SIMULATING LARGE FLOW NETWORKS IN UNDERGROUND MEDIA

by

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SUMMARY

This paper presents a new model for simulating the flow behavior describing the efficiency of oil recovery in underground geologic formations. Simulation of the oil displacement problems involve the solution of large numbers of simultaneous equations incorporating the properties of both the fluid and the rock as well as the well patterns.

This paper describes in detail the equations used for simulation and the development of the new simulation technique. The application of the new method incorporates a statistical representation of the rock containing the oil and application of flow equations to ascertain the oil recovery efficiency.

The oil recovery efficiency is measured by the areal sweep obtained from each separate well pattern by computerized particle tracking of the fluid.

The goal is to simulate various oil recovery methods for comparison. This provides a means for selecting a most optimum technique which results in maximum oil recovery and minimum cost.

A number of figures are provided to show the results of the simulation technique. Tables and figures show comparisons of the oil recovery efficiency using the new simulation method.

INTRODUCTION

Muskat presented the areal sweep efficiencies when flooding or cycling five-spot, direct line-drive and staggered line-drive patterns several decades ago[1]. These areal sweep efficiencies represent the area swept at breakthrough in water flooding or fluid displacement patterns for the case of uniform homogeneous media, mobility ratios of one and gravity and capillary effects are neglected. It is well known that petroleum reservoirs are quite heterogeneous. Core analyses show that rock permeabilities may differ greatly from foot to foot. The basic areal sweep efficiencies which are near 72 percent for the five-spot pattern, have been sustained through the literature from Muskat's time to the present day. The last few reservoir engineering text books have all presented sweeps for uniform homogeneous media.

Improvements in estimating areal sweeps have been directed primarily toward sweeps for mobility ratios other than one. A great host of data have been presented in the literature on this topic[2,3,4,5,6,7]. In recognizing that reservoir rock is heterogeneous, a study was made to determine the possible effect of such heterogeneous media on the areal sweep efficiencies. The present work is restricted to that of the two-dimensional case showing the areal sweep for three displacement patterns consisting of heterogeneous rock. It is well known that in a heterogeneous reservoir the fluid will tend to go around the tight zones and that fluid will flow more easily in the regions of high permeability. However, the full extent of the effect of this meandering through the reservoir has not been determined in a quantitative manner.

In making this study it was presumed that a reservoir pattern such as a five-spot pattern, direct-line drive square or staggered-line drive pattern could be represented by a matrix of several hundred rock blocks. Each block would have a different permeability. In separate studies these permeabilities might range between 50 and 100 millidarcies or between 1 and 100 millidarcies or between 1/10 and 100 millidarcies. The permeabilities themselves are given by a distribution curve which may be obtained by core analysis. Each block in the pattern would constitute one entry from a particular permeability distribution curve. It is realized, of course, that these blocks could be distributed differently in the reservoir. For this reason a random number generator was used to vary the position of these blocks in the flooding pattern. In the usual case 400 blocks were considered to make up the pattern, corresponding to a 20 x 20 grid.

MATHEMATICAL DEVELOPMENT

The partial differential equation that governs the flow of fluid in heterogeneous media having rectangular coordinates is:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial P}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial P}{\partial y}\right) + q(x, y) = 0.0 \qquad (1)$$

where $q(x, y)$ represents fluid added or withdrawn at any point x, y.

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It is common practice to solve the flow equation by setting up a system of difference equations that can be solved by several different techniques simultaneously for pressures at grid intersections.

The second derivatives are replaced by second differences such that at any point x, y the 2nd derivative can be replaced by:

$$\frac{K_{x+1/2}\left[\frac{P_{x+\Delta x}-P_x}{\Delta x}\right]-K_{x-1/2}\left[\frac{P_x-P_{x-\Delta x}}{\Delta x}\right]}{\Delta x} \tag{2}$$

Assuming $\Delta x=\Delta y$ the equation in explicit form becomes:

$$K_{x-1/2}P_{x-\Delta x}+K_{x+1/2}P_{x+\Delta x}+K_{y+1/2}P_{y+\Delta y}$$
$$+K_{y-1/2}P_{y-\Delta y}=K_{x+1/2}+K_{x-1/2}\left[P_x\right]$$
$$+K_{y+1/2}+K_{y-1/2}\left[P_y\right] \tag{3}$$

If the permeability at each individual grid point, x, y be denoted by $K_{x,y}$ then equation (3) becomes

$$P_{x,y}=\left[K_{x-1/2,y}\cdot P_{x-\Delta x,y}+K_{x+1/2,y}\cdot P_{x+\Delta x,y}\right.$$
$$\left.+K_{x,y+1/2}\cdot P_{x,y+\Delta y}+K_{x,y-1/2}\cdot P_{x,y-\Delta y}\right]$$
$$\div\left[K_{x-1/2,y}+K_{x+1/2,y}+K_{x,y-1/2}+K_{x,y+1/2}\right] \tag{4}$$

Once the pressure distribution for the entire grid has been calculated, the fluid movement may be determined. The path of a single molecule can be determined by computing the x and y components of velocity at each grid intersection, eg

$$V_x=\left[\frac{\Delta P_x}{\Delta x}\right]\frac{K_x}{\mu\emptyset} \tag{5}$$

The velocity of a molecule of fluid may be determined by bi-linear interpolation between velocities at four surrounding points. The new x, y position of the molecule is then

$$x_{t+\Delta t}=x_t+v_x\cdot\Delta t$$
$$y_{t+\Delta t}=y_t+v_y\cdot\Delta t \tag{6}$$

where the maximum useable time increment Δt is limited by the maximum velocity or distance permitted for a particular point.

This method can become very time consuming if a large grid network is studied. The problems are magnified when a large permeability difference exists between contiguous points.

The particle velocity tracking method may be used with a small Δt so that point movement is restricted within a block of one permeability. However, when the particle traverses a boundary into a block having a greatly different permeability the velocity is greatly different and the traversed distance becomes subject to judgement on what to use for K, ΔP and Δt. No choice can be defended soundly; it is a matter of judgement and compromise.

A new method for directly calculating streamlines is proposed here:

Recalling the flow equation:

$$q=1.127\cdot A\cdot\frac{k}{\mu}\cdot\frac{\Delta P}{\Delta L} \tag{7}$$

Refer now to Figure 1. For steady-state incompressible flow there is a constant, but at present unknown, rate q flowing between two points such as S_o and S_1. The pressure change must equal to zero round some point S_o so that:

$$\Delta P_1=\frac{q_1}{cK_1},\ \Delta P_2=\frac{q_2}{cK_2}\ \cdots\ \Delta P_n=\frac{q_n}{cK_n} \tag{8}$$

These rates can be put in terms of stream functions as follows:

$$S_o-S_1=q_1$$
$$S_o-S_2=q_2$$
$$\text{or}\quad\vdots \tag{9}$$
$$S_o-S_n=q_n$$

The sum of the pressure drops about a point is zero, i.e.,

$$\sum_{i=1}^{4}\Delta P_i=0$$

$$\sum_{i=1}^{4}\Delta P_i=0=+\frac{S_3-S_o}{K_3}-\frac{S_o-S_2}{K_2}-\frac{S_o-S_1}{K_1}$$
$$+\frac{S_4-S_o}{K_4} \tag{10}$$

The streamline value at point S_o is then:

$$S_o = \left[\frac{S_3}{K_3} + \frac{S_2}{K_2} + \frac{S_1}{K_1} + \frac{S_4}{K_4} \right]$$

$$\div \left[\frac{1}{K_3} + \frac{1}{K_2} + \frac{1}{K_1} + \frac{1}{K_4} \right] \quad (11)$$

The resulting value determined at each grid intersection in the system then becomes the stream function value at that point. These values may then be plotted to show the streamlines or paths of a single molecule of fluid going from one input well to an output well.

MODEL SIZE STUDY

In setting up and testing the new method, a confined pattern was used for the five spot, direct and staggered line drives and these are the patterns petroleum reservoir engineers are familiar with. It is realized that the straight lines of the boundaries will not be streamlines in the heterogeneous reservoir, yet confined patterns have been used for earlier studies when the boundary was known to not be a streamline[8]. The total input rate was set at 1000 barrels per day for convenience. A uniform flow was assumed for the initial guess of the streamlines and equation (11) was solved to acceptable convergence criteria by successive overrelaxation. The criteria were considered satisfied when the total change in magnitude of all points was less than .001 percent and the algebraic sums of the pressure drop about a point less than .000001 psi.

We used the same number of streamlines as the number of points alongside and the area between streamlines was calculated. The sweep efficiency is directly related to the product of the smallest area between streamlines times the number of streamlines divided by the total area.

This new method was used on grid sizes ranging from 15 by 15 to 40 by 40 to study the effect of the number of elements in a pattern on the areal sweep. The number of elements ranged from 225 to 1,600. The average areal sweeps for these different size five spots ranged from 30 to 37 percent as shown in the table below:

EFFECT OF MODEL SIZE ON
AREAL SWEEP EFFICIENCY

Model Size	Number of Blocks	Number of Streamlines	Areal Sweep
15x15	225	15	30.2 percent
20x20	400	20	33.0
30x30	900	30	29.6
40x40	1600	40	37.0
Average Sweep (all models) -----			32.5

From 15 to 40 streamlines were used for this special study of the effect of pattern size. It should be realized that although the block permeabilities were taken from the same distribution curve, there was no way to have identical positioning of permeabilities in patterns of different sizes. Since uniform random generation was used to randomly position the permeability blocks there is some small probability that any one particular distribution may have a line of contiguous high or low permeability blocks. This could cause a fast breakthrough area or a slow area. Therefore, these same permeability range blocks were positioned by random generation five different times and the breakthrough area was calculated for each of these five distributions. Because of the possibility of a line of high or low permeability blocks at any one random generation, the average of the five distributions is used. This provides an average estimate of areal sweep efficiency for a particular core analysis datum. For a given distribution and an infinite number of runs one would eventually arrive at a most probable sweep and subsequently a range of sweeps having various probabilities.

For our study as will be seen in subsequent figures, we did not find the sweep to differ more than ten percent of the total area for any one permeability distribution and the same pattern studied.

Since within the same pattern and permeability distribution the normal spread in areal sweep was only a maximum of seven percent of the total area for four different grid sizes, the grid size appears to not significantly influence the outcome of this new method. Since the computation time is proportional to the number of elements in the pattern, we used a 20 x 20 or 400 block system for our basic patterns. This represents a compromise between the 225 and 1600 block systems studied, but is believed sufficient to illustrate this new technique.

The new technique is also valuable in that it shows that adjacent well patterns can and will behave differently due to rock heterogeneities. For a particular permeability distribution there is a most likely areal sweep for a pattern and possible sweeps having certain probabilities. The latter would be determined after a large number of studies.

MODEL APPLICATION

The direct streamline method was applied to three well known reservoir patterns: five-spot, direct-line drive (square) and staggered line drive patterns. Each pattern was simulated with three different permeability ranges. The ranges were (a) 100 to 50 md, (b) 100 to 1.0 md and (c) 100 to 0.1 md. These distributions were used along with a random process to distribute the permeabilities throughout a 20 x 20 matrix yielding a 400 block system.

Each permeability distribution was obtained from a distribution curve similar to the one shown in Figure 2. This curve was constructed from the tabular core data found in Table I. This curve is not the straight line possibly desired by reservoir engineers doing a field study, and therefore, could be assumed to be a very likely real world example. The simulated heterogeneous rock matrix can then be formed by picking permeability values from these curves using a random number process.

An example of these randomly chosen contiguous permeability blocks can be seen in Figure 3. The permeabilities ranged from (100 to 0.1) millidarcys. The direct streamline method was then applied to each pattern over each range allowing areal sweep calculations to be made and tabulated.

FIVE-SPOT PATTERN

Figure 2 and Table I show a permeability distribution ranging from 100 to 0.1 md. From a mathematical standpoint it is quite easy to change the "Y" scale on Figure 2 so that the minimum permeability is 50 md and keep the distribution curve the same. This means that all the block permeabilities would range between 100 and 50 md. This would result in a fairly uniform reservoir.

Figure 4 shows the streamlines for the five-spot pattern when the permeability ranged between 100 and 50 md. The ratio between the highest to lowest permeability was 100 ÷ 50 = 2.0. The areal sweep for this pattern was found to be 68 percent. This is somewhat lower than the 72 percent to be expected for the uniform homogeneous five-spot pattern.

Figure 5 shows the streamlines for the five-spot pattern when the permeability ranged between 100 and 0.1 md. The streamlines for heterogeneous rock were found to be greatly different than those for the homogeneous or near homogeneous system. Note that a streamline does not go directly across from the input to an output well as does a streamline for the homogeneous case. The large areas between streamlines found near the central part are regions of low permeability.

The convergence of streamlines in an area is indicative of a region of high permeability. The areal sweep for this heterogeneous pattern was near 25 percent. It is possible to have the same exact block permeabilities, but rearranged so that they appear in a different location of the pattern. This was done several times and the areal sweeps ranged from 21 to 28 percent. See Table II.

Figure 6 shows a plot of the data on the range of sweep efficiencies achieved for various permeability ratios. When the permeability ratio (highest to lowest permeability) was two, the areal sweep ranged between 66 and 70 percent for five different studies. When the permeability ratio was 100 to one, the areal sweeps ranged between 52 and 58 percent. For a permeability ratio of 1000 to one; that is, the actual permeabilities ranged between 100 and 0.1 md, the areal sweeps were between 21 and 28 percent.

DIRECT LINE DRIVE

Figure 7 shows the streamlines of the direct-line drive pattern for the permeability range of (100 to 50) millidarcys. The pattern appears to be fairly smooth and symmetrical like the homogeneous case. This is understandable in that the permeability range is small, only two to one. However, when this permeability range is widened to (100 to 0.1) millidarcys the resulting streamlines are distorted considerably as can be seen in Figure 8. One may notice the spread and convergence of the streamlines in various areas. This is due to the low and high permeability blocks located within the matrix. See Table II for the areal sweeps.

Figure 9 shows a plot of the areal sweeps for the direct-line drive (square) pattern for the three permeability ratios of 2/1, 100/1, and 1000/1. It should be noted here that the triangle at 56 percent represents the sweep found by Muskat in the homogeneous case. Recalling the areal sweeps for the three permeability ranges for the five-spot pattern, it is noted that in general the direct-line sweeps are smaller for two of the three ranges. The average sweep for the (100 to 50) md range being 55 percent, 46 percent for the (100 to 1.0) md range, and approximately 36 percent for the (100 to 0.1) md range. The direct-line drive pattern appears to be less affected by the larger heterogeneities ranges than the five-spot pattern. This is reflected in the larger areal sweep of 36 percent as compared to a 25 percent sweep for the five-spot. However, for nearly homogeneous patterns the five-spot sweeps are definitely superior.

STAGGERED-LINE DRIVE

Figure 10 illustrates the streamlines generated by a staggered-line drive pattern over a permeability range of (100 to 50) millidarcys. This small permeability range is responsible for the symmetrical streamlines. As already illustrated in the other patterns, the larger permeability range as seen in Figure 11 generates very unsymmetrical streamlines reflecting the tight and permeable zones that had been randomly distributed.

Figure 12 shows the areal sweeps for the staggered-line drive over the three permeability ranges. Again, the triangle in the upper left-hand corner represents Muskat's areal sweep of near 77 percent for the homogeneous case. The average areal sweep for the (100 to 50) md range was 76 percent, 65 percent for the (100 to 1.0) md range, and 26 percent sweep for the (100 to 0.1) md range. The two smaller permeability ranges resulted in a larger areal sweep for the staggered-line drive than the five-spot or direct-line drive patterns. However, for the wide permeability range of (100 to 0.1) md, about the same areal sweep was obtained for the staggered-line drive and the five-spot patterns, but both gave smaller sweeps than the direct-line drive square pattern.

DISCUSSION AND CONCLUSION

A study has been made to estimate the areal sweep efficiencies when flooding or cycling in heterogeneous rock patterns. It was found that the areal sweep depended greatly on the permeability ratio. Areal sweeps for very heterogeneous five-spot patterns were reduced to near 25 percent or about one-third of the sweep expected in homogeneous media.

With the same permeability distribution and same permeability range the areal sweep could easily change a few percent depending on the actual distribution of the permeability blocks within the matrix.

This study was made using the confined or normally studied patterns. For this study the boundaries form streamlines. For the heterogeneous case it is clear the boundaries would not be streamlines, but since reservoir engineers were familiar with confined patterns, it was believed desirable to study the streamlines in a confined system. This is similar to a study by Caudle, Erickson and Slobod when they studied areal sweeps beyond the well patterns [8].

The heterogeneous staggered line drive pattern gave surprisingly low areal sweeps, but on reflection this could be due to the probability of a lower sweep when two wells were available for early breakthrough.

The direct-line drive (square) pattern is normally considered to have an inferior areal sweep to both the five-spot and staggered-line drive patterns, yet, for very heterogeneous systems the direct-line drive (square) pattern was superior to both.

Heterogeneous rock systems provide a meandering set of streamlines resulting in extremely low areal sweeps in some cases. The effect of a specific heterogeneity may possibly be studied prior to a fluid displacement program to explore the range of areal sweeps to be expected.

It is known from a study of logs and cores that some oil bearing strata are stratified and different permeability distributions exist within each stratum. It would quite likely be beneficial to expand the method to study such systems.

It is desired to emphasize that the specific results in this paper are useful only in a qualitative sense to show the adverse effect on sweep efficiency of a heterogeneous reservoir at a mobility ratio of one. Vertical crossflow, that is, three dimensional models have not been studied.

It is believed that each reservoir should be studied in detail with a series of runs to show the most probable and likely range of areal sweeps to be expected.

SYMBOLS

k = permeability in md.
A = area, ft^2
μ = viscosity, cp
ΔL = length change, feet
ΔP = pressure
q = flow rate, bbls/day
p = pressure, psi
x = x coordinate
y = y coordinate
Δx = grid length x direction
Δy = grid length y direction
ϕ = porosity
V = velocity
Δt = time increment

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TABLE I

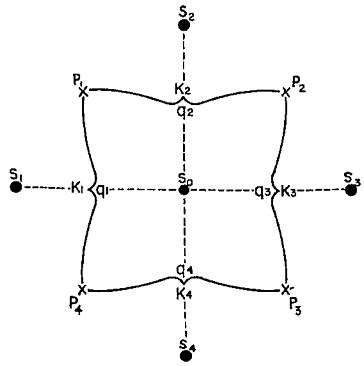
CORE DATA TABLE

Classmark (Millidarcies)	Percent Sample Class	Cumulative Percent
0.1	9	9
0.9	3	12
3.4	7	19
19.2	5	24
28.0	12	36
41.3	26	62
59.0	1	63
72.0	14	77
83.0	3	80
99.0	20	100

TABLE II

PERCENT AREAL SWEEPS

Study No.	Permeability Range-md. 50-100	1.0-100	0.1-100
	Five-Spot Pattern		
1	66	55	28
2	70	54	21
3	68	58	26
4	68	56	24
5	69	52	23
	Direct-Line Drive(Square Pattern)		
6	55	48	38
7	56	46	34
8	54	44	37
9	57	47	36
10	55	45	37
	Direct-Line Drive (d/a=2)		
11	75	68	44
12	73	64	41
13	72	69	39
14	75	62	46
15	74	65	45
	Staggered-Line Drive (d/a=1.0)		
16	78	67	26
17	76	64	28
18	75	62	24
19	77	66	25
20	76	65	23



assuming that the pressure drop around a point (S_0) is equal to 0, i.e. $\sum_{i=1}^{4} \Delta P_i = 0$, and:

$$q = -1127\left[\frac{Kh}{\mu} A\left(\frac{\Delta P}{\Delta L}\right)\right]$$

FIGURE 1

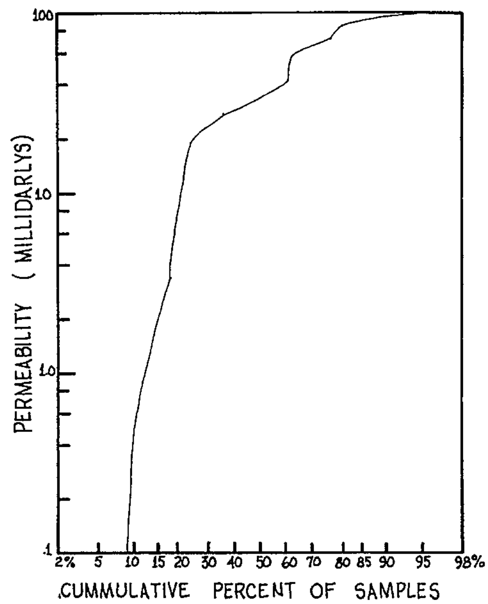
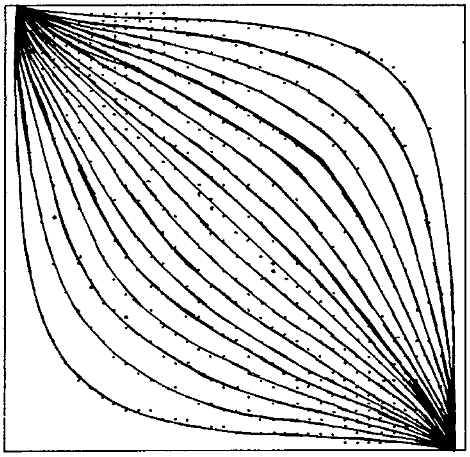


FIGURE 2

0	0	0	0	0	0	0	0	0	0
0	39.	2.3	3.2	2.6	41.	31.	30.	83.	0
0	62.	19.	18.	2.1	32.	97.	.93	39.	0
0	.70	59.	1.5	35.	4.7	32.	90.	3.5	0
0	98.	.85	69.	88.	92.	8.5	90.	26.	0
0	91.	61.	93.	24.	87.	87.	1.4	98.	0
0	32.	35.	37.	33.	66.	.13	.10	92.	0
0	97.	88.	91.	71.	69.	.14	.88	.175	0
0	71.	.49	.96	41.	38.	86.	18.	83.	0
0	0	0	0	0	0	0	0	0	0

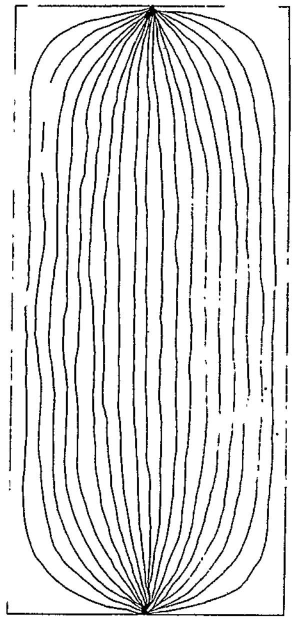
ILLUSTRATIVE PERMEABILITY DISTRIBUTION RANGE (0.1 TO 100 md)

FIGURE 3



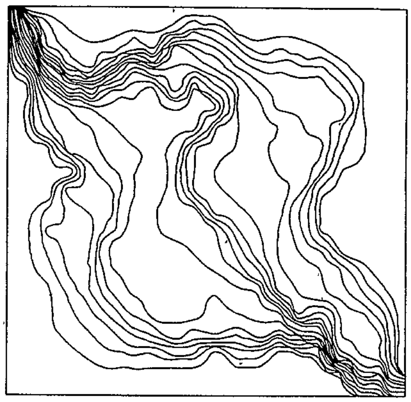
FIVE-SPOT WELL PATTERN STREAMLINES PERMEABILITY RANGE (50 TO 100 md)

FIGURE 4



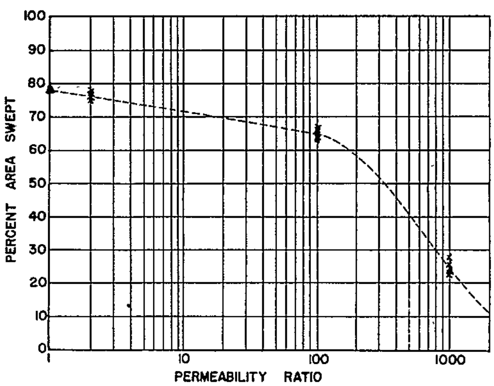
DIRECT LINE DRIVE STREAMLINES PERMEABILITY RANGE (50 TO 100 md)

FIGURE 7



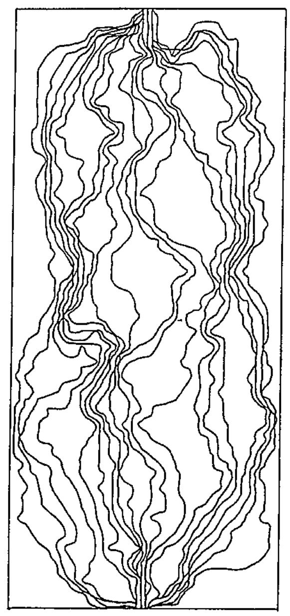
FIVE-SPOT WELL PATTERN STREAMLINES PERMEABILITY RANGE (0.1 TO 100 md.)

FIGURE 5



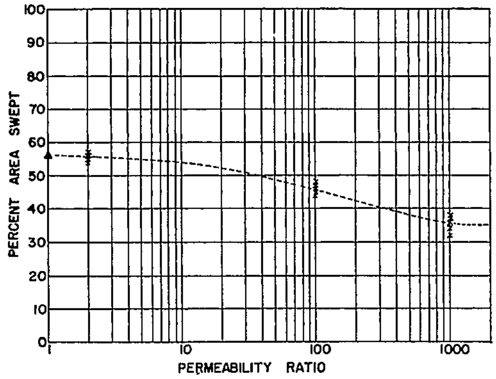
STAGGERED LINE DRIVE

FIGURE 6

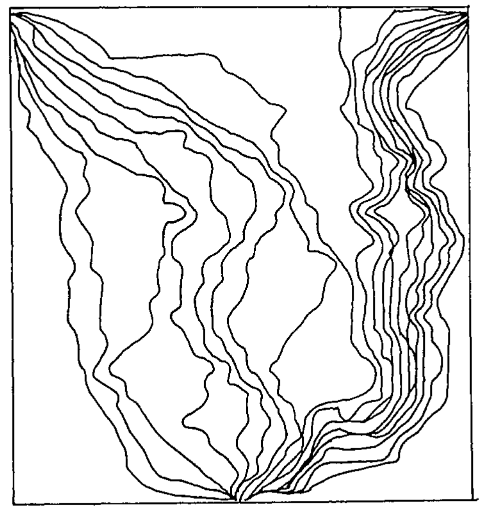


DIRECT LINE DRIVE STREAMLINES PERMEABILITY RANGE (0.1 TO 100 md)

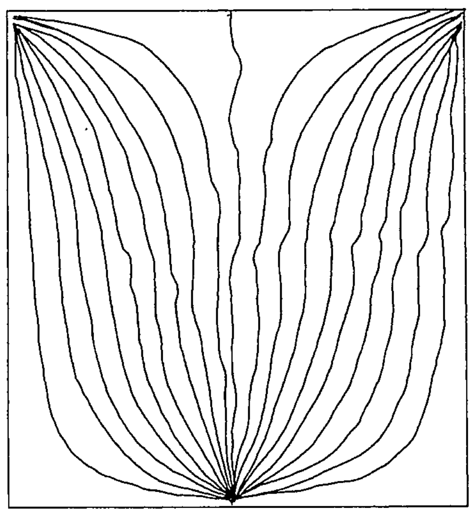
FIGURE 8



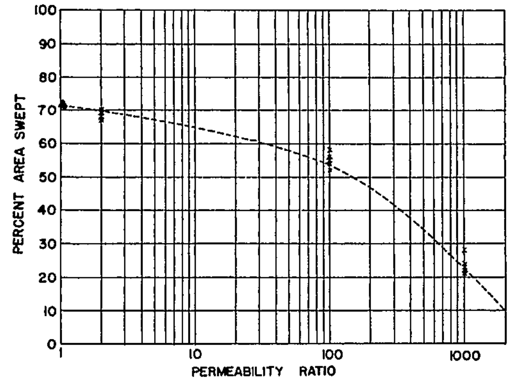
DIRECT LINE DRIVE
FIGURE 9



STAGGERED LINE DRIVE STREAMLINES PERMEABILITY RANGE (0.1 TO 100 md)
FIGURE 11



STAGGERED LINE DRIVE STREAMLINES PERMEABILITY RANGE (50 TO 100 md)
FIGURE 10



FIVE-SPOT WELL PATTERN
FIGURE 12