ABSTRACT

THE DEVELOPMENT OF A STATE-SPACE MODEL OF A FIRM

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This study constructs what is essentially an accounting-type model of a firm to simulate the flow of the various products thru the stages of the basic line processes and the flow of resources into the products. The basic resources are labor, material and facilities. These resources flow both directly into the product at various production stages and indirectly thru the various overhead and service functions which in turn inject their resources into the product. To manipulate these flows and their costs, management uses certain control variables such as cost reduction and sales promotion activities. Figure 1 illustrates the gist of the model. Figure 2 presents a simplified close-up view of the flow of product and resources into and out of a given production sector.

The methodology used was originally developed for analyzing system models of electrical networks. Like an electrical system, a firm can also be conceptualized as a system of interacting components. The system components and their interfaces can be represented by a linear graph of points (vertices) and connecting lines (edges). The behavioral characteristics of a component can be specified by a pair of complementary variables associated with each edge. The X variable is called the propensity variable and is usually regarded as the "cause" or "result" of the flow or Y variable. In electrical processes, X and Y can be voltage and current respectively. In this model of the firm, X and Y are imputed cost and flow of units thru production. Figure 3 is the system graph of the model of the firm.

The theory gives explicit procedures for developing a set of simultaneous algebraic and/or differential or difference equations showing the interdependence of a set of variables which characterize the observable behavior of the system. These equations are composed of the system's component characteristic equations and the constraint equations from the interconnection pattern of the linear graph. The reduction of this set of equations to a minimal set of first ordered difference or differential equations produces the state model.

Its basic form is: $\psi(n+1) = P\,\psi(n) + QE(n) + SF(n)$ where ψ is the state vector, P is the transition matrix, Q is the excitation matrix, E is the control vector, F is a vector of known functions of time and S is its coefficient matrix. Thus next period's state is expressed as a function of the current state, the control vector and known functions of time.

The model can be used for heuristic simulation by solving for $\psi(n)$. This solution is:

$$\psi(n) = P^{n}\,\psi(0) + \begin{bmatrix} P^{n-1}Q & P^{n-2}Q & \cdots & P^{o}Q \end{bmatrix} \begin{bmatrix} E(0) \\ E(1) \\ \vdots \\ E(n-1) \end{bmatrix} + G(n)$$

$$\text{where } G(n) = \begin{bmatrix} P^{n-1}S & P^{n-2}S & \cdots & P^{o}S \end{bmatrix} \begin{bmatrix} F(0) \\ F(1) \\ \vdots \\ F(n-1) \end{bmatrix}$$

The firm is described by the flows and costs of all the resources and products as measured at various points or sectors within the firm. By following certain methodological procedures, specific flow and cost variables are selected to comprise the state vector. These particular flow and cost variables have no significance to the layman other than that given these variables, one can solve for every other flow and cost variable in the system.

Thus by starting out with the present flows and costs within the firm, one can try out different control strategies and simulate what the flows and costs of resources and products would be at any sector of the firm and at any future point in time. Considerable practical utility is found for both the methodology and the model. Relative to other techniques, the methodology provides a convenient formal and generalized procedure for developing a model of the firm.

The model has many managerial uses such as forecasting costs, load, resource requirements or cash flow and analyzing total impact of alternative decisions. The fact that the model automatically takes into account all the critical interactions of the firm, assists the executive in an area where he has traditionally felt impotent.

The methodology used allows not only simulation but also stability and control analysis. The stability of the model can be determined analytically without having to resort to heuristic simulation.

The state controllability of the system can also be analyzed. A system is said to be state controllable if by means of a series of control signals, it can be brought from any initial state to any desired state in a finite period of time. If a system is found to be state controllable, the values of the control signals necessary to bring the system to a desired state can be computed.

For the model described in this paper, the necessary parametric data was estimated and substituted into the model. Heuristic simulation was performed. This prototype model was found to be stable and controllable.

The most promising facet of the approach used in this study is its generalized analytical capabilities. If it can assist management planning by indicating the control values that will lead to the attainment of the objectives of the firm, this will be an invaluable contribution. As management pins down more of the true cause and effect relationships in the firm, the control strategy from the model will play an ever increasing role in guiding their key decisions.

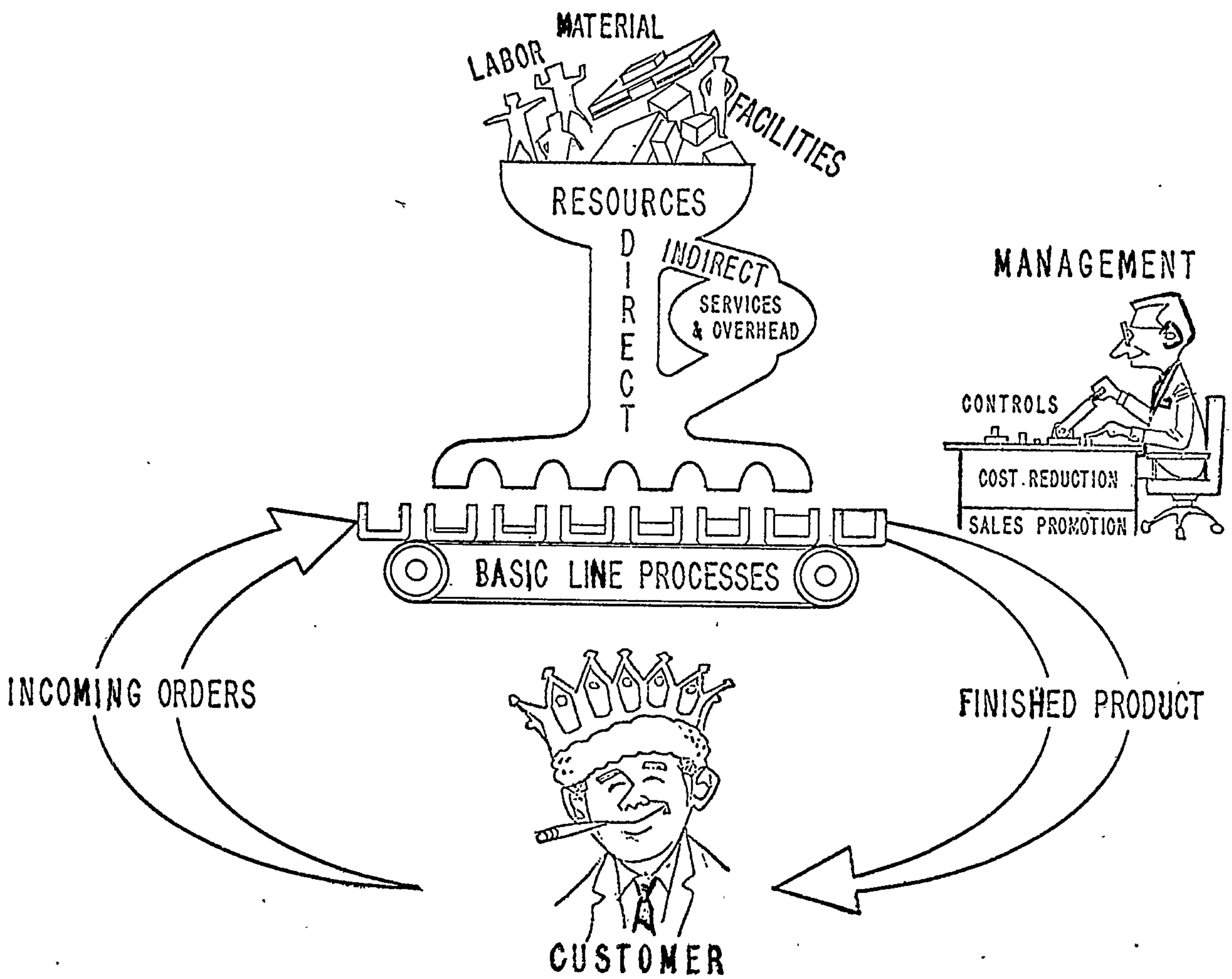

FIGURE 1

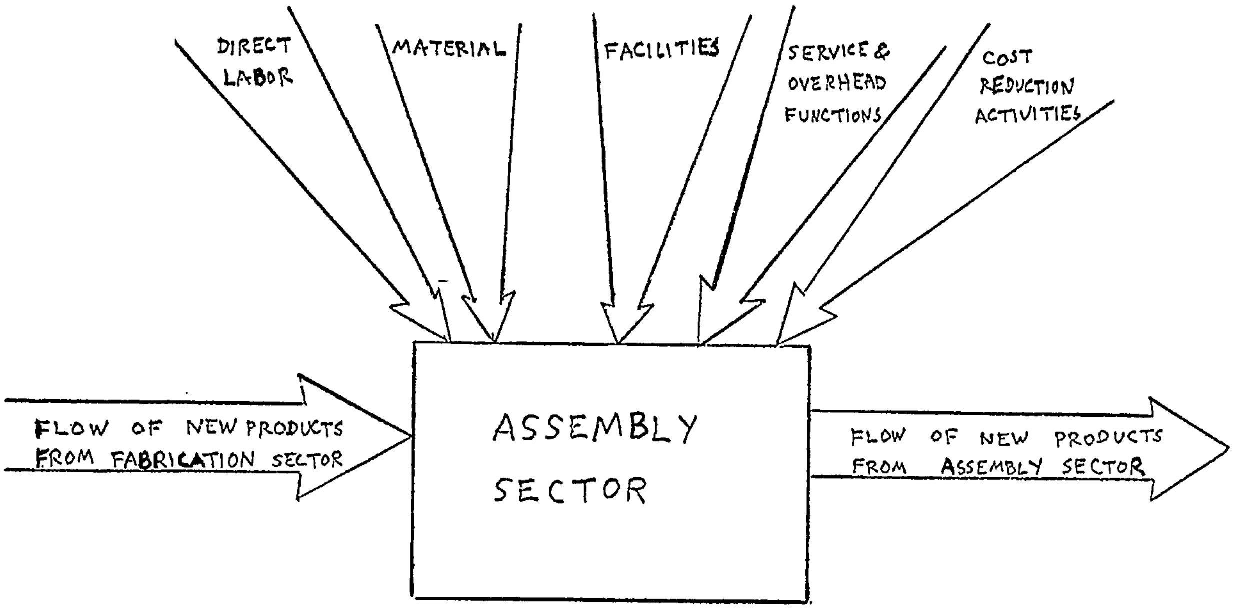

FIGURE 2

SYSTEM GRAPH OF THE MODEL OF THE ENTERPRISE

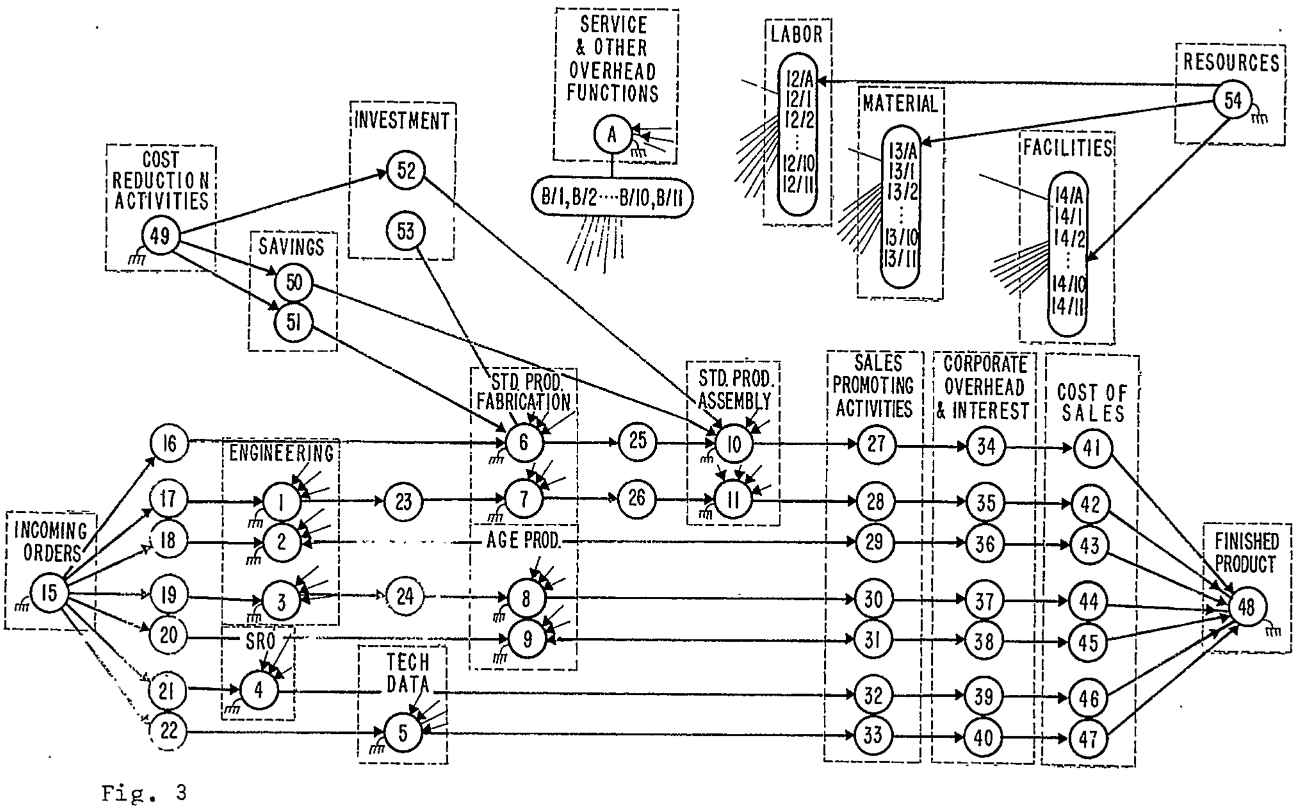

Fig. 3