coefficients that are computed from the global variables, (a), and the underlying cash flow conservation equations.

(d) Include user imposed constraints, such as compensating balances and/or a lower bound on the cash level.

(e) Calculate the optimal timing and size of cash transfers, and the marginal values of changes in exogenous cash flows, compensating balance levels and lower bounds on the cash level.

(f) Organize the output in a graphical or tabular display as specified by the user.

APL was selected as the program language for two reasons. First, the interactive aspect of the language allows users to employ the programs frequently and within the short timespan available for making cash management decisions. Second, the compact, array oriented mathematical structure of *APL* is ideally suited for constructing the appropriate tableau, for solving the optimization problem, and for quickly modifying the model's assumptions and parameters through user defined global variables.

The simulation model is built around a linear program that calculates optimal cash transfers and stock levels. The interactive computer program allows a decision maker to simulate the impact of changes in predicted cash flows, cash management objectives, institutional constraints, and the length of the planning horizon on cash transfer decisions and on future stocks of cash and the income producing asset. The program package can be used in several ways.

(a) To find feasible cash flow transfer schedules that will meet the organization's commitments. This scheduling problem can be considered in the long run (monthly periods), in the short run (daily periods), or over a sequence of unequal periods (days, weeks, months, quarters, years).

(b) To estimate the costs of cash management, and to project the future stocks of cash and the income producing asset.

(c) To determine the costs of institutional arrangements such as compensating or minimum balance agreements.

(d) To examine the impact of smoothing cash flows.

(e) To forecast periods in which short-term borrowing will reduce costs.

(f) To identify unforeseen investment possibilities.

(g) To test the effect of alternate cash management objectives on cash management policy.

(h) To calculate the cash transfers and stock levels of cash and the income producing asset that result when the worst possible cash inflow occurs.

(i) To gauge the impact of alternate cash flow forecasts on costs and cash transfer decisions.

(j) To discover a suitable planning horizon for cash management planning.

III. Formulation

This section formulates the cash flow model and indicates some of the features that the simulation program can accommodate.

The fundamental cash flow conservation equation

(1) $X[J] = X[J-1] + S[J] + U[J] - V[J]$ for $J = 1, 2, \ldots, T$

with $X[0] = XQ$ a user supplied global variable.

Period J is an interval of time from instant J-1 to instant J. The variables in (1) are defined as follows: $X[J-1]$ is the cash level at the end of period J-1 ; $U[J] - V[J]$ is the net amount of cash transferred into, $U[J] > 0$ for an inflow and $V[J] > 0$ for outgo, the cash account at the beginning of period J, (at time J-1). $S[J]$ is the net inflow of cash during period J and $X[J]$ is the cash level at the end of period J, (at time J). The initial cash level $X[0]$, and the exogeneous cash inflows $S[J]$ are known. If a lower bound determined by the global variable $L\underline{X}$, is placed or the cash account then equation (1) continues to hold although $X[J]$ in (1) must be interpreted as the cash level in excess of the lower bound; i.e. the actual cash level minus $L\underline{X}$. A compensating balance requirement is described by a constraint on the total cash holding.

(2) $X[1] + X[2] + \ldots + X[T] \geq (T \times (A\underline{X} - L\underline{X}))$

Here $A\underline{X}$ is a global variable describing the average level in the cash account. The simulation program ignores the average balance constraint if $A\underline{X} < L\underline{X}$.

The cash management objective is selected through a user defined global variable $O\underline{E}$. The four possibilities are:

$O\underline{E}$ ←1: Minimum sum of all future opportunity and transfer costs

←2: Minimum discounted sum of all future cash transfers

←3: Maximum end of period cash and securities

←4: Maximum present worth of future cash and securities

Other objective functions can easily be handled. The model is able to simulate the policy implications of following any specified objective.

Other features of the package allow for

unequal planning periods, and allow for multiple average balance constraints. For example, the planning period could consist of eight weekly periods followed by four monthly periods. If the organization is constrained on an average monthly balance then there are a total of six balance constraints. The first involves $X[1]$ through $X[4]$, the second $X[5]$ through $X[8]$ and the last four simply state that $X[J] \geq AX$ for $J \geq 9$. The complete list of user supplied global variables is described below.

- XQ - initial cash stocks
- YQ - initial stock of income-producing assets
- CX - variable unit "opportunity" cost of holding cash
- CU - variable unit cost of transferring assets to cash
- CV - variable unit cost of transferring cash to assets
- AX - the average cash level or compensating balance
- LX - lower bound for cash stocks
- OE - an integer denoting the choice of objective function
- D - discount factor
- R - unit period interest rate
- DP - a vector giving the number of days, weeks or months in each accounting period
- BP - the number of periods that apply to a compensating balance restriction

To avoid complications we shall concentrate our analysis on the special case $DP = 1$, $OE = 1$, and BP equals the length of the planning horizon.

IV. Marginal Analysis

This section describes in detail the marginal cost information derived in the simulation when the cash management objective is the minimization of total opportunity and transfer cost. A similar analysis is possible for the other cash management objectives mentioned in Section III. The information derived in this section will be useful in interpreting the numerical examples that are presented in Section V.

The primal linear program minimizes the total opportunity and transfer cost subject to the flow equations (1) and the total balance restriction (2). The dual linear program associates variables $P[1], P[2], \ldots, P[T]$ with the flow equations and a variable Q with the balance condition. These variables must satisfy the dual feasibility conditions

$$(3) \quad \begin{array}{ll} \text{(i)} & -CU \leq P[J] \leq CV \qquad J = 1,2, \ldots, T \\ \text{(ii)} & P[J] - P[J+1] \leq CX - Q \end{array}$$

Where we assign $P[T+1] = 0$.

In addition, a primal solution (X,U,V) and dual solution (P,Q) are optimal if and only if they satisfy the complementarity conditions

(i) $-C\underline{U} < P[J] < C\underline{V} \Rightarrow V[J] = U[J] = 0$

$$V[J] > 0 \Rightarrow P[J] = C\underline{V}$$

(4) $$U[J] > 0 \Rightarrow P[J] = -C\underline{U}$$

(ii) $P[J] - P[J+1] < C\underline{X} - Q \Rightarrow X[J] = 0$

$$X[J] > 0 \Rightarrow P[J] - P[J+1] = C\underline{X} - Q$$

The variable $P[J]$ is the rate of change of the minimal cost with respect to changes in $S[J]$. With this interpretation of P in mind it is easy to derive (3) and (4). Increase $S[J]$ by a small amount E. This change can be offset by a corresponding increase in $V[J]$. The increase in cost is $E \times C\underline{V}$ which must be larger than the increase in the minimal cost $E \times P[J]$. Since $E > 0$, it follows that $C\underline{V} \geq P[J]$. If, $V[J] > 0$ then the argument above could be repeated with E negative. Then $V[J] > 0$ could be decreased. The increase in cost remains $E \times C\underline{V} \geq E \times P[J]$. However, since $E < 0$, it follows that $C\underline{V} \leq P[J]$. Therefore $V[J] > 0$ implies $C\underline{V} = P[J]$.

A similar argument holds for $U[J]$. If $S[J]$ decreases by $E > 0$ then this decrease can be offset by a corresponding increase of E in $U[J]$. The increase in cost is $E \times C\underline{U} \geq -E \times P[J]$. Therefore $C\underline{U} \geq -P[J]$. However when $U[J] > 0$, the same argument can be repeated with $E < 0$. It follows that $U[J] > 0$ implies $C\underline{U} = -P[J]$.

The variable Q measures the rate of increase in optimal cost per unit of increase in the total balance constraint (2). Suppose, that $S[J]$ is increased by E, $S[J+1]$ is decreased by E, and the total balance requirement is increased by E. If $E > 0$ this change can be offset by increasing $X[J]$ by E. The increase in cost is $E \times C\underline{X}$ which must exceed the increase in the optimal cost $E \times (P[J] + Q - P[J+1])$. Division by E yields (3ii). If $X[J] > 0$, the same argument holds with $E < 0$. Therefore, $X[J] > 0$ implies $C\underline{X} - Q = P[J] - P[J+1]$.

It is useful to interpret $C\underline{X} - Q$ as an opportunity cost in a related optimization problem that obeys the flow conservation constraints (1) but ignores the total balance constraint (2). If (X,U,V) solve the original cost minimization problem subject to (1) and (2), then (X,U,V) will solve the related problem with opportunity cost $C\underline{X} - Q$ and no total balance constraint. In this way, Q acts as an incentive for holding cash. If the incentive is just right then the opportunity loss will be reduced so that the optimal program automatically satisfies the total balance restriction.

V. Examples

Several simulations are shown below. The data and length of the planning horizon were selected to illustrate the output and to ease interpretation of the results. In the output format, the first row numbers the time periods, the second row contains the cash inflows (S), the third the asset to cash transfers U, the fourth the cash to asset transfers V, the fifth the cash levels X, the sixth the level of the interest earning account, the seventh the costs incurred in each period, and the eighth and final row contains the dual variables P. The final column contains averages. To obtain the total cost, the average cost should be multiplied by the number of periods. The number in the last column, row eight is Q, the increase in optimal cost of increasing the *total* cash holdings by one unit. In addition, each of the three examples is preceded by a data statement that gives the relevant values of the global variables.

In each example the following global variables remain constant.

XQ ↔ 5000   YQ ↔ 20000   CU ↔ .03

CU ↔ .04   R ↔ .1   OE ↔ 1

BP ↔ 6   DP ↔ 1

The exogenous cash flow S remains the same in each example.

S ↔ ¯20000 8000 5000 ¯15000 10000 20000

In the first example, LX ↔ 0 , AX ↔ 5000, and CX ↔ .1 . The result is shown in the tableau at the top of the next page. Note that P[2] = .04 and P[4] = ¯.03 . This implies total costs could be reduced by delaying one unit of input from period 2 to period 4. In contrast, costs are increased if inflow is delayed from period 1 to period 2. Note that $Q = .06 < .1 = CX$. Raising the total balance constraint will increase the holding of cash in some period. However, the cost of this is less than the opportunity cost since the timing of additional holdings can be selected to save on transactions costs.

The second tableau sets AX ↔ 0 , and LX ↔ 5000 . The result average balance is 5000 , however, costs increase due to a loss of flexibility. The optimal policy matches the inflow with the transfers into the asset account.

In the third example LX ↔ 0 , and AX was increased to 10000 . The value of Q in the first tableau indicates that average costs should increase by $5000 \times .06 = 300$. The increase is actually 490 . Note that the value of Q increases to .08 . This measure is a loss in flexibility due to the increase in the total balance constraint.

With the LX ↔ 0 , and the compensating balance constraint AX ↔ 5000 , the cost CX was reduced to .05 . This resulted in no policy change over Tableau 1 although Q dropped to .02 .

EPOPTDATA

XQ	YQ	LX	AX	CX	CU	CV	R
5000.000	20000.000	0.000	5000.000	0.100	0.030	0.040	0.100

PRINT EPOPT S

TIME PERIOD	1	2	3	4	5	6	AVG
NET CASH FLOW	¯20000.00	8000.00	5000.00	¯15000.00	10000.00	20000.00	1333.33
ASSETS TO CASH	15000.00	0.00	0.00	7500.00	0.00	0.00	3750.00
CASH TO ASSETS	0.00	5500.00	0.00	0.00	10000.00	0.00	2583.33
CASH STOCKS	0.00	2500.00	7500.00	0.00	0.00	20000.00	5000.00
ASSET STOCKS	7000.00	13200.00	14520.00	8472.00	19319.20	21251.12	13960.39
CASH COSTS	450.00	470.00	750.00	225.00	400.00	2000.00	715.83
MARGINAL COST	¯0.03	0.04	0.00	¯0.03	0.04	0.03	0.06

EPOPTDATA

XQ	YQ	LX	CX	CU	CV	R
5000.000	20000.00	5000.000	0.100	0.030	0.040	0.100

PRINT EPOPT S

TIME PERIOD	1	2	3	4	5	6	AVG
NET CASH FLOW	¯20000.00	8000.00	5000.00	¯15000.00	10000.00	20000.00	1333.33
ASSETS TO CASH	20000.00	0.00	0.00	15000.00	0.00	0.00	5833.33
CASH TO ASSETS	0.00	8000.00	5000.00	0.00	10000.00	20000.00	7166.67
CASH STOCKS	5000.00	5000.00	5000.00	5000.00	5000.00	5000.00	5000.00
ASSET STOCKS	2000.00	10200.00	16220.00	2842.00	13126.20	34438.82	13137.84
CASH COSTS	1100.00	820.00	700.00	950.00	900.00	1300.00	961.67
MARGINAL COST	¯0.03	0.04	0.04	¯0.03	0.04	0.04	0.00

EPOPTDATA

XQ	YQ	LX	AX	CX	CU	CV	R
5000.000	20000.00	0.000	10000.000	0.100	0.030	0.040	0.100

PRINT EPOPT S

TIME PERIOD	1	2	3	4	5	6	AVG
NET CASH FLOW	¯20000.00	8000.00	5000.00	¯15000.00	10000.00	20000.00	1333.33
ASSETS TO CASH	15000.00	0.00	0.00	2000.00	0.00	0.00	2833.33
CASH TO ASSETS	0.00	0.00	0.00	0.00	500.00	0.00	83.33
CASH STOCKS	0.00	8000.00	13000.00	0.00	9500.00	29500.00	10000.00
ASSET STOCKS	7000.00	7700.00	8470.00	7317.00	8548.70	9403.57	8073.21
CASH COSTS	450.00	800.00	1300.00	60.00	970.00	2950 00	1088.33
MARGINAL COST	¯0.03	0.01	¯0.01	¯0.03	0.04	0.02	0.08

VI. Summary

We have described a two asset cash flow simulation model. Given a sequence of future cash inflows, institutional restrictions, and a cash management objective the model projects future cash transfers, the costs of these transfers and future cash and income-producing asset stocks.

The model assumes a deterministic inflow of cash. In many cash management problems with planning horizons of one year or less and planning periods of one week or more, the random component of cash inflow is relatively small. Cash flows that occur in the relatively near future are predictable in magnitude. By concentrating on the nonstationary deterministic component of cash flows, the model schedules the size and timing of major cash transfers in order to meet institutional requirements and optimize cash management objectives.

The assumption of a single income producing asset is a useful first approximation to the multiple asset case. In particular, if various assets have larger yields for longer holding periods, then the cash transfers and asset levels calculated by the simulation program can be used as inputs to a multiple asset scheduling subproblem.

Several uses for the simulation package were outlined in Section II. There is another, perhaps more important, use of the model. The flexibility and interactive nature of the model gives the user a feel for the cash flow process and the interaction of different policy variables. The decision maker, by using such a simulation model, learns how to analyze cash measurement policies. In addition to the greater confidence which usually results from the use of such interactive models the user should also be able to detect the sensitivity of new policies to various institutional assumptions and management objectives. Hopefully this may lead to recommendations as to how the cash management process may be better organized and controlled.