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MULTI-OBJECTIVE PROBABILISTIC BRANCH-AND-BOUND FOR BIKE-SHARING STORAGE PROBLEM

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ABSTRACT

This study addresses bike-sharing system (BSS) storage level management using a multi-objective simulation optimization (MOSO) approach. We consider two objectives: minimizing customers unable to rent bikes and those unable to return bikes due to full stations. A discrete-event simulation of Taipei's top ten stations evaluates system performance over one day. We develop a discrete version of the multi-objective probabilistic branch-and-bound (MOPBnB) algorithm to approximate the Pareto optimal set for this discrete MOSO problem. Convergence analysis and numerical results demonstrate the algorithm's effectiveness in identifying trade-offs, providing practical insights for BSS storage management.

1 INTRODUCTION

Bike-sharing systems (BSS) have become widely utilized worldwide as a convenient mode of urban transportation. An essential operational challenge within BSS is the management of rack inventories to ensure satisfactory service levels for users. While most existing studies address this storage level management problem using a single-objective cost framework (Swaszek et al. 2019), this approach may limit the insight available to decision-makers. In this study, we model and evaluate the storage level management problem in BSS using discrete-event simulation and further formulate it as a multi-objective simulation optimization (MOSO) problem. By identifying the Pareto optimal set, the MOSO framework enables a more comprehensive understanding of the trade-offs inherent in system performance. To address this problem, we extend the multi-objective probabilistic branch-and-bound (MOPBnB) algorithm originally developed for continuous domains (Huang and Zabinsky 2014) and propose a discrete version of the MOPBnB algorithm tailored to the BSS storage level management context.

2 METHODOLOGIES

The multi-objective simulation optimization (MOSO) problem for storage level management in BSS considered in this study is formulated as: $\min_{x\in\Omega} g_i(x) = E[G_i(x)]$, $i = 1,2$, where Θ denotes the feasible solution space bounded by box constraints. The objective of this study is to approximate the Pareto optimal set $P(\Theta)$, which consists of all non-dominated solutions. The objective functions $G_i(x)$ are evaluated using a discrete-event simulation model.

In contrast to prior studies that aggregate multiple service objectives into a single weighted cost function, we directly consider the following two objectives: (1) minimizing the number of customers unable to rent a bike due to insufficient bike availability, and (2) minimizing the number of customers unable to return a bike due to full docking stations. The simulation considers the top ten stations in Taipei City, with bike rental and return requests generated using exponential inter-arrival times and uniformly distributed processing times. The system is simulated over a one-day period. Rental and return events are assumed to wait for a maximum duration drawn from a uniform distribution; if this maximum waiting time is reached, the event is recorded as a failed rental or return attempt. It is further assumed that customers unable to rent will balk and leave the system, whereas customers attempting to return a bike will continue to seek available docks until successful.

To approximate the Pareto optimal set for this discrete MOSO problem, we employ a discrete version of the multi-objective probabilistic branch-and-bound (MOPBnB) algorithm, extending the original MOPBnB developed for continuous domains (Huang and Zabinsky 2014). The discrete MOPBnB iteratively samples solutions to construct a sampled non-dominated set and partitions the solution space into subregions categorized as either pruned or current. Current subregions are those that may contain elements of the Pareto optimal set, while pruned subregions are discarded from further consideration. The algorithm proceeds in three phases: (1) Sampling and Dominance Evaluation, (2) Subregion Updating, and (3) Branching and Termination. The discrete MOPBnB allocates a varying number of simulation replications across subregions, prioritizing undecided current subregions to improve the precision of dominance evaluations. Notably, unlike the original continuous MOPBnB, the discrete version permits revisiting current subregions, enhancing the exploration and refinement of the Pareto front approximation in the discrete setting.

3 RESULTS AND DISCUSSIONS

Both theoretical convergence analysis and numerical experiments are conducted to evaluate the performance of the discrete MOPBnB algorithm. First, the convergence analysis establishes that the current subregions identified by the discrete MOPBnB converge to the Pareto optimal set as the number of iterations $k \to \infty$. Second, numerical experiments are performed using the storage level management problem in BSS as the test environment. Table 1 presents the results obtained after 125 iterations of the discrete MOPBnB, yielding 41 non-dominated solutions. The number of customers unable to rent a bike (Cost_NoCar) ranges from approximately 20 to 190, while the number of customers unable to return a bike (Cost_NoPosition) ranges from approximately 6,000 to 26,000. The trade-off relationship between the two objectives is clearly demonstrated: as the serial number of non-dominated solutions increases, Cost_NoCar increases while Cost_NoPosition decreases.

The notably larger magnitude of Cost_NoPosition compared to Cost_NoCar results from the modeling assumption that customers unable to return a bike continue to search for an available docking position rather than leaving the system. For the simulation, the rack capacities at the ten stations are set as (61, 49, 99, 99, 64, 49, 52, 81, 28, 30). As the serial number of the non-dominated solutions increases, the overall storage levels across stations generally decrease. However, certain stations, such as the 4^{th} and 8^{th}, tend to maintain relatively higher storage ratios across the solution set.

Table 1: Part of numerical experiment results.

No.	Station Storage Level	Cost_NoCar	Cost_NoPosition
1	(56,39,75,98,53,47,51,79,27,25)	20.10	25715.35
11	(29,38,37,56,50,22,43,73,28,16)	54.35	13233.35
21	(30,26,25,62,47,0,44,77,24,13)	96.00	9795.20
31	(14,29,19,51,54,1,23,67,14,12)	126.85	7435.40
41	(6,26,29,52,52,0,16,51,12,2)	183.20	5981.25

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