A SIMULATION STUDY OF THE SIZE DISTRIBUTION OF FIRMS

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ABSTRACT

Simulation models for alternative stochastic processes of firm growth are presented. Gibrat, Mansfield, and size-independent growth mechanisms are investigated, both with and without inclusion of birth and death processes. The resulting firm size distributions are compared with the trend of observed distributions in the high-concentration region, and with minimum efficient scales of plant in the low-concentration region. Size-independent growth combined with certain birth and death criteria leads to skewed firm size distributions, suggesting that entry and exit conditions can play a role in contributing to the skewed distributions observed in practice. This latter development makes possible the use of stochastic models to generate families of firm-size distributions which match empirical observations.

1. Introduction

Firm-size distribution in many industries is known to be highly skewed. Firms in the high-concentration tail of such typical skewed distributions control a disproportionately large percentage of their industry's total sales, whereas the low-concentration tail contains a relatively larger number of firms, each accounting for a substantially smaller portion of industry sales. Table 1 shows an example of this situation as encountered in the steel industry, using production capacity data rather than sales data.* The frequency with which such patterns are encountered in empirical investigations occurs for many other economic variables as well.[9] This suggests the existence of some underlying mechanism which explains the regularity of the findings. Such a mechanism would, in general, depend upon the phenomenon being examined, although there might be parallel mechanisms at work in other areas.

Producer	Capacity°
U. S. Steel	38.7
Bethlehem	17.1
Republic	11.3
Jones & Laughlin	8.5
National	6.8
Youngstown	5.2
Armco	4.8
Inland	4.2
Colorado Fuel & Iron	3.8
Wheeling	3.4

Table 1. Capacities of 10 Leading Steel Producers on January 1, 1954

°Millions of Tons per Year

It might be thought that economic theory, when brought to bear on the question of firm-size distribution, would lead to satisfactory interpretation of its shape. Yet this is not the case. Microeconomic theory in this area is based upon cost curves and optimal scales of plant. As Simon and Bonini have pointed out, such static analysis can indicate the minimum size of firm in an industry in certain circumstances, but makes no predictions about the distributions of firms by size.[9] The result of this is that investigators have turned to alternative theories based on stochastic models of the processes by which firms change in size.

One goal of the modeling process is to search for a preferably "weak" (i.e., relatively unrestrictive) set of assumptions which are consistent with the known facts in as wide a range of cases as possible and which, despite their weakness, lead to model output in agreement with empirical observations. For the case at hand, such processes as changes in size of existing firms in an industry, entry of new firms, and death of old firms are relevant contributors to the explanation of

*Alternatively, the measure of concentration might be assets, profits, etc.

the behavior of firm size. Each of these three processes, in turn, can generally be described by independent sets of assumptions. Hence, a wide variety of alternatives exists for constructing stochastic models for prediction of firm-size distributions.

Our objective is to investigate a combination of birth-growth-death assumptions which are economically realistic and to compare their predicted firm-size distributions with each other and with the general trend of observed distributions. One of the growth mechanisms explored, here termed Mansfield growth, has to the best of our knowledge not been previously used in the modeling process. It is a "weaker" assumption than the more customary Gibrat condition, and produces highly skewed firm-size distributions. In addition, a particular birth-death process used in conjunction with size-independent firm growth is shown to lead to moderately skewed size distributions. This suggests that, if a combination of mechanisms is in effect and if their relative contributions to the overall evolution of size distributions are appropriately weighted, then a wide variety of empirical distributions can be rationalized by the stochastic modeling approach.

2. Summary of Some Previous Findings

Some background information concerning previous work on the size distribution of firms, much of it pertinent to later discussion of the models being presented in this paper, will now be summarized.

1. It is well known that when the fraction of total industry sales accounted for by the largest firms is plotted on a double-logarithmic scale against their rank in the industry, a linear relationship is frequently observed.

2. The law of proportionate effect has often been invoked as the mechanism by which firms in an industry change in size. This law, formulated by Gibrat,[2] states that the distribution of percentage changes in firm size from period to period is independent of firm size. That is, a firm which had, say, 20,000 units in sales in a given year has as good a chance of growing by 15% in the next year as a firm which had, say, 2,000 units in sales in the given year. This law is defensible on at least two grounds. First, it has been found in some cases to be in agreement with empirical data.[3] (This is not true in all instances, however. See Points 6. and 7. below.) Second, under an often-used assumption of constant returns to scale, it would be expected that a firm's increase or decrease in size would be proportional to its present size.

3. Under the assumption of Gibrat's law of proportionate effect, coupled with the assumption that new firms enter the smallest size-class in the industry at a relatively constant rate, the Yule distribution can be derived.[8] In the upper tail, the Pareto distribution is a good approximation to the Yule. (For an excellent summary of the properties of the Yule, Pareto, and other discrete or discretized distributions, see Reference 6.) The Pareto distribution, converted to logarithms, yields a linear relationship consistent with the empirical observations noted in Point 1. above.

4. When Gibrat's law is used without the assumption of new-firm birth, the log-normal distribution can be developed.[9]

5. When the strong form of Gibrat's law stated in Point 1. above is replaced with a weaker form to the effect that total change in size of firms within various size strata is independent of strata, it is also possible to develop the Yule, Pareto, and log-normal distributions.[4]

6. Ijiri and Simon point out that there is serial correlation in growth rates of firms.[4] In the source cited, they propose a model with serial growth correlation and the possibility of new-firm birth, but not old-firm death, and show that the resulting firm-size distribution is approximately linear on a double-logarithmic graph.

7. Mansfield takes objection to Gibrat's law on three counts.[5] He points out that the law of proportionate growth can be formulated in a variety of ways, depending on how the death process and the size-independent assumption are viewed. In the paper cited, these three alternatives are considered and their validity is seriously questioned on the basis of empirical evidence:

   a. Suppose that the law of proportionate effect applies to firms in their last years before death. In seven of ten cases examined by Mansfield, this supposition fails to hold.

   b. Assume that the law applies only to firms not leaving the industry. For four of ten cases examined, the evidence fails to support the assumption.

   c. Postulate that the law is restricted to those firms operating above some minimum efficient size. Here, the postulate fails to hold in six of ten cases examined.

Mansfield further points out that smaller firms tend to have more variation in their growth rates than larger ones. He suggests that the distributions of the percentage change in firm size should have a common mean but that their standard deviation should vary inversely with the size of firm.

8. In a study of the percentage change in size of 369 firms among the Fortune 500 beween 1954 and 1960, Scherer found the percentage changes to be normally distributed with a mean of 6% and a standard deviation of 16%.[7]

9. Bain estimated minimum feasible plant size (i.e., the size beneath which unit costs are no longer approximately constant but rise rapidly), as a function of percent of industry sales.[1] A portion of his results is shown in Table 2.

3. Characteristics of the Growth, Birth, and Death Processes Used Here

Growth Processes

Gibrat Growth In Gibrat growth, each firm's size in the next year is the product of a sample drawn from the N(1.06,0.16) distribution (the normal distribution with mean 1.06 and standard deviation 0.16) and the firm's sales in the current year. Use of the N(1.06,0.16) distribution is consistent with Sherer's findings reported in Section 2, Point 8.

Industry	Minimum Feasible Plant Size*
Flour and Milling	0.05 to 0.25
Cement	0.4 to 0.7
Distilled Liquors	0.2 to 0.3
Petroleum Refining	0.4 to 0.9
Rubber Tires and Tubes	0.35 to 0.7
Rayon	1.0 to 3.0
Soap and Glycerin	0.2 to 0.3
Cigarettes	1.0 or less
Fountain Pens	1.3 to 2.5
Typewriters	5.0

Table 2. Estimated Minimum Feasible Plant Sizes in Various Industries

*As Percent of National Market

Mansfield Growth In Mansfield growth, each firm's size in the next year is determined as it is for Gibrat growth, except that sampling is from the N(1.06,SIGMA) population, where SIGMA varies inversely with the firm's fraction of total industry sales in the preceding year. Let FXN be the firm's fraction of total industry sales. Then SIGMA varies with FXN this way:

For $FXN \leq 0.10$, SIGMA = 0.32
For $0.10 < FXN < 0.34$, SIGMA = 0.42 - FXN
For FXN $\geq$ 0.34, SIGMA = 0.08

Hence, in Mansfield growth, the standard deviation of the percentage-change population is 0.32 for firms having 10% or less of the market, 0.08 for firms having 34% or more of the market, and varies linearly between 0.32 and 0.08 for market shares between 10% and 34%. This functional dependence follows the qualitative suggestion made by Mansfield as reported in Section 2, Point 7. Note that for both Gibrat and Mansfield growth, change in industry size is the sum of changes in sizes of individual firms making up the industry (except as modified when death occurs; see "Death Process" below).

Size-Independent Growth When growth of firms is size-independent, the industry's size in the next year is taken as the product of a sample from the N(1.06,0.16) population and the industry's total sales in the current year. This change in size is then distributed over the industry's firms by the following procedure. A pre-scaled change in firm size is determined, firm-by-firm, by sampling from the symmetric triangular distribution with vertices at (0,0), (.5,2), and (1,0), then multiplying the resulting factor (a number between 0 and 1) by the industry's change in size. The ratio of total industry change to the sum of such pre-scaled firm changes is then used as a factor to normalize the pre-scaled changes.

Birth Processes

Whenever the growth in total industry sales from one year to the next is 14% or more, one firm enters the industry the following year. Hence, growth of the industry as a whole by a sufficiently large amount acts as a signal to potential entrants that the industry is profitable. Note that, for Gibrat and size-independent growth, this sets the level of industry attractiveness at a year-to-year growth rate of one half or more standard deviations above the mean. The supporting economic theory rests on two assumptions. First, the industry involves all firms operating above the sales level at which minimum long-run average cost is in effect; and second, the cost curve is horizontal for all relevant ranges of output above that point. Alternatively, these assumptions can be replaced by assuming that firms practice sales-maximization or some form of satisficing which puts emphasis on sales.

When a firm enters the industry, its sales during its first year of life are determined by either one of two approaches:

Expected-Condition Entry In expected-condition entry, the new-born firm is given sales in its first year equal to those initially assigned to firms existing in year one, but compounded by 6% from year one to the year of entry. Hence, because each firm initially in the industry is given sales of 1,000 units, a firm entering in year 5 would be assigned initial sales of $1{,}000(1.06)^4$. This time-dependent initial sales concept assumes that the minimum point of long-run average cost rises over time as firms grow. The rise in initial-year sales is compounded at 6%, in parallel with average annual firm growth.

The new firm's initial-year sales are not simply added to the industry's total sales for the year. Rather, firms previously in the industry are forced to "give up" a portion of their sales that year to fully account for sales captured by the entering firm. This reflects the increased competition offered by the entrant. In expected condition entry, established firms give up sales to the newcomer in proportion to their share of the market. This is equivalent to arguing that existing firms feel the effect of the newcomer in a manner relative to their size.

Current-Condition Entry In current-condition entry, sales of the new-born firm in its first year are set equal to 75% of the average firm's sales in that year. This approach ties entering firm sales to the industry's actual current experience. In contrast, the expected-condition approach ties new-firm sales to the expected current experience of existing firms. If, in the expected-condition approach, only a few or perhaps not any firms have experienced the expected growth pattern, then the entering firm starts with an unrealistically high rank. This is not likely to happen with Gibrat or Mansfield growth, but can happen with high probability when the size-independent growth mechanism is invoked. Hence, current-condition entry is designed to correspond to the nature of size-independent growth.

As in the case of expected-condition entry, previously existing firms are forced to contribute a portion of their sales in the year of entry to the entrant. For current-condition entry, however, old firms contribute uniformly to the pool that makes up the entrant's initial sales. This assumes that the new firm will be relatively more competitive with existing small firms than with larger ones. The contrast between "current-condition entry with uniform take-away" and "expected-condition entry with proportional takeaway" should be noted.

Death Process

The minimum efficient scale of plant in the first year of the simulation is assumed to be 750 sales units. (Recall that in year one of the simulation, each existing firm is given sales of 1,000 units, or 33% above this minimum scale.) It is also assumed that the minimum efficient scale grows at a compound rate of 3% per year. Whenever a firm's sales in a given year slip below this minimum, the firm immediately leaves the industry. This criterion is applied, however, only after a firm is three or more years old.

Note that death can also occur independent of minimum plant scale, whether or not a "formal" death process is in effect. Sales can fall to or beneath zero in the case of Mansfield growth, where small firms sample from the N(1.06,0.32) distribution and a negative percentage change can result. This might also happen on occasion when an existing firm is forced to yield its alloted quota of sales to an entering firm.

4. Conditions Under Which the Simulation Runs Were Made

The various growth-birth-death combinations used in the simulation runs reported here are presented in Table 3.

These were the several parameter values in effect for each combination studied:

1. There were 25 firms initially in the industry.

2. Each of the 25 starting firms was assigned sales of 1,000 units for the first year of the simulation.

3. The industry was allowed to mature for 100 years.

4. For each growth-birth-death combination studied, the industry was allowed to mature 25 times.

The random variables whose distributions were estimated are listed below:

1. Relative share of market for each of the 12 top-ranked firms.

2. Relative share of the market for each of the 5 firms with lowest rank.

3. Number of firms in the industry.

4. Number of original firms still in the industry.

As indicated above, each of the distribution estimates is based on a sample of size 25. Estimates of the distributions were made at years 20, 40, 60, 80, and 100.

5. Discussion of Simulation Results

Tables 4. through 9. display selected statistics from the various models. The information in these tables will be briefly discussed, table-by-table.

Table 4 shows results produced for Gibrat growth without death, with birth but no death, and with both birth and death. Note these points:

a. Growth of the top-ranked firm continues throughout the 100 years when neither birth nor death are allowed (the "1" columns). On the other hand, the fifth-ranked firm's market share is between 6 and 7 percent throughout the simulation.

b. The differential effect of birth without death (the "2" columns) is to increase competition and reduce market shares of firms in the industry. The effect is most pronounced for the top-ranked firm. At year 100, the top firm has 22.6% of the market, vs. 32.9% with no birth allowed. Lower-ranked firms are not as severely affected. The results for all ranks appear to have stabilized by year 60.

c. The differential effect of death on the growth-birth combination is to increase the market share of the surviving firms (the "3" columns). By year 80, the top-ranked firm's share appears to have stabilized about mid-way between its share without birth or death and its share with birth but no death. Change in the other firms' shares, though modest, is in an upward direction. In fact, the lower-ranked firms do somewhat better on balance than when neither birth nor death are allowed.

Table 5 is strictly analogous to Table 4., except that the growth process is Mansfield. The pattern of results resembles Table 4:

Combination	Growth Process*	Birth Process*	Death Process*
1	Gibrat	No Birth	No Death
2	Gibrat	Industry-Independent	No Death
3	Gibrat	Industry-Independent	As Described
4	Mansfield	No Birth	No Death
5	Mansfield	Industry-Independent	No Death
6	Mansfield	Industry-Independent	As Described
7	Size-Independent	No Birth	No Death
8	Size-Independent	Industry-Dependent	As Described

Table 3. Growth-Birth-Death Combinations Modeled

*See Section 3. for an Explanation of Terms

Year:	20			40			60			80			100		
Combination:	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Rank 1:	12.9	11.6	14.9	20.0	17.5	21.1	26.0	22.0	24.0	27.3	21.0	27.1	32.9	22.6	27.5
2:	9.3	8.6	11.6	11.4	10.7	13.3	13.0	11.5	14.3	15.0	12.4	15.1	15.0	13.5	13.9
3:	7.8	7.4	9.6	9.3	8.7	10.3	9.7	8.9	10.6	10.4	9.1	10.0	9.6	9.5	10.3
4:	6.7	6.5	7.8	7.7	7.3	8.7	7.6	7.1	8.9	7.6	7.7	7.7	7.5	7.7	8.1
5:	6.1	6.0	7.2	6.3	6.3	7.4	6.1	5.8	7.2	6.7	6.5	6.5	6.2	6.1	6.7

Table 4. Partial Results for Combinations 1, 2, and 3 (Gibrat Growth): Percent Share of Market for 5 Top-Ranked Firms

Year:	20			40			60			80			100		
Combination:	4	5	6	4	5	6	4	5	6	4	5	6	4	5	6
Rank 1:	25.8	23.2	28.9	38.7	29.8	37.5	45.5	41.7	43.7	49.9	43.0	40.9	55.9	48.7	40.5
2:	16.0	13.8	19.1	22.8	19.3	24.1	26.7	23.3	25.5	32.0	24.6	27.3	32.4	23.8	28.8
3:	11.6	10.3	13.7	12.0	12.1	14.7	13.3	11.5	13.6	10.7	13.3	13.9	7.6	11.2	13.3
4:	7.8	8.3	10.1	7.9	8.3	8.8	5.1	6.7	7.6	3.4	6.4	7.0	2.3	7.2	7.1
5:	6.7	6.9	7.9	4.8	6.1	5.0	3.0	4.5	3.8	1.5	4.7	4.8	0.7	4.4	4.4

Table 5. Partial Results for Combinations 4, 5, and 6 (Mansfield Growth): Percent Share of Market for 5 Top-Ranked Firms

a. When neither birth nor death is allowed, the top-ranked firm continues to grow throughout the simulation (the "4" columns). The top firm has 55.9% of the market by year 100, in contrast to 32.9% for the Gibrat-equivalent case. This is consistent with the relative stability of the percentage-change population from which the top-ranked firm is sampling at least by year 40 (market share exceeds 34%, dropping the standard deviation to 0.08).

b. The differential effect of birth is to decrease the market share of the top two firms (the "5" columns). In contrast with the equivalent Gibrat case, however, the firms ranked 3, 4, and 5 have experienced a noticeable improvement in their market share. The reason for this is that there are fewer firms still in the industry in later years with Mansfield growth than with Gibrat. With a standard deviation of 0.32, some small Mansfield firms are eventually dropping to zero in sales and therefore "dying". This is true even though the formal death mechanism (minimum feasible scale of plant) is not in effect. In fact, at year 100 there are only 9 firms left on average in the Mansfield industry (data not shown in tables) even though, with birth and theoretically no death, there should be at least 25.

c. When the growth-birth combination is augmented by the death process (the "6" columns), firms ranked 2 and 3 enjoy an improvement in market share, firms 4 and 5 are unchanged, and the top-position firm loses ground. On average, there are 8.4 firms in the industry in this case (data shown in Table 8.) vs. 9 with the death process excluded. There appears to be no necessary reason why ranks 2 and 3 have grown at the expense of rank 1. The result might be attributable to randomness. The standard deviation at year 100 of the market shares of firms ranked 1, 2, and 3 is 8.7, 7.7, and 5.6, respectively (data not shown in tables).

Table 6 shows the effects of size-independent growth without birth-death, and with birth-death. The no-birth, no-death case (the "7" columns) bears out that, for the growth mechanism in use, the firms have little tendency to separate into sharply differentiated ranks. When birth and death are introduced (the "8"

Year:	20		40		60		80		100	
Combination:	7	8	7	8	7	8	7	8	7	8
Rank 1:	4.9	7.7	5.5	7.9	6.0	9.0	6.7	11.1	6.8	12.4
2:	4.7	7.1	5.2	7.5	5.6	7.8	6.1	8.9	6.2	10.1
3:	4.6	5.2	5.0	6.6	5.3	7.0	5.7	8.3	5.9	9.2
4:	4.5	5.1	4.9	5.5	5.2	5.9	5.4	7.2	5.6	8.6
5:	4.5	5.0	4.8	5.3	5.0	5.7	5.3	6.0	5.3	8.2

Table 6. Partial Results for Combinations 7 and 8 (Size-Independent Growth): Percent Share of Market for 5 Top-Ranked Firms

Year:	20			40			60			80			100		
Combination:	3	6	8	3	6	8	3	6	8	3	6	8	3	6	8
Rank Last-4:	4.0	8.0	6.2	3.6	16.6	5.4	2.9	17.8	5.6	2.2	14.4	7.7	1.5	11.1	8.4
Last-3:	3.6	6.2	5.6	3.2	10.3	5.2	2.4	9.5	5.0	1.8	6.3	6.0	1.3	7.5	7.0
Last-2:	3.2	4.5	3.7	2.8	5.4	4.4	2.1	4.9	4.3	1.6	3.6	5.5	1.1	2.3	6.4
Last-1:	2.8	3.6	3.5	2.2	3.7	3.3	1.6	3.1	3.1	1.3	2.2	4.5	0.8	1.5	5.9
Last:	2.2	2.8	3.3	1.8	2.0	3.0	1.2	1.4	2.8	1.0	1.3	3.0	0.5	0.9	5.5

Table 7. Partial Results for Combinations 3, 6, and 8:
Percent Share of Market for 5 Smallest Firms

Year:	20			40			60			80			100		
Combination:	3	6	8	3	6	8	33	6	8	3	6	8	3	6	8
A:	16.4	9.8	22.9	15.3	8.1	22.9	15.8	7.6	21.6	17.0	7.9	18.8	18.5	8.4	14.8
B:	15.9	7.7	18.4	13.9	5.0	15.6	13.1	3.8	13.0	12.7	3.3	11.0	12.5	2.8	8.4

Table 8. Partial Results for Combinations 3, 6, and 8:
A: Number of Firms in Industry; B: Number of Original Firms Still in Industry

columns), a stronger tendency toward moderate skewness is evident. With the growth process used, combination 8 measures the ability of the imposed birth-death processes to induce skewness. Of course, the dominant concentration arising with Gibrat and especially Mansfield growth does not arise here.

Table 7 shows the market shares of the 5 lowest-ranked firms with birth and death in effect for each of the three different growth processes. For Mansfield growth (the "6" columns), the fourth firm from the bottom has a substantial market share. This is because the "fourth from the bottom" is also approximately the "fourth from the top" in the Mansfield case, where only 8 to 10 firms survive from year 20 forward. Other entries in Table 7. are in the 1 to 5 percent range, with some falling below 1 percent. These values should be compared with those in Table 2., where market shares corresponding to minimum feasible plant size are shown for various industries. Table 2. supports Table 7. fairly well, especially those parts of Table 7. corresponding to years later in the simulation. It might be argued, however, that the death condition used here was generally too severe.

Table 8 shows the number of firms in the industry as a function of time, and how many of those firms were initially in the industry when it was born. Note that from year 20 forward, all combinations have fewer than the initial 25 firms remaining. Original firms have the best chance of long-run survival with a Gibrat growth process (the "3" columns). As would be expected, Mansfield growth has eliminated about 70% of the original firms by year 20 (the "6" columns). Original firms still make up over 75% of the industry at that time, however. At year 100, 65% of the industry consists of original firms for Gibrat, 35% for Mansfield, and 57% for size-independent growth processes.

Table 9 is included to give some general indication of the dispersion of typical mean values displayed in Tables 4. through 8. There is considerable variation in the samples used to compute the reported means. Hence, an industry rigidly adhering to one of these birth-growth-death combinations might well evolve a concentration pattern strongly dissimilar to the mean patterns reported here.

Figure 1 displays the results in the upper tail of the distribution for model combinations 3, 6, and 8 on a double-logarithmic graph. Also shown in Figure 1. is the corresponding portion of the steel industry data from Table 1. The tendency toward a linear relationship indicated under Point 1., Section 2, is evident. Also note:

a. Gibrat growth is the middle-of-the-road case, producing a "good" approximation to a

Combination:	3		6		8	
	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
Rank 1:	24.0	7.1	43.7	12.0	9.0	7.6
2:	14.3	3.2	25.5	5.6	7.8	5.5
3:	10.6	2.2	13.6	5.9	7.0	3.7
4:	8.9	1.8	7.6	4.0	5.9	2.1
5:	7.2	1.1	3.8	1.7	5.7	2.0

Table 9. Partial Results for Combinations 3, 6, and 8:
Sample Mean and Standard Deviation Associated with Percent
Market Shares for 5 Top-Ranked Firms at Year 60

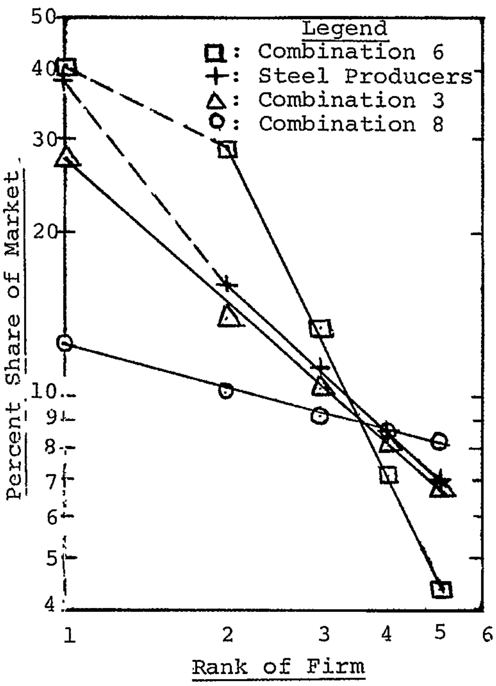

Figure 1. Double-Log Plot of Selected Statistics

linear relationship. The steel industry data parallels the Gibrat prediction quite well for firms ranked 2, 3, 4, and 5.

b. Mansfield growth produces the greatest skewness. If the line based on firms ranked 2 through 5 were extended, however, a relative market share exceeding 100% would be predicted for the top firm.

c. The size-independent growth and Mansfield growth bracket the Gibrat situation. Except for the top Mansfield firm, the numeric Gibrat results could be constructed with a combination of the Mansfield and size-independent results. With a different functional relationship between the standard deviation of the percentage-change population and market share in the Mansfield case, the top Mansfield firm could very likely also be brought into line.

6. Summary and Conclusions

1. Mansfield's qualitative suggestion that the standard deviation of the distribution of percentage change in firm size should vary with firm size has been given a quantitative interpretation and implemented. The resulting stochastic model produces highly skewed firm-size distributions. Mansfield's proposed growth mechanism is more realistic than that of Gibrat. It is also more flexible because of the ways in which standard deviation can be functionally related to the market share of firms.

2. A particular birth-death mechanism has been shown to induce moderate skewness in firm-size distributions.

3. Weighted combinations of Mansfield growth and size-independent growth can be used to explain a wide variety of empirically found firm-size distributions. The value in combining the two alternatives is that the underlying assumptions can be given economic interpretations. For example, the birth and death processes in the models discussed here are based on economic interpretations of level of industry attractiveness and minimum efficient scale of plant, respectively.

3. The differential effect of new-firm entry into an industry is to lower concentration, as is expected.

4. The differential effect of death is to raise concentration without restoring it to its levels before birth and death were imposed.

5. A model or models of this type could be used by an individual firm within an industry to help assess its future market position under alternative hypotheses.

6. It is claimed that the models presented here are more useful than their predecessors. They produce similar results based on weaker, more realistic assumptions. The various model components are all interpretable in economic terms.

10. Biographies

Alan R. Beckenstein is a student in the Ph.D. program in Economics at The University of Michigan, where he is also a Research Affiliate of the Brookings Institution. His Doctoral dissertation will be done in the general area of multi-plant operating strategy. Before beginning his Ph.D. studies, he was with the consulting firm of R. Shriver and Associates.

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11. References

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