Proceedings of the 2025 Winter Simulation Conference
E. Azar, A. Djanatliev, A. Harper, C. Kogler, V. Ramamohan, A. Anagnostou, and S. J. E. Taylor, eds.

OPTIMAL PLACEMENT OF SENSORS AT MINIMAL COST FOR CAUSAL OBSERVABILITY IN DISCRETE EVENT SYSTEMS

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ABSTRACT

This study addresses the problem of optimal sensor placement in discrete-event systems modeled by partially observable Petri nets, with the dual objectives of ensuring causal observability and minimizing total installation cost. The system is algebraically reformulated as a descriptor system, which leads to a combinatorial, nonlinear, and non-convex optimization problem. To overcome these difficulties, two metaheuristic approaches, simulated annealing and genetic algorithms, are proposed and evaluated on a complex manufacturing system. The results obtained highlight the effectiveness and scalability of both methods, underscoring their strong potential for industrial applications.

1 INTRODUCTION

In automated industrial or transportation systems, supervision and safety depend on sensors that monitor system components. However, installing sensors on every component is rarely feasible. Technical factors like physical inaccessibility or limited sensor interfaces, along with economic factors such as installation and maintenance costs, make full direct observation of the system impractical. Therefore, the challenge is to select the minimum number of sensors from potential locations to ensure system observability.

This work addresses this issue using a modeling framework based on Partially Observable Petri Nets (POPN), which provide a compact and expressive representation of discrete-event systems (DES). The POPN formalism distinguishes between observable and unobservable places and transitions, allowing partial information to be incorporated directly into the model structure. The POPN is reformulated as a discrete-time descriptor system (DTDS) to analyze observability conditions, where system evolution is described by a linear equation with a generalized state vector. Structural observability is achieved if the corresponding augmented matrix has full rank. The objective is to find the sensor placement matrix that ensures observability at minimal installation cost, resulting in a binary, nonlinear, and nonconvex combinatorial optimization problem. Due to its complexity, classical methods are unsuitable, so Simulated Annealing and Genetic Algorithm are applied and validated on a complex manufacturing system case study.

2 PROBLEM FORMULATION

We consider systems modeled by POPN, and be reformulated as DTDS. The evolution

$$E\Pi_{k+1} = A\Pi_k, \quad \psi_k = C\Pi_k \tag{1}$$

where:

- $\Pi_k \in \mathbb{R}^{n+m}$ is the generalized state vector, which combines the place markings μ_k and the transition firing vector τ_k, and the output vector ψ_k corresponds to the subset of observable variables,
- $E = \begin{bmatrix} I_n & -\Delta \end{bmatrix}$, with $\Delta \in \mathbb{R}^{n\times m}$ the incidence matrix of the Petri net, and $A = \begin{bmatrix} I_n & 0 \end{bmatrix}$,
- $C \in \{0,1\}^{s\times(n+m)}$ is the sensor placement matrix, Let n be the number of places, m the number of transitions, and s the number of sensors to be placed on either places and transitions.

Two objectives are considered :

1. **Causal observability**, which is ensured if the matrix $\begin{bmatrix} E \\ C \end{bmatrix} \in \mathbb{R}^{(n+s)\times(n+m)}$ is of full rank:

$$\text{rank}\left(\begin{bmatrix} E \\ C \end{bmatrix}\right) = n+m \tag{2}$$

2. **Minimal total cost**, where each variable (place or transition) $j \in \{1,\ldots,n+m\}$ is associated with a known cost $c_j > 0$. The total cost is given by:

$$\text{Cost}(C) = \sum_{j=1}^{n+m} c_j \left(\sum_{i=1}^{s} C_{i,j}\right) \tag{3}$$

The problem is subject to the following constraints:

- Each sensor observes exactly one variable: each row of C contains a single one,
- Each variable is observed at most once: each column of C contains at most one,
- The matrix C is binary: $C_{i,j} \in \{0,1\}$ for all i, j.

This yields a non-linear, non-convex combinatorial problem, where the classical methods struggle due to the binary nature of C, a non-convex rank constraint and an exponentially large search space.

3 METHODS AND RESULTS

Given the problem's complexity, exact methods become computationally prohibitive for large systems. Metaheuristics offer a suitable alternative, and in this work, two approaches are proposed:

1. **Simulated Annealing (SA)**: starts from a single solution and stochastically explores its neighborhood, applying a temperature-dependent acceptance rule to avoid getting trapped in local minima.
2. **Genetic Algorithm (GA)**: maintains a population of candidate solutions and uses selection, crossover, and mutation operators to gradually evolve the population toward better configurations.

A case study on a complex manufacturing process demonstrates the effectiveness of both methods, evaluated in terms of solution quality, convergence speed, and robustness. SA converges faster through efficient local exploration, providing quick feasible solutions, while GA, benefiting from population diversity, explores a wider search space to deliver more accurate and cost-effective results.

4 CONCLUSION

This study proposes an efficient framework for optimal sensor placement in systems modeled by POPN, incorporating both structural (observability) and economic (cost) constraints. The metaheuristic approaches developed, SA and GA, successfully address the combinatorial and nonconvex nature of the problem. Simulation results validate their applicability to large-scale systems. Future work will explore extensions to incorporate fault tolerance, redundancy, and sensor reliability metrics.

REFERENCES

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Bouchra Mamoun, Fayssal Arichi, Mohamed Djemai, and Brahim Cherki. Optimal sensor selection and placement for discrete event systems: An overview. In *2024 IEEE International Multi-Conference on Smart Systems & Green Process (IMC-SSGP)*, pages 1–6. IEEE, 2024.