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# SIMULATION CASE STUDIES ON AN ADVANCED SENSITIVITY ANALYSIS FOR NEW EXTENDED BUS TYPES IN THE MODERN POWER SYSTEMS

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## ABSTRACT

A simulation analysis in power system planning and operations considers the sensitivity of power flow, stability, and security of the system in response to changes in system states. Traditionally, these analyses have the power system defined by three bus types; a generator bus ($PQ$), a load bus ($PV$) and the reference bus ($V\theta$). As the contribution from variable renewable energy sources (such as wind and solar) increase along with proliferation of new electronic control devices, these traditional bus types are no longer sufficient. Reliance on the traditional bus types for sensitivity analysis may not guarantee simulation accuracy and may lead to poor decisions, thereby jeopardizing system reliability and stability. To address this issue, this paper proposes new bus types (such as $PQV, Q\theta, QV, PQV\theta$) that arise in modern power systems, and determines the corresponding parametric sensitivity calculation formulas based equations of the new bus types. The updated sensitivity analysis is implemented on two simulation case studies; the IEEE 14-bus test system, and on an example of power system with 262 buses. Results show that the use of the traditional bus types leads to significant error, whereas the use of extended bus types produces more accurate results.

## 1 INTRODUCTION

In numerical and simulation modeling, sensitivity analysis is a method that measures how changes in input variables can lead to impacts on the output variables (Saltelli 2002), and has been widely applied in many applications ranging from engineering to social networks to finance. (Kurtc, Köster, and Fischer 2021; Razavi, Jakeman, Saltelli, Prieur, Iooss, Borgonovo, Plischke, Piano, Iwanaga, Becker, et al. 2021).

In the power systems context, sensitivity analysis is used to identify influential components on system performance, including the generation of slow oscillations (Zhang, Mahmud, Govaerts, Chen, Xu, and Xiong 2020), real-time contingency analysis (Mitra 2016), and voltage stability assessment (Djari, Benasla, and Rahmouni 2017). To guarantee system stability, reliability, and security, load flow analysis is an important approach to determining how the power system performs during normal and contingency operating conditions in both operations and planning phases (Carpentier 1962; Molzahn, Hiskens, et al. 2019; Bent, Toole, and Berscheid 2011). Specifically, load flow analysis solves the steady state operational node voltages and branch power flows across the network, using the nonlinear real and reactive power balance equations for the system.

Sensitivity analysis in power systems analysis is used to calculate changes in branch flows, power losses and bus voltage due to variations in generation and loads. For example, to analyze the sensitivity of reactive power injection, the variation in voltage with respect to changes in reactive power injection has been widely used to calculate sensitivity parameters. Sensitivity analysis is a major criterion for determining the load flow impacts given small variations on boundary conditions (Peschon, Piercy, Tinney, and Tveit

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1968; Chang, Liu, and Yang 1992; Chen, Domínguez-García, and Sauer 2013; Gruosso, Netto, Daniel, and Maffezzoni 2019; Olabode, Okakwu, Alayande, and Ajewole 2020).

More specifically, during the load flow analysis, there are four systematic parameters under each bus: nodal active power injection $P$; nodal reactive power injection $Q$; nodal voltage magnitude $U$; and nodal voltage phase angle $\theta$. Each of the variables could be either known or unknown, thus there are up to 16 different bus types, as shown in Fig. 1, where different bus types are named under the variables that are given. In traditional power systems, the sensitivity parametric analysis uses three bus types, $PQ$, $PV$ and $V\theta$, where $PQ$ bus is considered as a bus connected with loads; $PV$ bus is considered as a bus connected with a generator; $V\theta$ bus is considered as a reference bus with the voltage magnitude and phase angle being selected as 1 p.u. and 0, respectively. The traditional power equipment modeling can be classified into these bus types. With the current transition underway and increasing integration of renewable resources and advanced power equipment ( e.g., electronic control devices), extended bus types are required. In addition, the increasing computational complexity of the uncertainties from large-scale, non-linear and dynamic power systems, add to the challenge. As a result, the traditional sensitivity analysis method is no longer sufficient for reliable analysis.

| $P$ | $Q$ | $U$ | $\theta$ | Bus type | $P$ | $Q$ | $U$ | $\theta$ | Bus type |
|---|---|---|---|---|---|---|---|---|---|
| √ | — | — | — | $P$ | — | — | — | — | 0 |
| √ | — | — | √ | $P\theta$ | — | — | — | √ | $\theta$ |
| √ | — | √ | — | $PV$ | — | — | √ | — | $V$ |
| √ | — | √ | √ | $PV\theta$ | — | — | √ | √ | $V\theta$ |
| √ | √ | — | — | $PQ$ | — | √ | — | — | $Q$ |
| √ | √ | — | √ | $PQ\theta$ | — | √ | — | √ | $Q\theta$ |
| √ | √ | √ | — | $PQV$ | — | √ | √ | — | $QV$ |
| √ | √ | √ | √ | $PQV\theta$ | — | √ | √ | √ | $QV\theta$ |

Figure 1: All available bus types including traditional bus types and extended bus types.

The literature in recent years has acknowledged the need for extended bus types in load flow analysis, online system security, and voltage stability assessment. Extended bus types were proposed in (Lu Wang 1990), which included the discussion of seven new types of buses. Papers (Chunlei, Hongbo, and Guoyu 1997; Pan and Zhang 2008; Barik and Das 2020) proposed a $PQV$ bus type to model a unified power flow controller for load flow calculation. (Kumari and Sydulu 2006) applied a $PQV$ bus into the load flow model considering static VAR compensator, with specified nodal voltage magnitude, active and reactive power. In (Variz, Pereira, Martins, et al. 2003), a generator bus controlling the voltage at a remote bus is defined as $P$ bus and the remote bus is defined as $PQV$ bus in the load flow model. The $P$ bus and $PQV$ bus were also introduced into load flow model for secondary voltage regulation in (Berizzi, Bovo, Delfanti, Merlo, and Tortello 2006). In addition, other extended bus types were also proposed, for example a power flow method that includes $Q\theta$ node was applied in (JIANG, WU, ZHANG, WANG, and SUN 2007) to reconstruct online security network modeling. In (Jaiju, Zhenstang, and Xiaong 1995; Li, Ding, and Du 2009), a $P_{i_f}$ bus was defined to deal with the excitation generators where the field current is beyond limits. A bus-type extended power flow with solvability criteria and steady-state voltage stability margins were discussed in (Liu, Zhai, Zhang, Yang, and Liu 2015; Ghiocel and Chow 2013; Wang, Xu, Liu, and Zheng 2018). Based on that, an extended-power load flow which considered branch power as substitutes to deal with the missing power injection caused from the strict requirement of traditional bus types was presented in (Lei 2013). A bus-type extended continuation power flow model was proposed and utilized to compute the voltage stability critical point and identify the type of bifurcation with different remote voltage control modes in (Zhao, Zhou, and Chen 2013).

The literature highlights the fact that sensitivity analysis is one of the key tools to ensure system reliability, stability and security. However, when new extended bus types are included in the power system, the accuracy of the existing methods should also be considered. In addition, the sensitivity analysis is a linear approximation of the true sensitivity that could be achieved from a perturbation analysis. However, the perturbation analysis is computationally too expensive to do on large-scale power systems. Therefore, to solve the computational tractability, this paper proposes an advanced sensitivity parametric analysis based on new extended bus types with linearized approximations of the power flow equations. The corresponding theoretical formulas for extended bus types are derived for load flow calculation and adjustment. The method is then demonstrated and compared with the traditional method through simulation case studies on the IEEE 14-bus test system and a larger practical power system.

The remainder of this paper is organized as follows. Background on traditional sensitivity analysis methods are given in Section II. The advanced sensitivity analysis based on new extended bus types are presented in Section III. Subsequently, the simulation results are then tested via IEEE 14-bus system and a 262-bus practical power system in Section IV. Conclusions and practical applications are discussed in Section V.

## 2 Traditional Sensitivity Analysis

The primary bus types considered in sensitivity analysis and distribution factors are presented in this section within the framework of the load flow power analysis problem.

It is assumed that the corrective equation of reactive power in *PQ* decoupling analysis method on all nodes are expressed as

$$-\begin{bmatrix} \boldsymbol{L}_{DD} & \boldsymbol{L}_{DG} \\ \boldsymbol{L}_{GD} & \boldsymbol{L}_{GG} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{U}_D \\ \Delta\boldsymbol{U}_G \end{bmatrix} = \begin{bmatrix} \Delta\boldsymbol{Q}_D \\ \Delta\boldsymbol{Q}_G \end{bmatrix}, \tag{1}$$

where the subscripts $G, D$ represent the generator bus and load bus, respectively. In traditional load flow calculation analysis, the generator bus and load bus are usually modelled as *PV* and *PQ* bus types, respectively. Matrix $\boldsymbol{L}$ is the parameter-based iteration matrix of reactive power that considers all nodes in *PQ* decoupling analysis method for load power calculation.

The sensitivity analysis between terminal voltage on *PV* (generator) node, $\boldsymbol{U}_G$, and nodal voltage on *PQ* load node, $\boldsymbol{U}_D$, is shown as follows:

$$\begin{array}{l} \Delta\boldsymbol{U}_D = \boldsymbol{S}_{DG}\Delta\boldsymbol{U}_G \\ \boldsymbol{S}_{DG} = -\boldsymbol{L}_{DD}^{-1}\boldsymbol{L}_{DG} \end{array}. \tag{2}$$

When considering the reactive power on a generator bus and nodal voltage on a load bus, we assume that the inverse matrix of the parameter matrix shown in (1) is expressed as:

$$-\begin{bmatrix} \boldsymbol{L}_{DD} & \boldsymbol{L}_{DG} \\ \boldsymbol{L}_{GD} & \boldsymbol{L}_{GG} \end{bmatrix}^{-1} = \begin{bmatrix} \boldsymbol{R}_{DD} & \boldsymbol{R}_{DG} \\ \boldsymbol{R}_{GD} & \boldsymbol{R}_{GG} \end{bmatrix}. \tag{3}$$

Then the relationship between reactive power on *PV* generator node, $\boldsymbol{Q}_G$, and nodal voltage on *PQ* load node, $\boldsymbol{U}_D$, is shown as follows:

$$\Delta\boldsymbol{U}_D = \boldsymbol{R}_{DG}\Delta\boldsymbol{Q}_G. \tag{4}$$

When considering reactive power on *PV* (generator) node, $\boldsymbol{Q}_G$, and terminal voltage on *PQ* load node, $\boldsymbol{U}_G$, is expressed as:

$$\Delta\boldsymbol{U}_G = \boldsymbol{R}_{GG}\Delta\boldsymbol{Q}_G. \tag{5}$$

And finally, the sensitivity relationship between nodal voltage on *PQ* load node, $\boldsymbol{U}_D$, and transformer tap, $\boldsymbol{t}$, is shown as follows:

$$\begin{array}{l} \Delta\boldsymbol{U}_D = \boldsymbol{T}_{Dt}\Delta\boldsymbol{t} \\ \boldsymbol{T}_{Dt} = -\boldsymbol{L}_{DD}^{-1}\left[\dfrac{\partial\Delta\boldsymbol{Q}_D}{\partial\boldsymbol{t}^T}\right] \end{array}. \tag{6}$$

As previously argued, modern power systems require new bus types that extend beyond the characteristics of the traditional types. The details of the extended bus types are described in Section 3.

## 3 Sensitivity Analysis for Extended Bus Types

For reactive power iteration of the extended bus type load flow calculation and adjustment, all the nodes are divided into the following four types:

1. $\boldsymbol{Q}$ is known, $\boldsymbol{U}$ is unknown;

2. $\boldsymbol{Q}$ is unknown, $\boldsymbol{U}$ is unknown;

3. $\boldsymbol{Q}$ is known, $\boldsymbol{U}$ is known;

4. $\boldsymbol{Q}$ is unknown, $\boldsymbol{U}$ is known.

The corrective equation of reactive power on all nodes is:

$$-\begin{bmatrix} \boldsymbol{L}'_{11} & \boldsymbol{L}'_{12} & \boldsymbol{L}'_{13} & \boldsymbol{L}'_{14} \\ \boldsymbol{L}'_{21} & \boldsymbol{L}'_{22} & \boldsymbol{L}'_{23} & \boldsymbol{L}'_{24} \\ \boldsymbol{L}'_{31} & \boldsymbol{L}'_{32} & \boldsymbol{L}'_{33} & \boldsymbol{L}'_{34} \\ \boldsymbol{L}'_{41} & \boldsymbol{L}'_{42} & \boldsymbol{L}'_{43} & \boldsymbol{L}'_{44} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{U}_1 \\ \Delta\boldsymbol{U}_2 \\ \Delta\boldsymbol{U}_3 \\ \Delta\boldsymbol{U}_4 \end{bmatrix} = \begin{bmatrix} \Delta\boldsymbol{Q}_1 \\ \Delta\boldsymbol{Q}_2 \\ \Delta\boldsymbol{Q}_3 \\ \Delta\boldsymbol{Q}_4 \end{bmatrix}, \tag{7}$$

where the subscripts $1,2,3,4$ represent the type of nodes described above, respectively. In traditional power flow calculation, generator node $G$ is modelled as $PV$ node or $V\theta$ node, which refers to the $4^{th}$ bus type; while the load bus $D$ is modelled as $PQ$ node, which refers to the $1^{st}$ bus type.

Since some parameters are known and some are unknown, the actual corrective equation of reactive power for extended bus-type load flow calculation is characterized as:

$$-\begin{bmatrix} \boldsymbol{L}'_{11} & \boldsymbol{L}'_{12} \\ \boldsymbol{L}'_{31} & \boldsymbol{L}'_{32} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{U}_1 \\ \Delta\boldsymbol{U}_2 \end{bmatrix} = \begin{bmatrix} \Delta\boldsymbol{Q}_1 \\ \Delta\boldsymbol{Q}_3 \end{bmatrix}. \tag{8}$$

Based on (7) and (8), different types of sensitivity parametric analysis for extended bus type load flow calculation are described and shown in the following.

When it comes to the sensitivity factor between terminal voltage on generator and nodal voltage on load, since the terminal voltage value on generator node is given while the reactive power injection is not given, thus the generator node is considered as the $4^{th}$ bus type. However, for the load bus, since the nodal voltage is not given, it could be either considered as $1^{st}$ bus type or the $2^{nd}$ bus type according to the known or unknown value of the reactive power injection, respectively.

Here the $\Delta\boldsymbol{Q}_1, \Delta\boldsymbol{Q}_3, \Delta\boldsymbol{U}_3$ from (7) are all equal to 0. By extending the first and third rows in (7), we obtain the following (in matrix form):

$$\begin{bmatrix} \boldsymbol{L}'_{11} & \boldsymbol{L}'_{12} \\ \boldsymbol{L}'_{31} & \boldsymbol{L}'_{32} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{U}_1 \\ \Delta\boldsymbol{U}_2 \end{bmatrix} = -\begin{bmatrix} \boldsymbol{L}'_{14} \\ \boldsymbol{L}'_{34} \end{bmatrix} \Delta\boldsymbol{U}_4. \tag{9}$$

The inverse matrix of the parameter matrix shown in (9) is expressed as:

$$\begin{bmatrix} \boldsymbol{L}'_{11} & \boldsymbol{L}'_{12} \\ \boldsymbol{L}'_{31} & \boldsymbol{L}'_{32} \end{bmatrix}^{-1} = \begin{bmatrix} \boldsymbol{R}'_{11} & \boldsymbol{R}'_{12} \\ \boldsymbol{R}'_{31} & \boldsymbol{R}'_{32} \end{bmatrix}, \tag{10}$$

then according to (9) and (10), the sensitivity analysis between terminal voltage on generator and nodal voltage on load is then achieved as follows, respectively:

$$\Delta \boldsymbol{U}_1 = -\left(\boldsymbol{R}'_{11}\boldsymbol{L}'_{14} + \boldsymbol{R}'_{12}\boldsymbol{L}'_{34}\right)\Delta \boldsymbol{U}_4, \tag{11}$$

$$\Delta \boldsymbol{U}_2 = -\left(\boldsymbol{R}'_{31}\boldsymbol{L}'_{14} + \boldsymbol{R}'_{32}\boldsymbol{L}'_{34}\right)\Delta \boldsymbol{U}_4. \tag{12}$$

If the reactive power $\boldsymbol{Q}$ is given, then the sensitivity is calculated using (11), otherwise it is calculated with (12).

When we analyze the sensitivity factor for reactive power on a generator node, the reactive power is given while the terminal voltage is unknown, thus the generator bus is referred as the $1^{st}$ type and the load bus is still considered $1^{st}$ or the $2^{nd}$ bus type. By dividing the $1^{st}$ bus type into a generator bus and a load bus, (8) becomes:

$$-\begin{bmatrix} \boldsymbol{L}'_{GG} & \boldsymbol{L}'_{GD} & \boldsymbol{L}'_{G2} \\ \boldsymbol{L}'_{DG} & \boldsymbol{L}'_{DD} & \boldsymbol{L}'_{D2} \\ \boldsymbol{L}'_{3G} & \boldsymbol{L}'_{3D} & \boldsymbol{L}'_{32} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{U}_G \\ \Delta \boldsymbol{U}_D \\ \Delta \boldsymbol{U}_2 \end{bmatrix} = \begin{bmatrix} \Delta \boldsymbol{Q}_G \\ \Delta \boldsymbol{Q}_D \\ \Delta \boldsymbol{Q}_3 \end{bmatrix}. \tag{13}$$

Let us assume that the inverse matrix of the parameter matrix shown in (13) is:

$$\begin{bmatrix} \boldsymbol{L}'_{GG} & \boldsymbol{L}'_{GD} & \boldsymbol{L}'_{G2} \\ \boldsymbol{L}'_{DG} & \boldsymbol{L}'_{DD} & \boldsymbol{L}'_{D2} \\ \boldsymbol{L}'_{3G} & \boldsymbol{L}'_{3D} & \boldsymbol{L}'_{32} \end{bmatrix}^{-1} = \begin{bmatrix} \boldsymbol{R}''_{GG} & \boldsymbol{R}''_{GD} & \boldsymbol{R}''_{G2} \\ \boldsymbol{R}''_{DG} & \boldsymbol{R}''_{DD} & \boldsymbol{R}''_{D2} \\ \boldsymbol{R}''_{3G} & \boldsymbol{R}''_{3D} & \boldsymbol{R}''_{32} \end{bmatrix}. \tag{14}$$

If the reactive power on load bus is given, it is then referred as the $1^{st}$ bus type. The corresponding sensitivity relationship between nodal load voltage and reactive power on generator is:

$$\Delta \boldsymbol{U}_D = \boldsymbol{R}''_{DG}\Delta \boldsymbol{Q}_G. \tag{15}$$

Similarly, if the reactive power on load bus is unknown, it is then referred as the $2^{nd}$ bus type. The corresponding sensitivity parametric analysis between nodal load voltage and reactive power on generator is:

$$\Delta \boldsymbol{U}_2 = \boldsymbol{R}''_{3G}\Delta \boldsymbol{Q}_G. \tag{16}$$

According to (13) and (14), the sensitivity analysis between reactive power and terminal voltage on generator is:

$$\Delta \boldsymbol{U}_G = \boldsymbol{R}'''_{GG}\Delta \boldsymbol{Q}_G. \tag{17}$$

When considering the sensitivity between nodal voltage on a load bus and transformer tap, the generator bus is still considered as *PV* bus type, (13) is then simplified as:

$$-\begin{bmatrix} \boldsymbol{L}'_{DD} & \boldsymbol{L}'_{D2} \\ \boldsymbol{L}'_{3D} & \boldsymbol{L}'_{32} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{U}_D \\ \Delta \boldsymbol{U}_2 \end{bmatrix} = \begin{bmatrix} \Delta \boldsymbol{Q}_D \\ \Delta \boldsymbol{Q}_3 \end{bmatrix}. \tag{18}$$

Assume that the inverse matrix of the parameter matrix in (18) is:

$$-\begin{bmatrix} \boldsymbol{L}'_{DD} & \boldsymbol{L}'_{D2} \\ \boldsymbol{L}'_{3D} & \boldsymbol{L}'_{32} \end{bmatrix}^{-1} = \begin{bmatrix} \boldsymbol{R}'''_{DD} & \boldsymbol{R}'''_{D2} \\ \boldsymbol{R}'''_{3D} & \boldsymbol{R}'''_{32} \end{bmatrix}. \tag{19}$$

Since the reactive power injection on the $3^{rd}$ bus type is constant, that is, $\Delta \boldsymbol{Q}_3 = 0$. If the $\boldsymbol{Q}$ on load bus is given, then according to (18) and (19), we get that:

$$\Delta \boldsymbol{U}_D = \boldsymbol{R}''_{DD}\Delta \boldsymbol{Q}_D. \tag{20}$$

The sensitivity analysis between nodal load voltage and transformer tap then becomes:

$$\begin{aligned} \Delta \boldsymbol{U}_D &= \boldsymbol{T}'_{Dt} \Delta \boldsymbol{t} \\ \boldsymbol{T}'_{Dt} &= \boldsymbol{R}''_{DD} \left[ \frac{\partial \Delta \boldsymbol{Q}_D}{\partial \boldsymbol{t}^T} \right]. \end{aligned} \tag{21}$$

If the reactive power $\boldsymbol{Q}$ on the load bus is unknown, then

$$\Delta \boldsymbol{U}_2 = \boldsymbol{R}''_{3D} \Delta \boldsymbol{Q}_D, \tag{22}$$

and the sensitivity parametric analysis between nodal load voltage and transformer tap is expressed as:

$$\begin{aligned} \Delta \boldsymbol{U}_2 &= \boldsymbol{T}''_{Dt} \Delta \boldsymbol{t} \\ \boldsymbol{T}''_{Dt} &= \boldsymbol{R}''_{3D} \left[ \frac{\partial \Delta \boldsymbol{Q}_D}{\partial \boldsymbol{t}^T} \right]. \end{aligned} \tag{23}$$

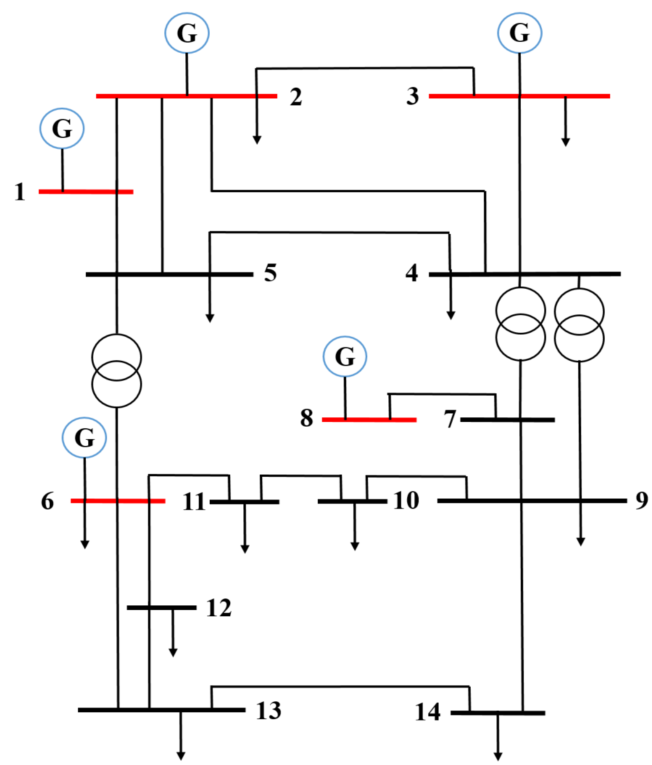


Figure 2: IEEE 14-bus test system.

## 4 Simulation Case Studies

In this section we use Matlab to run numerical simulations by applying the extended bus types in the analysis of two cases; first the IEEE 14-bus test system, and second a more realistic 262-bus network from China. The load flow calculation is run through interior-point method. Each of these case studies is described below.

### 4.1 Case Study 1: IEEE 14-bus test system

The *IEEE* 14-bus test system is selected with the corresponding topology shown in Figure 1 to validate the proposed formulation on a load flow adjustment with extended bus types.

From the original data of *IEEE* 14-bus test system shown in Figure 1, bus 1 is considered as a $V\theta$ node, busses marked as red that connected with generators are *PV* nodes, all the rest of busses are *PQ* nodes. Based on this, busses 7 and 8 are set as *PQV* node and *P* node, respectively. The following three sensitivity analyses are employed in this test system.

1. Traditional sensitivity analysis method: The formulas of traditional sensitivity analysis described in Section 2 are adopted to calculate the sensitivity factors.
2. Advanced sensitivity analysis with extended bus-types: The formulas for analysis with extended bus-type (described in Section 3) are used to calculate the sensitivity factor.
3. Perturbation method: Small perturbations are added to the base-case power flow solutions to analyze the sensitivity factor. This ensures that convergence errors from power flow calculations are not introducing errors in the parametric sensitivity analysis. The threshold of power flow convergence in perturbation calculation is set as $10^{-10}$.

Simulation results with selected busses based on the above three methods are shown in Table 1, where the first type of sensitivity analysis represents the sensitivity between generator terminal voltage and load nodal voltage; the second type of sensitivity analysis represents the sensitivity analysis between generator reactive power and load nodal voltage; the third type represents the sensitivity between generator reactive power and generator terminal voltage; and the fourth represents the sensitivity between load nodal voltage and transformer taps.

The perturbation method in this paper is calculated through the entire systematic non-linear AC power flow equations. Specifically, given a small perturbation in the system states (such as generator reactive power), instead of using sensitivity analysis to reveal the formula with the corresponding operational states (such as load nodal voltage), the variable with perturbation is substituted into the AC power flow equations to calculate the variations of the corresponding factors. In other words, the sensitivity estimated from perturbation method is achieved through the entire power flow calculation, thus is the most accurate way to characterize the power flow sensitivity factors. However, the perturbation method is computationally expensive, especially in a large-scale practical power system with high-dimensional system state variables. Therefore, the other two sensitivity analysis methods that include a linear approximation of the power flow equations are applied in the simulation case study. The methods are validated by comparison compared with the perturbation method.

Table 1 shows a comparison of sensitivity results given by the perturbation method, and the deviation of the two linearized approximations to the sensitivity. Taking the perturbation method as the true value, subsequent columns calculate the error resulting from the linearized approximations using the traditional (column 4) and extended (column 5) bus types. It is clear from Table 1 that the traditional sensitivity method leads to large margins of error, with some cases exceeding 200%. Conversely, the errors shown for the extended bus type method are significantly lower. This is because in traditional sensitivity analysis, the limited bus types do not capture the characteristics of the extended bus types defined in this paper. By allowing extended bus types, the system response is captured more accurately, and in some cases gives the same result as the more computationally expensive perturbation method. In particular, the average computational time for perturbation method, traditional sensitivity method, and extended new sensitivity method, across 4 types of sensitivity factor analysis parameters are 245s, 20s, and 16s. It is also worth noting that for every type of sensitivity considered in this 14-bus case, the extended bus types outperform the analysis conducted with the traditional bus types.

Table 1: Sensitivity Analysis Simulation Comparison Results on *IEEE* 14-bus Test System.

| **Sensitivity Factor Analysis Parameters** | | **Perturbation Method** | **Traditional Sensitivity Method** | | **Sensitivity Method for Extended Bus Type Load Flow** | |
|---|---|---|---|---|---|---|
| | | | Value | Relative Error | Value | Relative Error |
| **1st Type of Sensitivity Analysis** | Voltages between generator bus 2 and load bus 14 | 0.0310 | 0.1027 | **231.2903%** | 0.0283 | **8.7096%** |
| | Voltages between generator bus 6 and load bus 9 | 0.2546 | 0.3835 | 50.6284% | 0.2280 | 10.4477% |
| | Voltages between generator bus 6 and load bus 4 | 0.1183 | 0.1715 | 44.9704% | 0.1179 | **0.3381%** |
| | Voltages between generator bus 3 and load bus 14 | 0.0184 | 0.0629 | **241.8478%** | 0.0173 | **5.9782%** |
| **2nd Type of Sensitivity Analysis** | Reactive power on generator bus 2 and voltage on load bus 14 | 0.0010 | 0.0039 | **290%** | 0.0010 | **0%** |
| | Reactive power on generator bus 6 and voltage on load bus 9 | 0.0428 | 0.0801 | 87.1495% | 0.0424 | **0.9345%** |
| | Reactive power on generator bus 6 and voltage on load bus 4 | 0.0218 | 0.0358 | 64.2201% | 0.0219 | **0.4587%** |
| | Reactive power on generator bus 6 and voltage on load bus 11 | 0.1277 | 0.1534 | 20.1252% | 0.1243 | **2.6624%** |
| **3rd Type of Sensitivity Analysis** | Reactive power and voltage on generator bus 2 | 0.0327 | 0.0378 | 15.5963% | 0.0373 | 14.0672% |
| | Reactive power and voltage on generator bus 6 | 0.1680 | 0.2088 | 24.2857% | 0.1861 | 10.7738% |
| | Reactive power and voltage on generator bus 3 | 0.1041 | 0.1154 | 10.8549% | 0.1137 | **9.2219%** |
| **4th Type of Sensitivity Analysis** | Voltage on load bus 4 and transformer tap on branch 4-7 | 0.1880 | 0.1220 | 35.1063% | 0.2059 | **9.5212%** |
| | Voltage on load bus 9 and transformer tap on branch 4-9 | -0.1349 | -0.1980 | 46.7753% | -0.1422 | **5.4114%** |
| | Voltage on load bus 5 and transformer tap on branch 5-6 | 0.2063 | 0.2327 | 12.7968% | 0.2233 | **8.2404%** |

Table 2: Sensitivity Analysis Simulation Comparison Results on a Practical 262-bus Power System.

| **Sensitivity Factor Analysis Parameters** | | **Perturbation Method** | **Traditional Sensitivity Method** | | **Sensitivity Method for Extended Bus Type Load Flow** | |
|---|---|---|---|---|---|---|
| | | | Value | Relative Error | Value | Relative Error |
| **1st Type of Sensitivity Analysis** | Voltages between generator bus 159 and load bus 152 | 0.0632 | 0.0858 | 35.7594% | 0.0627 | **0.7911%** |
| | Voltages between generator bus 183 and load bus 249 | 0.1217 | 0.1387 | 13.9687% | 0.1193 | **1.9720%** |
| | Voltages between generator bus 186 and load bus 257 | 0.0634 | 0.0681 | 7.4132% | 0.0622 | **1.8927%** |
| **2nd Type of Sensitivity Analysis** | Reactive power on generator bus 159 and voltage on load bus 187 | $4.3326\times10^{-5}$ | $5.294\times10^{-5}$ | 22.1899% | $4.5927\times10^{-5}$ | **6.0033%** |
| | Reactive power on generator bus 183 and voltage on load bus 37 | 0.0013 | 0.0015 | 16.3432% | 0.001331 | **0.6716%** |
| | Reactive power on generator bus 15 and voltage on load bus 155 | $4.5778\times10^{-4}$ | $4.778\times10^{-4}$ | 4.3732% | $4.6667\times10^{-4}$ | **1.9419%** |
| **3rd Type of Sensitivity Analysis** | Reactive power and voltage on generator bus 2 | 0.006339 | 0.006444 | 1.6564% | 0.006377 | **0.5994%** |
| | Reactive power and voltage on generator bus 6 | 0.01199 | 0.01213 | 1.1676% | 0.01205 | **0.5004%** |
| | Reactive power and voltage on generator bus 3 | 0.005931 | 0.005989 | 0.9779% | 0.005949 | **0.3034%** |
| **4th Type of Sensitivity Analysis** | Voltage on load bus 4 and transformer tap on branch 4-7 | 0.0159 | -0.0003 | **101.8867%** | 0.0165 | **3.7735%** |
| | Voltage on load bus 9 and transformer tap on branch 4-9 | 0.0024 | 0.0004 | **83.3333%** | 0.0027 | 12.5000% |
| | Voltage on load bus 5 and transformer tap on branch 5-6 | 0.0976 | 0.0002 | **99.7950%** | 0.0953 | **2.3565%** |

### 4.2 Case Study 2: A 262-bus Power System

To evaluate performance of the proposed extension to the sensitivity analysis method on a power system of more practical size, a main grid network from a provincial dispatch control center in China is selected as a second case study. This network has 262 buses and 459 transmission lines. The original network topology and original bus types include, bus 41 as a slack bus; bus 189 is set as $PQV\theta$ bus; bus 212 is set as $P$ bus and bus 262 is set as $PQV$ bus. Results from selected buses are compared between traditional and the extended bus type sensitivity method and are shown in Table 2.

Similarly, the Table 2 shows that the proposed extended bus type sensitivity method exhibits improved accuracy not only in small test systems but also in the more realistic power system. Examination of results for the traditional sensitivity method shows that there are selected buses with relatively small differences from the perturbation method, though several of the selected buses show significant errors. Conversely, the results of the extended bus type analysis improve accuracy across all selected buses. Likewise, since this simulation is studied under a larger size of practical power system, the perturbation method is more computationally expensive. In particular, the average computational time for perturbation method, traditional sensitivity method, and extended new sensitivity method, across 4 types of sensitivity factor analysis parameters are 1560s, 302s, and 260s. Simulation results from both the small test system and large-scale practical power system have shown the efficacy and high accuracy of the proposed extended bus type sensitivity method.

## 5 Conclusions

Traditional power systems sensitivity analysis is based on linear approximations considering three classical bus types, that is, $PQ$, $PV$, and $V\theta$ buses. This paper proposes new bus types to represent situations that arise in modern power systems. These bus types are used to update the sensitivity analysis calculation. Simulation results on both an $IEEE$ 14-bus test system and a 262-bus practical power system have shown that, with the appropriate use of extended bus types in the modern power systems, the sensitivity analysis more accurately reflects the behavior of the system under simulated perturbations, which is the more accurate method to determine sensitivity. While the perturbation method is computationally expensive, an approximate method with extended bus types better represents these impacts with higher computational tractability.

For sensitivity analysis in the modern power systems, it is necessary to develop characterizations of the bus types seen in practical applications. The traditional sensitivity method can be applied if there are no extended bus types. Otherwise, the new proposed sensitivity method for extended bus types in this paper then becomes necessary for further power system analysis to guarantee system's reliability and stability. In addition, with the current transition underway and increasing integration of renewable resources and advanced power equipment (e.g., electronic control devices), extended bus types will be a more commonplace in the power system analysis and operation. Therefore, the extended sensitivity analysis will be considered as the main method to estimates the factors, and as a result, the traditional sensitivity analysis method to measure traditional bus types shown in Fig. 1 will be part of the new extended method, and thus can be also considered as a specific cased of the extended sensitivity analysis.

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