ACCOUNTING RATE OF RETURN
VS. TRUE RATE OF RETURN
CONSIDERING VARIABILITY AND UNCERTAINTY

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Abstract

An accounting rate of return and a defined true rate of return were assessed for a simulated firm composed of independent long-lived investment projects. The parameters of each individual investment project were determined by a Monte Carlo simulation technique. Differing degrees of environmental variability and uncertainty were represented by the simulation techniques used. Accounting rate of return, defined consistent with contemporary accounting practice, and a true rate of return, defined in economic terms, were contrasted. The efficacy of accounting rate of return as a surrogate for true rate of return was found to be a function of the degree of variability and uncertainty represented in the environment.

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The relationship between accounting rate of return (ARR) and true yield of a firm has been the subject of considerable research[1]. ARR has generally been defined consistent with accounting practice. True yield in such studies has been an economic concept. Previous researchers have contrasted accounting and economic measures in an almost endless variety of situations--and the accounting measures haven't always faired very well.

This paper starts with the conclusion that accounting measures of return are imprecise surrogates for the related economic concepts with which they are apparently to correspond. Such a conclusion, while disturbing to some, need not detract from the usefulness of accounting measures. It may be that period to period changes in accounting measures of return correspond closely with interperiod changes in true yield. The relationship between changes in ARR and changes in true yield is the subject of this paper.

Let us state the approach taken by means of an analogy. With a crude thermometer, we would not expect to accurately measure temperature, or even to record minor changes in temperature; but we would expect to be capable of identifying major changes in temperature. The precision of such a thermometer could be assessed by determining how violent temperature changes must be before they are capable of being recorded by the measuring instrument. In this paper, accounting and economic measures of return for a simulated firm, operating in a variable and uncertain environment, are defined and measured. Accounting measures can then be assessed in terms of the magnitude of actual underlying changes that are necessary to have a corresponding impact on the accounts.

In the next section of this paper, the model used to represent an enterprise, and the accounting and true yield measures are described. Particular emphasis is given to the means by which inter-year changes in the economic fortunes of the enterprise are induced. In the results section, the correlation between changes in the measured true yield and the measured ARR, as a function of inter-year variability, is presented. Finally, a macro-sensitivity analysis is presented to provide some indication of the generality of the results obtained.

The Model

As in most previous research, a firm will be envisioned as a collection of investment projects.[2] We will begin our description of the firm by describing an example project. This will be followed by a discussion of the generation and aggregation of projects.

An Example Project

It will help in describing projects to think of those aspects of a project which are known by management at the time it is undertaken, and those aspects which will be known at a later date. Consider the following example. At the time the investment project is undertaken, it is known that the investment outlay is $142,879 in then current dollars. The price index stands at 286. Management estimates the project will have a life of 8 years, after which time the salvage value will be $2,858. Straight line depreciation will be used for financial reporting purposes and sum-of-the-years-digits depreciation will be used for tax purposes. After the fact, the following information is known. The project actually lasted 10 years, at which time the salvage value was $8,200 but the price index stood at 410. The income tax rate was 50% throughout the project's life. The cash flow, expressed in real dollars on a before tax basis, generated from the investment each year and the associated price index is shown below.

Year	Real Dollar Before Tax Cash Flow	Price Index
1	$7549	297
2	6821	298
3	8749	298
4	6689	308
5	7842	322
6	6652	337
7	7010	356
8	8732	377
9	6497	389
10	8223	410

Real dollars refer to dollars with the purchasing power of an arbitrarily selected base year--the price index for that base year is 100. Given the <u>ex ante</u> and the <u>ex post</u> information, Tables 1, 2, and 3 can be prepared. Table 1 indicates what impact this project will have on accounting statements during its 10-year actual life and provides the very orthodox definition of ARR used in this study. Table 1 as well as Tables 2 & 3 are based on the following conventions. Investment outlays are made at the start of a year, other receipts and disbursements occur at the end of the year. Investments consist of depreciable assets only. The price-level index applies to the end of the year. Net receipts are either reinvested in other projects or distributed as dividends. With these conventions in mind, Table 2 can be seen as proof that the internal rate of return (IRR) for the example project is 4.56%. Table 3 presents earnings and asset values based on net present values using the IRR as a discount rate.[3] It presents measures of earnings and investment such that each year their ratio equals the IRR. Table 3 will be used below to define what will be called the true rate of return (TRR) for the firm.

True Rate of Return

From Table 1 and Table 3, the ARR and the IRR for this project can be compared. However, such a comparison is not very interesting. Rather, the comparison of the ARR and TRR for the firm is of interest. But if each project the firm undertakes has tables such as Table 1 and Table 3 prepared for it, the summation of appropriate table entries would provide the numerators and denominators of the two ratios of interest. This may be made clear by example. Assume the example project were undertaken in year 1961, and we are interested in the 1965 ARR and TRR for the firm as a whole. The numerator of the ARR would be the sum of appropriate entries from column (10) of tables like Table 1. The example project would contribute $3,893 to this sum. If each

TABLE 1

Calculation of Accounting Rate of Return for Example Project

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
			Reported Accounts and Ratios							
Years	Real dollar before tax cash flow	Price index (base = 100)	Current dollar before tax cash flow	Asset write-off using straight line depreciation	Asset balance-end of year	Average asset balance	Before tax profit	Provision for taxes	After tax profit	Accounting rate of return (10) ÷ (7)
1	7549	297	22444	17503	125376	134128	4942	2471	2471	1.84%
2	6821	298	20303	17503	107874	116625	2800	1400	1400	1.20%
3	8749	298	26069	17503	90371	99122	8566	4283	4283	4.32%
4	6689	308	20579	17503	72868	81620	3077	1538	1538	1.88%
5	7842	322	25288	17503	55366	64117	7785	3893	3893	6.07%
6	6652	337	22403	17503	37863	46614	4900	2450	2450	5.26%
7	7010	356	24926	17503	20360	29112	7423	3712	3712	12.75%
8	8732	377	32885	17503	2858	11609	15382	7691	7691	66.25%
9	6497	389	25270	0	2858	2858	25270	12635	12635	422.16%
10	10223	410	41896	2858	0	2858	39039	19519	19519	683.07%
				142879						

TABLE 2

Calculation of Internal Rate of Return for Example Project

Investment in Real Dollars = $142,879/2.86 = $50,000

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
			Accounts for Tax Purpose						
					Current Dollars				
Year	Real dollar before tax cash flow	Price index (base = 100)	Current dollar before tax cash flow	Asset write-off using sum-of-years digits depreciation method	Taxable income	Taxes paid	After tax cash flow	Real dollar after tax cash flow	Present value of (9) at 4.56%
1	7549	297	22444	31116	-8672	-4336	26780	9007	8614
2	6821	298	20303	27226	-6924	-3462	23765	7984	7303
3	8749	298	26069	23337	2732	1366	24703	8291	7252
4	6689	308	20579	19447	1132	566	20013	6505	5442
5	7842	322	25288	15558	9730	4865	20423	6333	5067
6	6652	337	22403	11668	10734	5367	17036	5058	3876
7	7010	356	24926	7779	17147	8573	16352	4599	3365
8	8732	377	32885	3889	28996	14498	18387	4882	3417
9	6497	389	25270	0	25270	12635	12635	3249	2174
10	10223	410	41896	2858	39039	19519	22377	5460	3495
									50000

TABLE 3

Calculation of True Rate of Return for Example Project

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Year	Real dollar after tax cash flow	True rate of return earnings .0456 x (6)	True rate of return capital recovery (2) - (3)	End of year investment $(5)_{t-1}$ - (4)	Average investment $(5)_{t-1}$	True rate of return (3) ÷ (6)
1	9007	2281	6726	43274	50000	4.56%
2	7984	1974	6010	37264	43274	4.56%
3	8291	1700	6591	30673	37264	4.56%
4	6505	1399	5106	25568	30673	4.56%
5	6333	1166	5167	20401	25568	4.56%
6	5058	931	4128	16273	20401	4.56%
7	4599	742	3856	12417	16273	4.56%
8	4882	566	4316	8101	12417	4.56%
9	3249	370	2879	5222	8101	4.56%
10	5460	238	5222	0	5222	4.56%
			50000			

project had a Table 1, all the contributions to accounting earnings would be known. Similarly, the denominator of the ARR is obtained by summing appropriate entries from column (7). The example project contributes $64,117. The numerator of the TRR is found by summing appropriate entries from column (3) of Table 3; and the denominator from column (6)[4]. The example project contributes $1,166 and $25,568 to the numerator and the denominator of the firm's 1965 TRR respectively. Thus, TRR as defined in this paper is the weighted average of the IRR's of the projects existing when the TRR is calculated. The weights used are asset values using the capital recovery method of depreciation[5].

To be more precise, let IRR_t be the internal rate of return for the project commenced in year t (this assumes 1 and only 1 project per year--an assumption relaxed later in this paper). Let $cash_{t,i}$ be the after tax, real dollar, cash flow in the i th year of the project commenced in year t. By convention, a positive value represents an inflow and a negative value (i=0) represents an outflow then, the true rate of return in year k is:

$$TRR_k = \frac{\sum_{t=1}^{k} IRR_t \sum_{i=0}^{k-t} - cash_{t,i}[1 + IRR_t]^{k-t-i}}{\sum_{t=1}^{k} \sum_{i=0}^{k-t} - cash_{t,i}[1 + IRR_t]^{k-t-i}}$$

TRR_k is totally expressable in terms of cash flows since IRR_t is the value for which

$$0 = \sum_i cash_{t,i}/[1 + IRR_t]^i$$

Another way of describing true rate of return and justifying that rather presumptuous label, is to take a brief look at the accounting problems of leasing companies. Two approaches have had widespread use in the field, the rental method and the financial method. The financial method, now in many cases required[6], is supported by analogy to economic concepts. In effect, TRR, as previously defined, is calculated using the financial method to account for non-lease investments. The financial method can only be used if all of the cash flows that will result from an investment are known. Such a requirement is approximately met in the leasing situation, and exactly met in a computer simulation. It could be met in the real world on a retrospective basis (indicating what the financial statements should have been) given adequate bookkeeping. Thus, the arguments for the validity of the TRR measure used in this paper are as strong as the arguments for the financial method of accounting for lessors. And a major problem from describing the TRR in terms of the capital recovery method of depreciation is countered. Some would argue that the capital recovery method of depreciation should use the firm's cost of capital as the discount rate; and not the IRR of the project with which the depreciable asset is associated. Such is not the case for the financial method of accounting for leases. The financial method uses the yield rate associated with each lease. Thus, the support for the financial method can be marshalled behind the TRR measure used in this study.

The Collection of Projects

Assume for the moment the existence of the simulated firm. Each year a certain after tax cash flow is generated (by convention, at the end of the year). The amount of cash to be invested (again by convention, at the start of the next year) is a random variable between 50% and 150% of the cash flow generated. If less than 100% is reinvested, it is assumed that the rest is distributed as dividends. If more than 100% is reinvested, it is assumed that the sale of common stock provided the necessary additional funds. A separate and independent (except that the firm is taxed as an entity) project is undertaken each year.

Other Model Parameters

The actual life of a project (L) is an integer random variable between 10 and 20 years. The estimated life of a project is an integer random variable between L-3 and L+3 years. Depreciation schedules for the duration of the actual life are based on the estimated life. For tax purposes, an accelerated depreciation method (sum-of-the-years digits) is used; for reporting purposes, straight line depreciation is used.

The real dollar actual salvage is a random variable ranging between 0 and 20% of the real dollar investment in the project. Let $S_{a,r}$ = actual salvage in real dollars. Let $S_{a,c}$ = actual salvage in current dollars--current meaning dollars as of the original investment. Let $S_{e,c}$ = estimated salvage in current dollars. This is the salvage value used in preparing depreciation schedules for both tax and reporting purposes. $S_{e,c}$ is a random variable ranging between .5 $S_{a,c}$ and 1.5 $S_{a,c}$.

The before tax real dollar cash return from the project takes one of two basic patterns, selected randomly, each with equal probability. One pattern is basically level over the life of the project. The other is a declining pattern, averaging a 5% reduction per year. Let the real dollar before tax cash flow in year t = R_t. Then:

$$R_{t+1} = R_t(1 - a)\, b$$

Where:

$$a \begin{cases} = 0 \text{ for level pattern} \\ = .05 \text{ for declining pattern} \end{cases}$$

b is a random variable ranging between .917 and 1.083.

R_0, the base used to calculate R_1, is a random variable ranging between A and B times the real dollar investment dividend by the project life. A and B are variables used to induce different degrees of uncertainty and variability into the simulation model. They are discussed later.

The Environment

The environment in which the simulated firm operates, has the following characteristics:

a) Accounting information is prepared on an historical cost basis. Accountants ignore price level changes. Inter-period tax allocations are made by accountants.

b) Inflation averages 3% a year. The price index in year t+1 is a random variable ranging between 1.00 and 1.06 times the price index in year t.

c) Taxes are assessed at a rate of 50% of taxable income. Taxable income is computed in the same manner as accounting income except different depreciation methods are used.

The Variables Manipulate

The variables of interest were inter-year changes in the measured rates of return and the degree of variability embodied in the simulation model. Variability was induced two ways. First, a pseudo random number generator was used to make Monte Carlo draws from specified distributions. The ranges of these distributions have been stated. Their form has not been specified. These distributions took 18 different forms to represent different degrees of uncertainty and variability. Limited uncertainty and variability was represented by symetrical distributions, tightly clustered at the mid-point of the ranges.

Different degrees of variability were represented by spreading out the distributions in steps, through uniform distributions, to U shaped distributions. More specifically, if a continuous random variable had a range from X_* to X^*, then it was selected as

$$X_* + (X^* - X_*)\ \tilde{r}$$

where $\tilde{r}$ is a random variable on the internal 0 to 1. $\tilde{r}$ was based on the beta distribution, calculated to approximate

$$\tilde{r} = F\ (\tilde{u}|\rho,\nu) \equiv \int_0^{\tilde{u}} f\ (t|\rho,_\nu)\ dt$$

where:

$$f\ (t|\rho,_\nu)dt \equiv \frac{t^{\rho-1}\ (1-t)^{\sigma-1}}{\int_0^1 t^{\sigma-1}\ (1-t)^{\sigma-1}dt}$$

$$\sigma = \nu - \rho$$

$$\rho = .5\nu$$

and either

$$\rho \geq 1 \text{ or } \rho,\sigma < 1$$

$\tilde{u}$ was an uniformly distributed random variable from a pseudo random number generator.[7] Since the cumulative beta function cannot generally be evaluated in terms of elementary functions, approximations were necessary. Specifically, $\tilde{r}$ was limited to 100 "equally likely" values for each ν, and Monte Carlo draws were taken from this list. Eighteen selected values of ν, ranging from .05 to 70 were used to induce different degrees of variability and uncertainty.

The second means by which variability and uncertainty was induced was by altering the range of the distribution of base year real dollar before tax cash flow. Previously this range has been referred to as A and B. At one extreme, this range was from 0 to 500% of annualized real dollar cost of the investment. The other extreme was 225% to 275%. Eight intermediate ranges were used, or 10 ranges for base year real dollar before tax cash flow in all. Each of the 10 ranges was used with each of the 18 probability distributions-- resulting in 180 simulation runs, each exhibiting different degrees of variability and uncertainty.

The Concepts of Variability and Uncertainity

It is important that the meaning of the terms variability and uncertainty be clearly stated. Variability merely means that the parameters of individual investment projects differ. No two projects are alike. Each has a different rate of return. Inducing more variability means that the projects tend to display greater differences. Uncertainty means that management does not know all of the parameters of an investment project at the time of its undertaking. Hence true rate of return cannot be calculated on a current basis. Only a surrogate, accounting rate of return can be so provided. There is considerable uncertainty surrounding individual investment projects at the time they are undertaken. Management is represented as not knowing what return would be forthcoming from individual investment projects. In fact, some projects had negative IRR's. Further, the life of a project could only be estimated. Similarly, salvage value is only an estimate. Thus a great deal of uncertainty is expressed at the investment project stage. Since the firm was envisioned as a collection of virtually independent investment projects, some smoothing occurred in aggregation. Because profitable projects were more likely to be undertaken than unprofitable projects, the simulated firm was not prone to bankruptcy. A greater degree of volitability could have been incorporated by allowing investment project parameters to drift over the life of the firm or to create dependencies between returns from existant investments and future investment decisions. Of course, either step might cause instability in the model. The model, as developed, was stable in the sense it produced mean reverting measures of return. None-the-less, the economic outlook for the firm, at any point in time, even tho not explosively unstable, was unknown because it was dependent upon the particular existant and future investment projects, which from the viewpoint of management well all uncertain ventures.

<u>Results</u>

The modeled firm was simulated 180 times, using each of the 18 forms for probability distributions and the 10 ranges described above, in combination. Each run simulated 200 years of firm. The ARR and the TRR were calculated for each year of each simulation. The ARR and TRR measures for one run are shown graphically in Figure 1.

Figure 1 is based on the most volatile situation modeled. This resulted in wide inter-year changes in ARR and TRR. The amount of variability and uncertainty is measured by the standard deviation of the TRR series--in this case .03813. Mere inspection of Figure 1 discloses that the modeled firm's economic performance was highly variable. The efficacy of ARR as a measure of return can be determined by contrasting ARR with TRR. More exactly, inter-year <u>changes</u> in ARR were correlated with inter-year <u>changes</u> in TRR[8]. For the simulation run depicted in Figure 1, the coefficient of correlation between changes in the measures of return is +.77407.

Figure 2 is based on the most minimal state of variability and uncertainty modeled. In this run, the modeled firms economic performance was

Figure I.

Accounting Rate of Return vs. True Rate of Return

Maximum Variability and Uncertainty

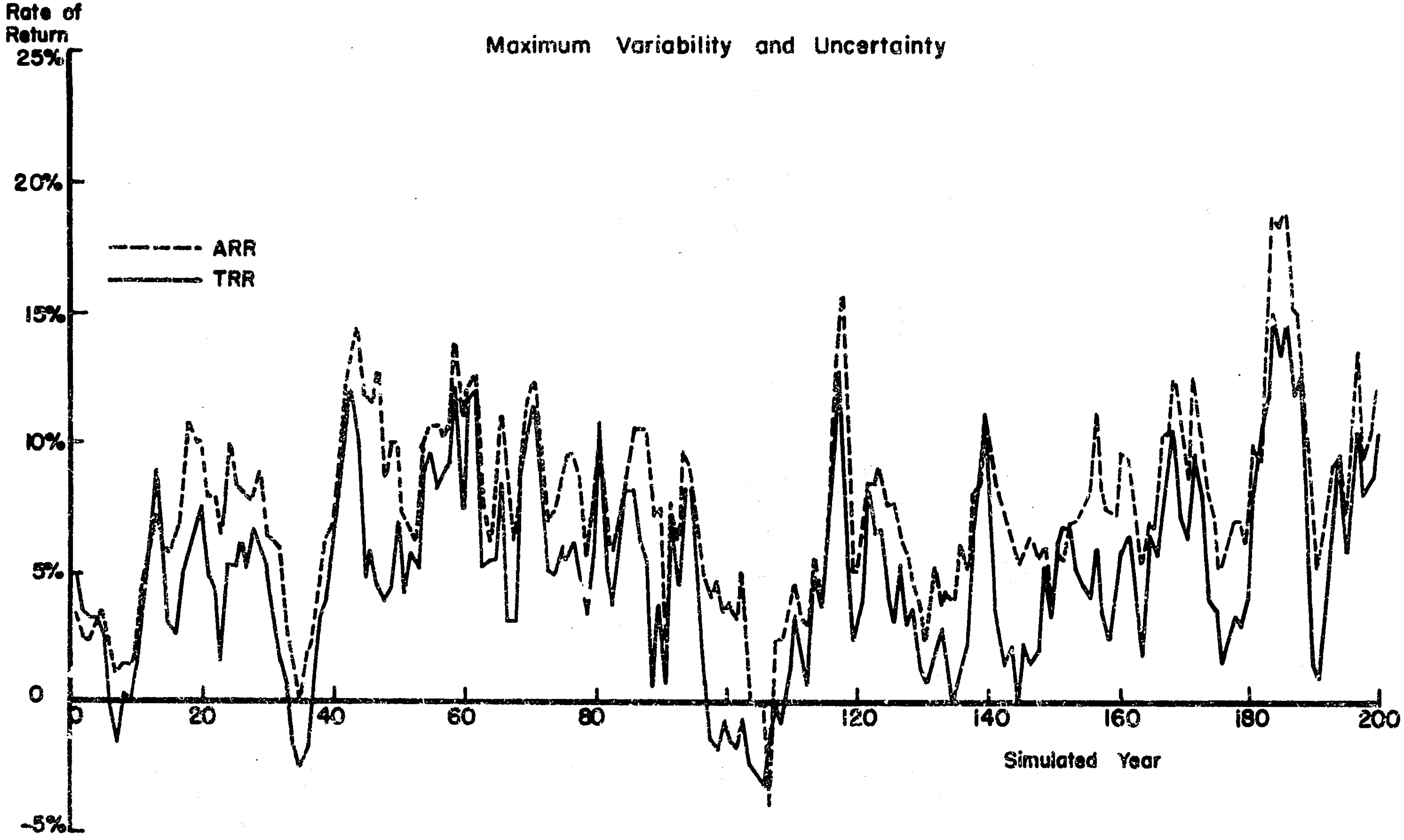
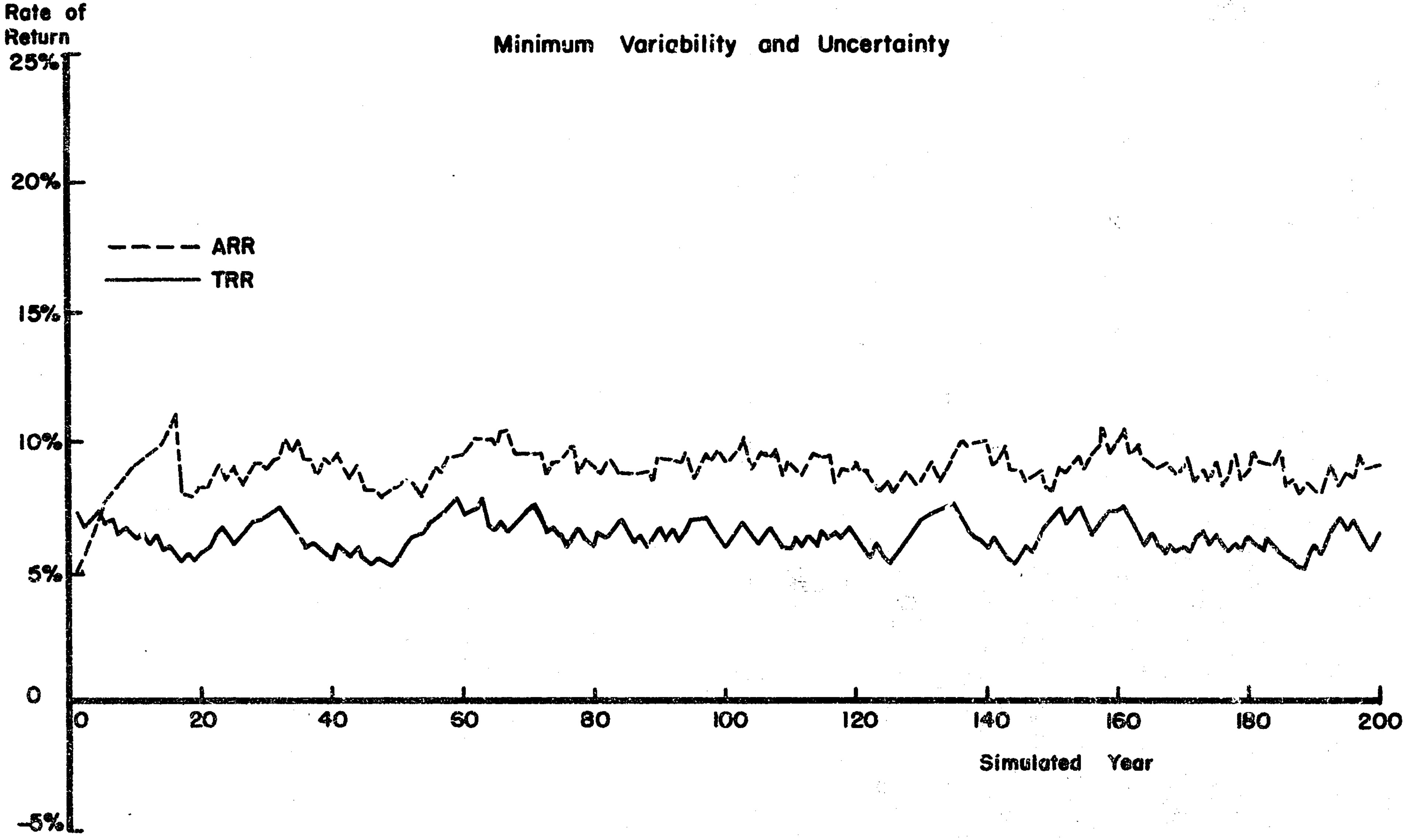


Figure 2.

Accounting Rate of Return vs. True Rate of Return

Minimum Variability and Uncertainty



considerably more stable. As in the case of Figure 1, the ARR is generally greater than the TRR, due in large part to the effects of inflation. However, transient conditions in the early years (roughly years 1 through 20 since the maximum project life was 20 years) were now evident. Therefore, only the last 180 years of the 200 years simulated in this and all other simulations runs were used for results purposes.

The standard deviation of the TRR series in Figure 2 is .00567, far less than .03713 for Figure 1. The coefficient of correlation between changes in ARR and changes in TRR is also far less in Figure 2 than it was in Figure 1; +.11577 vs. +.77407. Thus, the evidence from these two simulation runs is consistent with the crude measuring instrument hypothesis. The more variability and uncertainty, the better accounting measures serve as surrogates for their economic counterparts.

The evidence from all 180 runs is also consistent with the crude measuring instrument hypothesis. Table 4 summarizes all these runs. For each run, in the body of the table, is shown the standard deviation of the TRR series and the coefficient of correlation between inter-year changes in ARR and changes in TRR (the latter being in parentheses). Summary measures previously discussed for the run depicted in Figure 1 are in the upper right-hand corner of the body of Table 4. Figure 2 data is in the lower left-hand corner. The rest of the data in Table 4 is intermediate to the values in these two corners.

Table 4 is arranged such that the 18 entries in each column represent runs with constant ranges of base-year real dollar before tax return but different distributional forms for Monte Carlo draws. The 10 entries in each row have constant distributional forms but different ranges for base-year returns. Thus, the two methods for inducing variability and uncertainty form the rows and columns of the table. Generally, moving from left to right across the rows and from bottom to top in the columns means an increase in variability and uncertainty.

Table 4 is analyzed by rows and columns. Consider, for example, the 6th row of Table 4. The ten simulation runs reported in this row all used the same distributional forms for Monte Carlo draws, but different ranges for base-year return. The two concomitant observations for each of the 10 runs in this row are graphically depicted in Figure 3.

Inspection of Figure 3 shows that not only does increasing the range of base-year return increase the standard deviation of the TRR series, but it also increases the coefficient of correlation between inter-year changes in TRR and ARR. The coefficient of correlation between the concomitant observations in the 10 runs is +.93669. This value is shown for row 6 in the margin of Table 4. The correlation of concomitant observations for all the rows and columns are similarly indicated in Table 4. The conclusion being that the runs summarized in each row and column display the interesting relationship that the

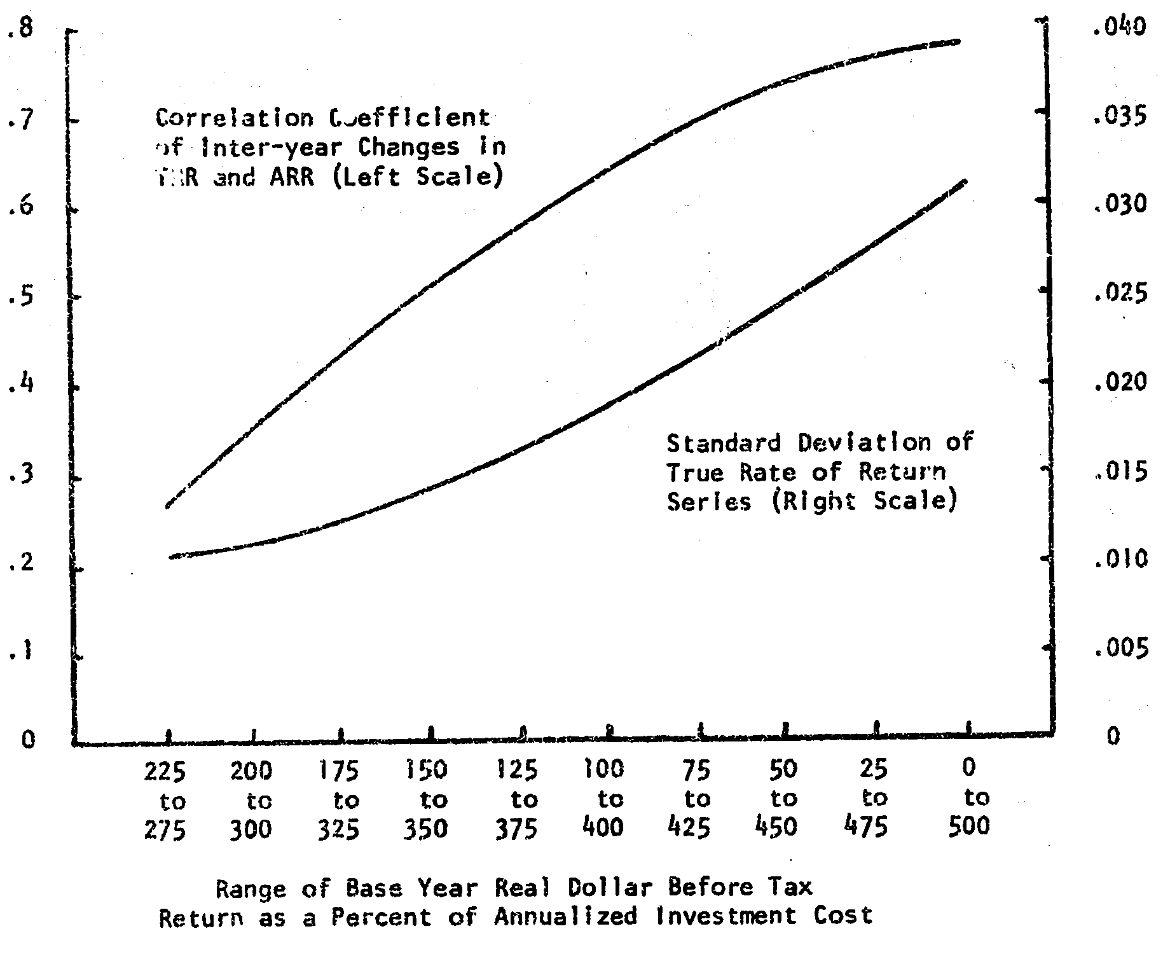
Table 4

Two Concomitant Observations for 180 Simulation Runs: Standard Deviation of True Rate of Return Series and, in Parentheses, Coefficient of Correlation between Inter-year Changes in True Rate of Return: Original parameters.

Distribution Parameter (ν)	225-275	200-300	175-325	150-350	125-375	100-400	75-425	50-450	25-475	0-500	Correlation Coefficient of Concomitant Observations in Row
	Range of Base Year Real Dollar Before Tax Return as a Percent of Annualized Investment Cost										
.05	.01211 (.26028)	.01286 (.34335)	.01447 (.43958)	.01672 (.52747)	.01940 (.60141)	.02241 (.6600[illegible])	.02568 (.70491)	.02919 (.73815)	.03297 (.76125)	.03713 (.77407)	.91870
.10	.01205 (.22673)	.01273 (.31575)	.01424 (.41751)	.01638 (.51118)	.01896 (.58914)	.02187 (.65082)	.02504 (.69815)	.02844 (.73343)	.03221 (.75836)	.03615 (.77312)	.91705
.20	.01200 (.235[illegible])	.01263 (.32566)	.01406 (.42708)	.01611 (.52018)	.01859 (.59766)	.02139 (.65905)	.02446 (.70626)	.02775 (.74156)	.03131 (.76663)	.03522 (.78169)	.91600
.40	.01162 (.25844)	.01226 (.33978)	.01362 (.43422)	.01554 (.52285)	.01787 (.59795)	.02049 (.65831)	.02335 (.70533)	.02643 (.74095)	.02935 (.76673)	.0333[illegible] (.78298)	92379
.60	.01121 (.26034)	.01185 (.33972)	.01319 (.43395)	.01505 (.52331)	.01730 (.59931)	.01984 (.66043)	.02259 (.70802)	.02556 (.74409)	.02673 (.77031)	.03218 (.78720)	.92690
.80	.01094 (.27465)	.01155 (.34299)	.01282 (.42889)	.01460 (.51351)	.01675 (.58761)	.01917 (.64862)	.02181 (.69711)	.02464 (.73462)	.02768 (.76261)	.03096 (.78167)	.93669
2	.00923 (.07642)	.00964 (.12890)	.01052 (.21083)	.01178 (.30340)	.01333 (.39357)	.01509 (.47473)	.01702 (.54455)	.01910 (.60300)	.02130 (.65104)	.02365 (.68984)	.96623
5	.00740 (.17897)	.00762 (.22639)	.00814 (.29155)	.00889 (.36382)	.00984 (.43525)	.01093 (.50125)	.01215 (.55979)	.01345 (.62045)	.01483 (.65360)	.01628 (.68996)	.96842
10	.00608 (.09700)	.00619 (.12820)	.00648 (.17514)	.00693 (.23190)	.00752 (.29292)	.00822 (.35392)	.00901 (.41211)	.00987 (.46592)	.01078 (.51463)	.01173 (.55812)	.98181
15	.00603 (.17476)	.00607 (.19198)	.00624 (.22136)	.00653 (.25983)	.00693 (.30404)	.00742 (.35097)	.00799 (.39822)	.00862 (.44410)	.00931 (.48730)	.01003 (.52781)	.98710
20	.00585 (.15778)	.00585 (.17448)	.00594 (.20068)	.00614 (.23429)	.00643 (.27296)	.00680 (.31447)	.00723 (.35695)	.00773 (.39898)	.00827 (.43355)	.00885 (.47801)	.98273
25	.005[illegible]6 (.14399)	.00587 (.16287)	.00595 (.18918)	.00612 (.22125)	.00636 (.25731)	.00666 (.29566)	.00702 (.33427)	.00743 (.37381)	.00789 (.41166)	.00838 (.44786)	.98132
30	.00577 (.12879)	.00578 (.14630)	.00586 (.17059)	.00600 (.20034)	.00621 (.23406)	.00648 (.27034)	.00679 (.30788)	.00715 (.34565)	.00755 (.38281)	.00798 (.41878)	.98390
35	.00576 (.16581)	.00576 (.18369)	.00582 (.20704)	.00594 (.23477)	.00612 (.26570)	.00635 (.29868)	.00663 (.33268)	.00694 (.36685)	.00730 (.40053)	.00768 (.43322)	.97968
40	.00574 (.17629)	.00576 (.19166)	.00582 (.21229)	.00594 (.23725)	.00610 (.26552)	.00631 (.29605)	.00656 (.32790)	.00685 (.36025)	.00717 (.39342)	.00752 (.42391)	.98453
50	.00576 (.11331)	.00575 (.12064)	.00578 (.13311)	.00586 (.15021)	.00598 (.17127)	.00613 (.19553)	.00632 (.22224)	.00555 (.25065)	.00680 (.28010)	.00709 (.31001)	.95957
60	.00568 (.11243)	.00568 (.11832)	.00572 (.12877)	.00579 (.14341)	.00589 (.16174)	.00603 (.18317)	.00620 (.20709)	.00640 (.23288)	.00662 (.25996)	.00687 (.28782)	.99354
70	.00567 (.11577)	.00567 (.11844)	.00571 (.12504)	.00577 (.13536)	.00586 (.14906)	.00598 (.16574)	.00613 (.18496)	.00630 (.20625)	.00649 (.22915)	.00671 (.25321)	.9373[illegible]
Correlation Coefficient of Concomitant Observations in Columns	.78393	.89674	.94587	.96250	.96566	.36253	.95596	.94706	.93589	.92142	

Figure 3

Graphical Depiction of Concomitant Observations Reported in Row 6 of Table 4



greater the degree of variability and uncertainty, the better accounting measures serve as surrogates for economic concepts.[9]

A Macro-Sensitivity Analysis

The parameters of the model were selected in what can only be called an arbitrary manner. The only defense that can be made for the particular values selected is that they were thought to be reasonable. Other values would also be reasonable. As a gross test of sensitivity of the model to parameter values, a new set of values were selected and the complete study rerun. The original and new parameter values are listed in Table 5. Results, using new values, are shown in Table 6.

It should be noted that in this version of the simulation, the single project per year requirement is removed, adding greatly to the computing time required.

Variability and uncertainty were again manipulated by altering the forms of probability distributions and the ranges of base-year return. Results using new parameters, as shown in Table 6, were similar to those previously obtained. Again, the greater the degree of variability and uncertainty in the ARR series, the higher the correlation between inter-year changes in the two return measurers.[10] Consistent results from this second version of the simulation allows at least a minor degree of confidence that there is some generality to the relationships found, but of course caution is warranted. At least the results are not dependent upon the original parameter values selected. Two sets of reasonable parameter values produced consistent results.

Discussion

This paper has treated accounting measures of return as crude surrogates for an economic concept. A method for assessing how crude the surrogate is has been developed, but more importantly, the Monte Carlo simulation technique has been brought to bear on an area of inquiry which has heretofore seen almost exclusive reliance upon certainty models.[11] One point that must be stated strongly is that certainty models are probably inadequate. The degree of environmental variability and uncertainty has been shown to be an important factor when contrasting accounting and economic measures.

Finally, some potential areas for further research can be identified. For example, one might ask if adjustments for price level changes in the accounts, as suggested by the accounting profession[12], would improve or detract from the correlation measures used in this study? Lead-log relationships and moving averages could be investigated. Economic cycles could be introduced into cash flow patterns. Thus, a host of additional research opportunities present themselves.

TABLE 5

Parameters Used in Simulation Model

Parameter	Original	New
Percent of available cash flow re-invested		
Maximum	150%	100%
Minimum	50%	50%
Number of investment projects undertaken annually	1	3
Cost of an investment as a proportion of current capital budget	100%	18.3 to 50%
Life of investment project		
Maximum	20 years	15 years
Minimum	10 years	5 years
Accountant's error in estimating life of investment project		
Maximum over estimate	3 years	4 years
Maximum under estimate	3 years	1 year
Actual salvage value in real dollars as a proportion of investment		
Maximum	20%	30%
Minimum	0%	10%
Accountant's error in estimating salvage		
Maximum over estimation	50%	25%
Maximum under estimation	50%	25%
Patterns of cash returns from investment		
Probability of level returns	50%	80%
Probability of decreasing returns	50%	20%
Average annual reduction with decreasing returns	5%	7%
Random deviations from pattern (annual)		
Maximum	8.3%	15%
Minimum	8.3%	10%
Price level changes (annual)	+6%	+10%
Maximum	0%	+ 2%
Income tax rate	50%	40%

Table 6

Two Concomitant Observations for 180 Simulation Runs: Standard Deviation of True Rate of Return Series and, in Parentheses, Coefficient of Correlation between Inter-year Changes in Accounting Rate of Return and Inter-year Changes in True Rate of Return: New Parameters.

Distribution Parameter (γ)	Range of Base Year Real Dollar Before Tax Return as a Percent of Annualized Investment Cost: 225-275	200-300	175-325	150-350	125-375	100-400	75-425	50-450	25-475	0-500	Correlation Coefficient of Concomitant Observations in Rows
.05	.01743 (.49420)	.01971 (.52079)	.02310 (.56307)	.02720 (.61083)	.03174 (.65693)	.03660 (.69777)	.04170 (.73213)	.04700 (.76000)	.05250 (.78185)	.05822 (.79811)	.97731
.10	.01703 (.48295)	.01925 (.50408)	.02257 (.54394)	.02661 (.59154)	.03111 (.63875)	.03593 (.68122)	.04098 (.71729)	.04623 (.74678)	.05167 (.77004)	.05733 (.78748)	.98100
.20	.01651 (.46683)	.01877 (.48217)	.02213 (.51792)	.02616 (.56308)	.03063 (.60920)	.03539 (.65151)	.04038 (.68802)	.04554 (.71826)	.05089 (.74243)	.05645 (.76079)	.98565
.40	.01636 (.39543)	.01845 (.41714)	.02156 (.46024)	.02532 (.51238)	.02951 (.56508)	.03400 (.61342)	.03871 (.65543)	.04360 (.69071)	.04867 (.71954)	.05394 (.74239)	.98523
.60	.01563 (.39330)	.01762 (.41369)	.02055 (.45327)	.02410 (.50188)	.02805 (.55199)	.03227 (.59912)	.03670 (.64113)	.04130 (.67732)	.04606 (.70771)	.05101 (.73259)	.98894
.80	.01509 (.45752)	.01710 (.48033)	.01998 (.51871)	.02343 (.56375)	.02724 (.60894)	.03131 (.65059)	.03556 (.68713)	.03998 (.71816)	.04454 (.74389)	.04926 (.76469)	.98625
2	.01223 (.44656)	.01373 (.46024)	.01581 (.48822)	.01827 (.52458)	.02100 (.56411)	.02390 (.60317)	.02694 (.63967)	.03010 (.67260)	.03335 (.70164)	.03670 (.72684)	.99536
5	.00957 (.33163)	.01040 (.35165)	.01159 (.38252)	.01304 (.42000)	.01457 (.46047)	.01644 (.50128)	.01831 (.54071)	.02025 (.57775)	.02226 (.61184)	.02432 (.64278)	.99618
10	.00822 (.31931)	.00880 (.34423)	.00960 (.37702)	.01058 (.41429)	.01168 (.45336)	.01288 (.49229)	.01416 (.52984)	.01550 (.56523)	.01689 (.59805)	.01832 (.62812)	.99426
15	.00761 (.25036)	.00804 (.27120)	.00865 (.29877)	.00940 (.33060)	.01026 (.36465)	.01121 (.39941)	.01223 (.43384)	.01330 (.46726)	.01441 (.49922)	.01556 (.52945)	.99607
20	.00733 (.30708)	.00768 (.31935)	.00817 (.33664)	.00878 (.35773)	.00949 (.38145)	.01028 (.40681)	.01112 (.43296)	.01202 (.45927)	.01296 (.48525)	.01393 (.51055)	.99883
25	.00692 (.30830)	.00723 (.32107)	.00765 (.33816)	.00817 (.35862)	.00878 (.38139)	.00946 (.40564)	.01019 (.43067)	.01097 (.45591)	.01178 (.48094)	.01263 (.50542)	.99849
30	.00644 (.33358)	.00670 (.34753)	.00708 (.36766)	.00754 (.38995)	.00809 (.41357)	.00870 (.43784)	.00936 (.46225)	.01007 (.48649)	.01081 (.51018)	.01158 (.53314)	.99500
35	.00619 (.28894)	.00645 (.30740)	.00680 (.32881)	.00724 (.35209)	.00775 (.37640)	.00832 (.40109)	.00893 (.42572)	.00959 (.44998)	.01027 (.47365)	.01099 (.49658)	.99415
40	.00597 (.25649)	.00619 (.27373)	.00650 (.29470)	.00689 (.31824)	.00735 (.34333)	.00786 (.36916)	.00842 (.39515)	.00902 (.42086)	.00966 (.44600)	.01031 (.47039)	.99498
50	.00598 (.23447)	.00618 (.24822)	.00645 (.26534)	.00678 (.28499)	.00718 (.30638)	.00762 (.32885)	.00811 (.35186)	.00863 (.37503)	.00918 (.39805)	.00975 (.42071)	.99698
60	.00605 (.26191)	.00622 (.27530)	.00645 (.29134)	.00673 (.30937)	.00707 (.32876)	.00745 (.34901)	.00787 (.36969)	.00832 (.39050)	.00880 (.41118)	.00930 (.43157)	.99585
70	.00607 (.22039)	.00623 (.23368)	.00643 (.25013)	.00669 (.26904)	.00699 (.28973)	.00732 (.31158)	.00769 (.33407)	.00809 (.35679)	.00852 (.37943)	.00897 (.40176)	.99697
Correlation Coefficient of Concomitant Observations in Columns	.92057	.92692	.94083	.95258	.95951	.96235	.96219	.95981	.95561	.94965	

Notes and References

1 A recent monograph on this subject contains much of the previously reported research. J. Leslie Livingstone and Thomas J. Burns, Income Theory and Rate of Return, College of Administrative Science Monograph S-2, The Ohio State University, 1971. An excellent survey article, also contained in the above monograph, is John Leslie Livingstone and Gerald L. Salamon, "Relationship Between the Accounting and Internal Rate of Return Measures: A Synthesis and An Analysis," Journal of Accounting Research, Vol. 8, No. 2, Autumn, 1970, pp. 199-216. A more recent, and perhaps the most comprehensive study to date is Thomas R. Stauffer, "The Measurement of Corporate Rates of Return: A Generalized Formulation," The Bell Journal of Economics and Management Science, Vol. 2, Autumn, 1971, pp. 434-469.

2 See Melvin N. Greenball, "The Accuracy of Different Methods of Accounting for Earnings - A Simulation Approach," Journal of Accounting Research, Vol. 6, No. 1, Spring, 1968, pp. 114-129, for a different model formulation.

3 Often referred to as the compound-interest method of amortization.

4 Of course the TRR could not be calculated until all of the required ex post information is known.

5 The capital recovery (or compound-interest) method requires an interest rate. The project IRR is used for the assets of each project. It should be noted that the use of project IRR's rather than a firm's cost of capital has been criticized in William J. Vatter, "Income Models, Book Yield and Rate of Return," The Accounting Review, Vol. 41, No. 4, October, 1966, pp. 681-698.

6 Accounting Principles Board Opinion Number 7, May, 1966.

7 The pseudo-random number generator was derived from P. A. W. Lewis, A. A. Goodman, and J. M. Miller, "A Pseudo-Random Number Generator for the System/360," IBM Systems Journal, Vol. 8, No. 2, 1969, pp. 136-146.

8 Evaluating the information content of changes in ARR rather than of ARR itself reflects the dominant role of the consistency convention in Accounting practice.

9 I would prefer to limit statiscal analysis to descriptive statistics since the assumptions of classical statistical methods are not met in such a computer simulation experiment. See R. W. Conway, "Some Tactical Problems in Simulation Method," Memorandum RM-3244-PR, The Rand Corporation, Santa Monica, California, October, 1962, as reported in Claude McMillan and Richard F. Gonzalez, Systems Analysis: A Computer Approach to Decision Models. Homewood, Illinois: Richard D. Irwin, Inc., 1965. However, there seems to be interest in tests of inappropriate null-hypotheses. In this vain, the following statistics are offered. For the 18 row r's, tests to reject the null hypotheses that r=0 resulted in t's ranging from 6.458 to 39.942; each with 8 degrees of freedom. For the 10 columns, t ranged from 5.050 to 14.866; each with 16 degrees of freedom.

10 For the 18 row r's, tests to reject the null hypotheses that r=0 resulted in t's ranging from 13.049 to 58.386; each with 8 degrees of freedom. For the 10 columns, t ranged from 9.880 to 14.161; each with 16 degrees of freedom. See Note 9.

11 Livingstone and Salamon op. cit. concluded their paper by noting "Like other studies cited, this one also assumes a constant IRR for all projects. While convenient, this assumption is certainly limiting and its removal would provide a better approximation to the real world. So, likewise, would the recognition of uncertainty."

12 See Accounting Principles Board Statement No. 3, "Financial Statements Restated for General Price-Level Changes," June, 1969.