SIMULATION OF INVENTORY SPACE FACTOR FOR VARIOUS OPERATING POLICIES

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Summary

An inventory space factor is a measure of the interactions between individual products competing for space in a warehouse. The value of this factor will be dependent on the operating environment of each warehouse. Such factors as fixed interval ordering, bulk arrivals and bulk receivals will affect the amount of warehouse space required to store the many types of items. This paper discusses a GPSS model of the factors which describe the operating conditions of normal warehouse, and the resulting space factor, which would then define the interaction of products. This factor would then be useful to estimate warehouse size requirements for various operating policies.

Introduction

There have been several inventory models which deal with the procurement and ordering problem under limited warehouse capacity conditions. In one of Western Electric's manufacturing locations, the lack of a quantitative method to evaluate the relationship between decisions made at purchasing, ordering, and warehousing caused serious shortages of space in the raw material warehouses. Engineers at the Western Electric Research Center were asked to investigate the situation and develop a method of providing a more meaningful relationship between costs in the functional areas involved. The outcome of this project was a dynamic programming model for solution of the multiple-item multiple source procurement problem with several warehouses.[1] This model chooses the optimal space allocation for each product, the best warehouse size under different warehouse configurations, the order quantity for each product, and the supplier for each product. In order to account for the independence of inventory patterns between products, a space factor is used to determine the required warehouse size. This can be formulated as:

$$\frac{q_i}{\alpha} = \mu_i$$

where:

q_i = order quantity of product i

α = space factor

μ_i = amount of effective space required by product i

The warehouse required can then be given as:

$$W = \sum_{i=1}^{N} \mu_i$$

where:

W = warehouse size

N = number of products

This space factor can be best illustrated by a simple example as shown in Figure 1, which exhibits the inventory patterns for three products with order quantities q_i. When asked to predict the warehouse size required to store these products there are several logical choices. A pessimistic approach would be to say that;

$$W_P = \sum_{i}^{N} q_i.$$

An optimistic approach would be to assume the average inventory of each being present so that;

$$W_O = \sum_{i=1}^{N} q_i/2.$$

Realistically, the independence between products can be accounted for by a space factor so that;

$$W_R = \sum_{i=1}^{N} \frac{q_i}{\alpha}$$

Since the order quantity will have great effect on the space utilization μ_i, the importance of using a meaningful space factor is highly significant. For this reason it was felt that further research should be undertaken into the components which describe a space factor for various situations.

Selection of Simulation Approach

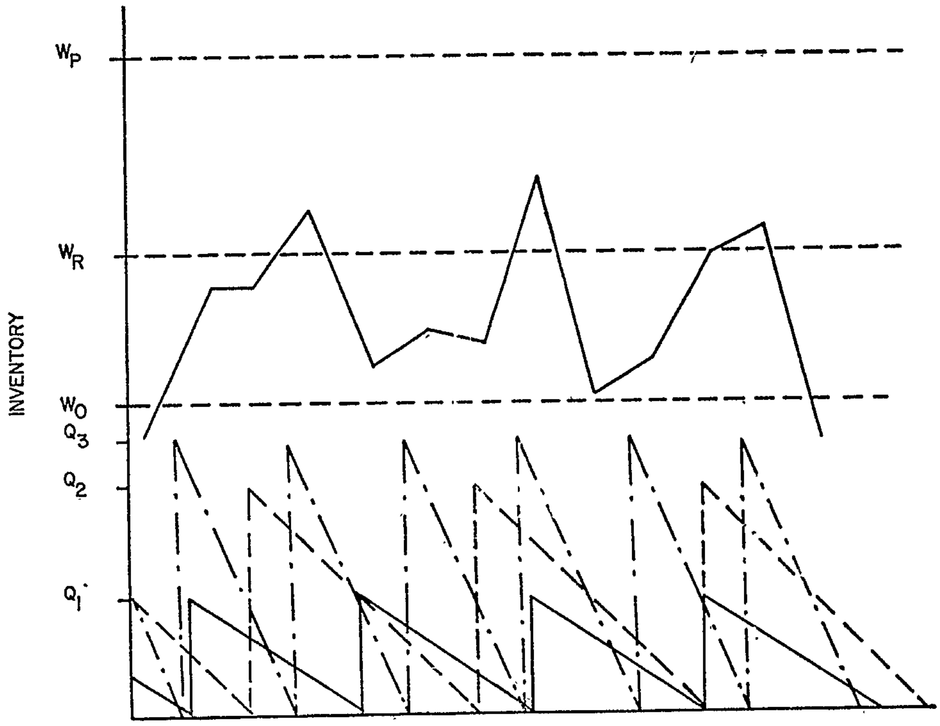
The existence of this space factor was also verified through sampling of existing warehouses and a study of inventory levels and order quantities, as compared to overall warehouse size. It was then necessary to describe the variables which could contribute to various space utilizations while the order quantities remained constant. The ability of simulation to aid in decisions which use data which is not describable by any of the standard distributions was the prime motivation for its selection. However, the ability of simulation to consider very complex situations, and to minimize the number of sometimes neutralizing assumptions in order to facilitate mathematical formulations, is certainly a factor of great importance. Another factor of less importance, but worth mentioning, was the feeling that a simulation model could be constructed in such a manner that it could be made readily understandable to people without previous modeling experience. Rather than developing a model which satisfied the conditions of the case which initiated the study, it was felt that a generic approach should be taken to the solution of the problem.

Selection of a Language

In order to accomplish the objectives of flexibility, simplification, and universality the selection of GPSS/360 was made as the medium for solution of the problem. The ability of GPSS to be readily understood was the prime reason for its selection, and the handling of many distributions can be accomplished easily be use of the FUNCTION card. Since the problem falls into the classification of inventory queuing, it was also felt that it readily satisfied the world view of GPSS, which handles transactions in terms of queuing, delays, and utilization.

Construction of the Model

The essential objective of the model is the measurement of the level of overall inventory with respect to independent actions of the individual items which comprise this inventory. Each item can be thought of as reacting to outside inputs by changing its level of inventory.



INVENTORY LEVEL OF THREE ITEMS
FIGURE I

The classification of the contributing factors into basic categories facilitated description of individual components. These general classes were: the ordering decisions, the arrival pattern, the storage pattern, and the withdrawal pattern. Enumeration of the items to be considered exhibits the large number of variable conditions which must be accounted for in any model. For ordering, there are several influencing factors:

1. The order quantities for each product.

2. The trigger level for each product that will start the inventory cycle.

3. The decision of what supplier to place the order with will have effect on the arrival pattern.

4. The inventory review policy, and whether it is constant, random, or at fixed intervals will affect the space utilization.

5. The amount of safety stock must be accounted for in some manner.

The arrival pattern is important because the arrival points are the times when inventory increases, and the upper range of total on hand inventory will dictate the warehouse capacity necessary to accommodate the system. Pertinent considerations for the arrival classification can be listed:

1. The lead time between the placing of an order and its arrival must be considered. It can be assumed to be deterministic, or variable according to some distribution.

2. Bulk arrivals of a number of products are very likely. A supplier might hold up delivery until he has assembled a number of orders, or the capacity of the delivery medium has been filled. Since this real world occurrence will have significant effects on the warehouse utilization, it must be considered.

The storage considerations were less numerous. The primary factor in this category would be the different amounts of space consumption by units of various products. The normal method of inventory competition is on a dollar basis, but in a warehouse cubic capacity consumption is the parameter of interest.

The withdrawal pattern is another point of interest. The production rate is the controlling variable with respect to withdrawal, however, it does not necessarily describe space utilization for several reasons:

1. The production rate can be assumed deterministic or variable according to some distribution.

2. Seasonal effects must be accounted for in a realistic manner.

3. The method of withdrawals, whether at a constant rate paralleling the production rate, or in bulk quantities corresponds to periodic activity or space limitations in the production area.

The existence of all of these factors exhibits the complexity of the ordering-warehousing-production system. In order to model the interactions between decisions and environments a methodology must be used which is capable of handling many types of distribution. It must also be able to reflect as many types of variable situations as possible in order to improve its validity. For this reason, considering the nature of the problem, the method of simulation was proposed. If we think of this as the QUEUE of the item we can then begin thinking about it in terms of GPSS. The sum of the individual queues for each product would then be the QUEUE for the warehouse. The model operates on each item within a separate loop. The only commonality is the entrance of the items into a QUEUE which encompasses all the items. The remainder of the model concerns the various methods of entering and leaving the QUEUES, and the measurement of the space factor in a dynamic situation. A short example (Figure 2) of the coding for one item with constant inventory monitoring will exhibit the concept quite clearly.

Figure 2

```
          GENERATE     , , 9
          TRANSFER     , PRO1
PRO1      QUEUE        1
          QUEUE        3
          SEIZE        1
          DEPART       1
          DEPART       3
          ADVANCE      250, 50
          RELEASE      1
          TEST G       Q1, 7, CEK1
          TERMINATE
CEK1      GATE LS      1, ORD1
          TERMINATE
ORD1      LOGIC S      1
          ADVANCE      2500
          LOGIC R      1
          SPLIT        15, PRO1
          TERMINATE
```

The initialization of the items in inventory is accomplished by the GENERATE blcok. The transactions then enter the two queues. QUEUE 1 is the queue for the item while QUEUE 3 is the queue for the warehouse. Each item in the system would have a similar set of blocks, but every set would have QUEUE 3 entered. The transaction then attempts to SEIZE the facility which represents utilization. The ADVANCE timing would correspond to the production rate. After each withdrawal a TEST is made of the items on hand to determine if a trigger level has been reached. If it has, a check is made at the GATE block to see if an order has already been placed. If no order has been placed a LOGIC switch is set and an ADVANCE is entered representing the lead time between ordering and arrival of the order. When the order arrives the LOGIC switch is reset allowing another order to be placed as required. The SPLIT block is used to send the required number of units to the QUEUEs of the product and the warehouse.

The concept used is not difficult. The model is now ready for exposure to various distributions of delay timing, by use of the FUNCTION block. It is also ready for consideration of other forms of ordering and withdrawal logic. For example, the constant inventory monitoring can be readily replaced by fixed interval monitoring by just a slight modification as shown in Figure 3.

Figure 3

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.
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.
RELEASE      1
TERMINATE
GENERATE     500
TEST G       Q1, 7, CEK1
TERMINATE
.
.
.
```

Instead of each order triggering a check of the inventory level, a transaction is created by the GENERATE block every 500 time units which triggers the TEST. Bulk withdrawals can be handled by use of STORAGE blocks with defined capacities which represents the amount withdrawn. Every time the storage is empty the required number are withdrawn from the QUEUE. Seasonal fluctuation can be handled by going to different distributions depending on clock time. Bulk receivals can be handled by accumulating information in a SAVEVALUE every time an order is ready for arrival, and then TESTing the SAVEVALUE for some criteria such as number ready for shipment.

One of the problems of many simulations is the determination of the time that meaningful information can begin to be extracted from the model. This model has the power of depicting a system at any point in time with respect to inventory level. The initial inventory can be set for each item by the GENERATE block. It is also possible to simulate the pending arrival of orders through use of the following set of blocks (Figure 4).

Figure 4

```
GENERATE     , , , 1
LOGIC S      1
TERMINATE
GENERATE     , , 342, 15
LOGIC R      1
TRANSFER     , PRO1
```

After 342 time units, 15 transactions will enter the queues for the item and warehouse. It is now possible to either simulate the starting condition or to actually sample the inventory system under study and use the initial conditions found at some point in time. For this reason, meaningful statistics can be gathered almost immediately.

Measurement of the Space Factor

The space factor is the parameter of interest of this model. Since it is so dynamic, a meaningful way must be employed which will measure it throughout the simulation. A method must be used which will allow the manager to make conclusions about the space factor for the system under study. That is, what α can be used which will specify the required size of the warehouse using the formula

$$W = \sum_{i=1}^{N} \frac{q_i}{\alpha}$$

The measurement of the space factor at different time periods was accomplished using a FVARIABLE block to make the calculation:

$$\alpha = \frac{\sum_{i=1}^{N} q_i}{W_t - \sum_{i=1}^{N} SS_i}$$

where:

w_t = number of items in the warehouse queue at time t

SS_i = safety stock for product i

A transaction was GENERATEd at frequent time intervals, which triggered the calculation of α. This value was then TABULATEd in a TABLE which collected the information on the number of times the space factor was at each possible value within its feasible range. The value for the space factor α, was specified as the highest number within the 95% value of the "cumulative remainder" column in the standard TABLE output. That is, 95% of the time the space factor specified would satisfy the physical constraints of a warehouse planned using that value. Of course, the 95% figure is arbitrary, and the value of α picked should be analyzed statistically by using several runs with different initial conditions and random number seeds, but the concept of how to collect the pertinent information is the item of interest of this paper.

Running the Program

Since each product requires a substantial number of lines to specify the characteristics of operation, some of the effort saving attributes of GPSS/360 can be resorted to in order to reduce the time involved for coding, keypunching, and data variation for sensitivity analysis. The logic of the individual areas can be coded using MACRO instructions to represent each. For example, there could be MACROs for; due to arrive, the production phase, the testing phase, the delay phase, and the arrival phase. It is then quite easy to have several options for each phase so that the appropriate MACROs can be used for each product, avoiding the need to make generalizing assumptions. In order to reduce the effort required to do sensitivity analysis by varying the data, matrix savevalues are used for the values of mean production time and deviation, mean arrival time deviation, trigger levels, and order quantities.

In order to handle a large system, space economies of GPSS/360 had to be used. The REALLOCATE block was used to take space from functions that are automatically allocated space. The liberated space was then given to COMMON in order to guarantee enough core for all of its functions. User chains were also used to minimize the number of transactions on the current events chain. Depending on the number of active transactions in the simulator, it might be necessary to REALLOCATE space to allow more live transactions. Using the 256K version of GPSS/360, 1200 transactions are automatically permitted. If the number has any chance of exceeding this amount, steps should be taken to provide the additional space.

The time required to formulate, collect data, and test the model was about two man-months of sporadic activity. The model runs on an IBM 360/50 using the 256K version of GPSS/360. The computer time required to analyze three years of activity for thirty-five items is about 7 minutes. More complicated tests of logic, such as for bulk withdrawals, and to allow multiple orders to be outstanding would sacrifice more time on the computer.

Results of the Simulation

The order quantities are an input to the simulation model, however, they are also an output of the dynamic programming model, discussed in the beginning of this project, which uses the space factor found in the simulation. There seems to be a conflict here similar to the proverbial cart before the horse. However, I think that the validity of both models can actually be improved by this method, rather than downgraded. The dynamic programming model uses deterministic information in order to find a solution. Even with the introduction of stochastic variables, there must be assumptions concerning the interactions between products. If the order quantities outputted by this model are subjected to the simulator the validity of the space factor used can be proven. Another point is that for many warehouses the general magnitude of ordering quantities on a percentage basis could be very helpful in the initial simulation to determine what space factor to use in the first D.P. run.

The simulator has the power not only to monitor the space factor, but can also be used to determine the necessary size of a proposed warehouse or storage facility, given that the order quantities and other information is known. By collecting cost information in a SAVEVALUE every time a test is made of the inventory level, a study on possible cost saving due to conversion from a constant to fixed interval policy can be easily accomplished. The ordering costs of an S-s policy can also be investigated, as well as the probability of stockouts with various safety stock policies. In reality, the model can be used as a generalized inventory-warehousing model.

Since each system has its own characteristics, any numerical results concerning the space factor would be almost meaningless as far as drawing conclusions about its value. However, a simple graph (Figure 5) will exhibit its potential, given the system under consideration. The value of the space factor for a normal distribution of the delivery internal with a mean of 6 weeks and a deviation varying from 0 to 2 weeks exhibits that the planned space factor would necessitate that one warehouse would have to be 18% larger than another provided the distribution could be found and used to plan the warehouse capacity.

Another result from the simulation was the fact that the number of products under consideration is a critical parameter concerning the value of the space factor. As the number of products increases, the overall inventory begins to approach the sum of the average inventory. For the thirty-five (35) products in the study, which encompassed a large portion of the space utilization in the warehouse, similations were run to see the value of the space factor if only some of the products were used in the planning model. Figure 6 exhibits the value of α as the number of products increases from 5 to 35. It can be seen that for a smaller number of items the use of the average value of inventory would be very misleading. However, for systems with a large number of items the use of $\alpha = 2$ would be sound, provided other characteristics (e.g., fixed interval ordering) of the system did not cause larger variations. Since many warehouses will have a large portion of space dedicated to only a small portion of items the simulation can have meaning even for warehouses with a large number of items.

Conclusion

The proposed model enables the manager to analyze the cost of various inventory policies. It also provides the ability to plan storage and warehouse facilities necessary to accommodate a large interacting system of products. It also allows the investigation of the value for the space factor used in other mathematical models, and provides for checking the validity of these mathematical models under more realistic conditions than must be generally assumed.

References

P. T. Lele and E. A. Siecienski, "Multiple-Item Multiple-Source Procurement Policy with Several Warehouses," 40th Meeting of Operations Research Society, Los Angeles, California, October, 1971.

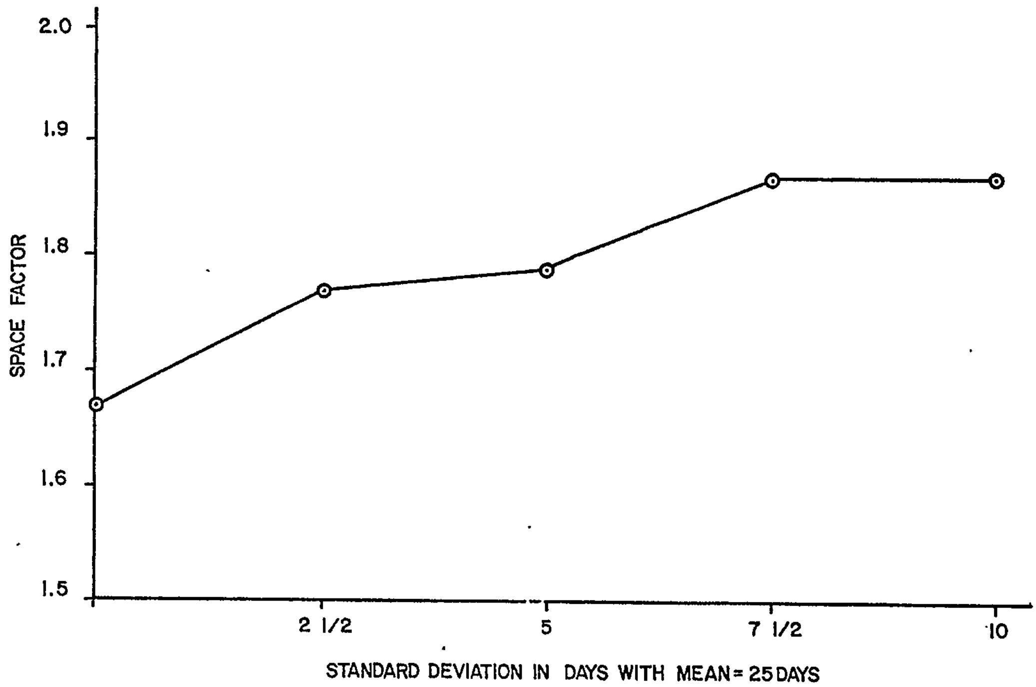

FIGURE 5

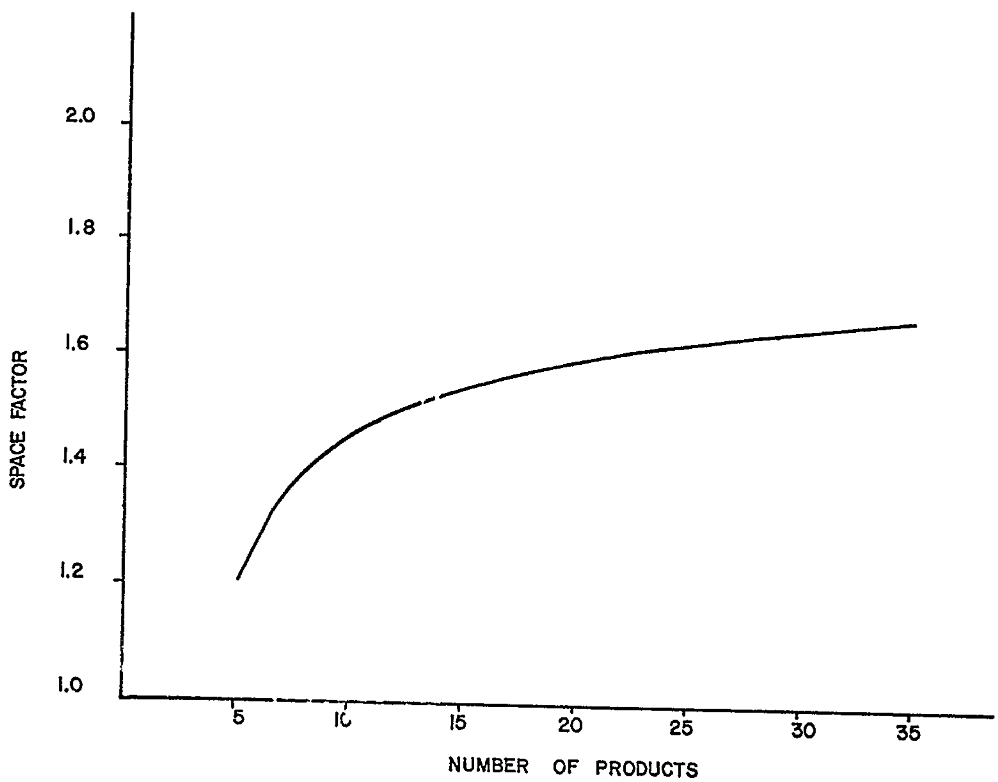

FIGURE 6